To summarize: this text will prove very useful to a beginning postgraduate student working in the area of nonlinear PDEs. The book is not perfect, both in the organization of the material and in the finer points of proofreading; it is to be hoped that all of these shortcomings will be corrected in subsequent printings.

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Pretzel, O. *Codes and algebraic curves* (Oxford Lecture Series in Mathematics and its Applications No. 8, Clarendon Press, 1998), xii + 192 pp., 0 19 850039 4, £35.

The algebraic-geometry codes known as geometric Goppa codes, discovered in 1981, have extraordinary error-correcting capacity. This text has the single specific aim of making them and their background comprehensible to those who, drawn by the impressive credentials, nevertheless are daunted by the formidable nature of the machinery their definition and properties involve. The author's intention is to provide a geometrically intuitive, but rigorous, approach that will harmonise with the modern algebraic one presented, for example, in the (undeniably superb) volume by H. Stichtenoth, *Algebraic function fields and codes* (Springer-Verlag, 1991), and will allow the reader to access to the relative sophistication of the latter. Arising out of an earlier text of the author, *Error correcting codes and finite fields* (Oxford, 1992), it is not in itself a general work on codes, but is, however, essentially self-contained.

The work is divided into two parts, each of which can be read largely independently of the other. Part I contains a transparent and frankly affine account of the concepts and theory of plane curves (with analogies to the theory of functions) up to the statement (only) of key theorems such as the Riemann-Roch theorem. This gives the reader a clear feel for divisors, their degree and dimensions of associated linear spaces, genus, etc. The geometric Goppa codes associated with *smooth* plane curves over a finite field F_q are then defined and their parameters and properties explained in terms of these numbers. A feature is the focus on specific examples, the (elliptic) cubic $x^3 + y^3 = 1$, the Klein quartic $x^3y + y^3 + x = 0$ and the (Hermitian) quintic $x^5 + y^5 = 1$ over F_{16}, for which a handy compact version is tabulated. There is also a full account of the error-processing algorithm of Skorobogatov-Vlădut (1990), which is fairly simple but does not allow correction up to the capability of the code, and that of Duursma (1993), which deals with this weakness but may not always be practical.

Part II contains the elements of the theory of function fields of one variable in the Chevalley-Deuring-Stichtenoth tradition but tailored to and simplified for the present context. This works well and all the theorems of Part I are duly justified. What I did miss here was a review of the definition of a geometric Goppa code in respect of a general function field and its first degree places. Though there would have been formal similarity to material in Part I, the increased scope to function fields, presented as fields of curves in higher dimensional space or as extensions of non-rational function fields, might have merited some space. This is particularly relevant to the final chapter which describes how the rates of geometric Goppa codes can approach (or even beat) the famous Gilbert (or Gilbert-Varshamov) lower bound: the codes in question cannot be those associated with plane curves. Indeed, recent work, such as that of Niederreiter and Xing, and that of Stepanov, tends to be in this direction. This having been said, it is conceded that plane curves yield plenty of good examples, and virtually all those in Stichtenoth's book come into this category.

I enjoyed reading this book and believe it is successful at doing its job. As a footnote, let me record that a preprint by Niederreiter, Xing and Lam, 'A new construction of algebraic-geometry codes', has recently come to hand. In it Goppa's construction is powerfully extended using higher degree places. This might be an incentive for the reader of Pretzel's text, or ammunition for a future edition.

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CVETKOVIĆ, D., ROWLINSON, P. AND SIMIĆ, S. *Eigenspaces of graphs* (Encyclopedia of Mathematics and its Applications, Vol. 66, Cambridge University Press, Cambridge, 1997), xiii + 258 pp., 0 521 57352 1, £45 (US$69.95).

It is over forty years since Collatz and Sinogowitz's seminal paper [1] on the eigenvalues of a graph. Since then, there have been over a thousand papers on the subject, as well as three important books by Cvetković, Doob and Sachs [2], Biggs [3], and Cvetković, Doob, Gutman and Torgašev [4]. These books reviewed the progress in the subject over the intervening years; in particular, the third edition of [2] describes recent developments up to 1995.

The book under review extends the subject further by concentrating on the eigenspaces of a graph. If G is a graph with vertex-set $\{v_1, \ldots, v_n\}$, then its adjacency matrix is the $n \times n$ matrix $\mathbf{A} = (a_{ij})$, where a_{ij} is the number of edges joining the vertices v_i and v_j. The *spectrum* of G is the set of eigenvalues of $\mathbf{A}$, and does not depend on the particular labelling of the vertices; the *index* of G is the largest eigenvalue of $\mathbf{A}$. Although the eigenvalues of certain families of graphs specify the graphs completely, this is far from being true in general; for example, Schwenk [5] has proved that almost no tree is determined by its eigenvalues. Such considerations lead to a search for sets of graph invariants that specify a graph uniquely, and in this book the authors obtain such a set of invariants – the distinct eigenvalues and a certain type of basis for $\mathbb{R}^n$.

There are nine chapters. The first two provide a masterly summary of earlier spectral results, designed to bring the reader up to speed on the results needed for later chapters; in particular, there is an introductory discussion of graphs that are characterized by their spectrum. In Chapter 3 the authors introduce several eigenvector techniques and use them to investigate the indices of various families of graphs. Chapters 4–6 inject a more geometrical flavour by discussing eigenspace invariants such as the *angles* of a graph. Although the eigenvalues and angles do not specify the graph completely, except in small cases and for particular families of graphs, they prove to be a most useful tool in the general discussion, and contribute to our understanding of strongly regular graphs, the graph reconstruction conjecture, and graph perturbations (changes in the spectrum caused by adding or deleting individual vertices and edges).

Chapters 7 and 8 form the core of the book. Chapter 7 continues the geometrical approach through the notion of a *star partition* of vertices, an important concept that enables one to construct natural bases (*star bases*) for the eigenspaces of a graph. In Chapter 8 a unique canonical star basis is obtained for each graph, and it is this basis, together with the distinct eigenvalues, that forms the complete set of invariants for the graph. The authors also present efficient algorithms for finding star bases for a given graph. The final chapter is a survey of some interesting results that are related to graph eigenspaces, but which do not fit readily into earlier chapters.

This book is highly recommended for anyone interested in learning about current trends in spectral graph theory, especially those developments of a more geometrical nature.

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