THE STABILITY OF AXISYMMETRIC GALAXY MODELS WITH ANISOTROPIC DISPERSIONS

Tim de Zeeuw, Marijn Franx, Jacques Meys, Karel Brink and
Harm Habing
Sterrewacht Leiden

The possible equilibrium configurations for elliptical galaxies and
the bulges of spiral galaxies are no longer thought to be confined to the
small class of axisymmetric systems with isotropic velocity dispersions.
Many of these systems are not supported by rotation but instead by aniso-
tropic velocity dispersions which are maintained by nonclassical integrals
of motion (e.g., Binney 1978), and may be triaxial. Few theoretical models
for such systems exist (Schwarzschild 1981). Here we discuss some axisym-
metric models that we have constructed by means of Schwarzschild's (1979)
selfconsistent method, and in particular their stability.

We consider oblate spheroidal models in which the density is con-
stant on spheroids of fixed eccentricity e, and is given by a power-law

$$\rho(\tilde{\omega},z) = \rho_n s^n = \rho_n \{\tilde{\omega}^2 + z^2/(1 - e^2)\}^{n/2}, \qquad 0 \le s \le a, \qquad (1)$$
$$= 0, \qquad\qquad\qquad\qquad\qquad a < s.$$

Here $(\tilde{\omega},\phi,z)$ are cylindrical coordinates, with the z-axis along the sym-
metry axis. The corresponding gravitational potential can easily be
derived. Within the boundary of the model the equations of motion for
this potential are invariant under the scaling transformation

$$\tilde{\omega} \rightarrow \alpha\tilde{\omega}, \qquad\qquad z \rightarrow \alpha z, \qquad\qquad\qquad (2)$$
$$\phi \rightarrow \phi, \qquad\qquad t \rightarrow \alpha^{-n/2}t.$$

This scaling property greatly reduces the numerical work necessary to
obtain a complete catalog of orbits within the model. It is sufficient
to compute orbits for only one energy; orbits at all other energies can
be obtained by simple scaling. This property of power-law density dis-
tributions was used by Richstone (1980) for the special case of the
logarithmic potential $(n = -2)$.

The results presented below are for the case $n = -1.8$, $e = 0.91$. The
corresponding mass model is thought to be representative of the bulge of
our own galaxy, up to about 300 pc from the center (e.g., Isaacman 1980).

A complete catalog of orbits was used to construct a number of equilibrium models with Schwarzschild's linear program. A model consists of a set of orbits with positive occupation numbers which reproduces the density distribution in which the orbits were calculated. For each model we have the freedom to choose the fraction of retrograde stars in each orbit since in an axisymmetric model there is no distinction between retrograde and direct motion around the symmetry-axis. All orbits turn out to have three isolating integrals of motion, so that the resulting models have anisotropic velocity dispersions. The primary difference from Richstone's (1980) models is the existence of a definite boundary, so that the problem becomes inherently two-dimensional.

We have used van Albada's axisymmetric N-body program (van Albada and van Gorkum 1977) to test the stability of the equilibrium models against axisymmetric perturbations. We find that models which have a local radial velocity dispersion below a certain critical value are unstable to such perturbations. We suspect that a generalized stability criterion for the local, anisotropic, velocity dispersion may exist, analogous to the result for infinitely thin disks (e.g., Toomre 1964). This stability of course depends only on the orbits that constitute the model, not on the fraction of retrograde stars in each orbit.

The models that are stable against axisymmetric perturbations may still be unstable to bar-like perturbations. This instability depends on the total angular momentum of the model (Ostriker and Peebles 1973), so now the fraction of retrograde stars is important. Using Vandervoort's (1982) refinement of the Ostriker-Peebles criterion, we find that some of our models should be stable even if all stars are direct, but that other models can be made stable only when a fraction of the stars is retrograde. We are currently testing Vandervoort's criterion with three-dimensional N-body calculations.

The stable models still show a wide variety in observable properties. The combination of linear programming and N-body methods used here will allow us to investigate the full range of stable dynamical models corresponding to the given density distribution.

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