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Correspondence

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Comments on "Buoyancy-driven lacustrine calving, Glaciar Nef, Chilean Patagonia" by C. Warren, D. Benn, V. Winchester and S. Harrison

Warren and others (2001) (hereafter referred to as WBWH) present a simple buoyancy-driven model in an attempt to quantitatively describe calving events for Glaciar Nef, a lacustrine glacier with its terminus in Lago Nef, Chilean Patagonia. The model proposed makes use of basic physical principles, primarily elementary beam theory, to determine the location of maximum tensile stress along the base of the glacier near its terminus. The location of maximum stress is then used to predict where the ice will most likely fail, leading to basal crevassing and, possibly, large iceberg calving from the ice snout. This information is further used to determine a possible calving rate and volume of ice discharged. However, there are inconsistencies between the physical model presented and the numerical results shown. The formulation of the model is valid, but the incorrect numerical results invalidate the conclusions made about the possibility of basal crevassing.

As previously stated, the model formulation presented in the original paper is correct. The inconsistency arises when the graphical results are compared with the mathematical formulation, where a discrepancy of almost an order of magnitude is found. One of the original authors (D.B.) was willing to provide the original work. It was here that an error was discovered in the application of the area moment of inertia of the glacier tongue. (Ice thickness $h_{\mathrm{i}} = h(x) = h_0 + x \tan\alpha$ in figure 9 of the original paper was incorrectly multiplied by 0.5, a quantity that is then cubed in computing I and thus σ_x (Equation (2).)

Briefly, the moment induced on a floating ice tongue whose thickness is less than the flotation height the water can support is given by

$$M = \int_0^x \sigma_z x' \mathrm{d}x' = \rho_{\mathrm{i}} g \left(\frac{h_0 x^2}{2} + \frac{x^3}{3} \tan\alpha - \frac{x^2}{2} h_n \right), \qquad (1)$$

where flotation thickness h_n and stress σ_z are given by equations (2) and (3), respectively, and all other variables are defined by figure 9 in the original WBWH paper.

To determine the tensile stress acting on the tongue, Equation (1) is inserted into the expression (Gere and Timoshenko, 1997)

$$\sigma_x = \frac{Mc}{I}, \qquad (2)$$

where c is the (vertical) distance from the neutral axis of the body to some point and I is the area moment of inertia. The maximum of Equation (2) will occur for the greatest value of c, which in this case would be all points on the plane where $c = 0.5h(x)$, or the base of the glacier. (Points along the surface of the glacier will have the same magnitude stress, but it will be compressive. The value of c is always taken as positive.) The formula for the moment of inertia of the system, a geometric property of the body, can be directly taken from Equation (6) in the WBWH paper.

Similar to the results presented in WBWH, but using this corrected formulation, the maximum tensile stress is determined for a range of terminal ice thicknesses and for two distinct values of the surface slope α.

The numerical results of Equation (2) are shown in Figure 1. Upon comparison to the original results presented, we see there is a large difference in the calculated maximum basal stress that can occur. The maximum stresses found in this paper are ~140 kPa, as compared to the ~1 MPa claimed in WBWH. By itself this discrepancy may not seem to mean much, but when this model is used to predict basal fracture locations, the model proves insufficient.

Two papers, Vaughan (1993) and Gagnon and Gammon (1995), present values for the tensile strength of glacial ice. For the purposes of their paper, WBWH use the results of Gagnon and Gammon (1995) to determine where tensile failure is most likely to occur as predicted by this model. The results presented by Gagnon and Gammon determine a ten-

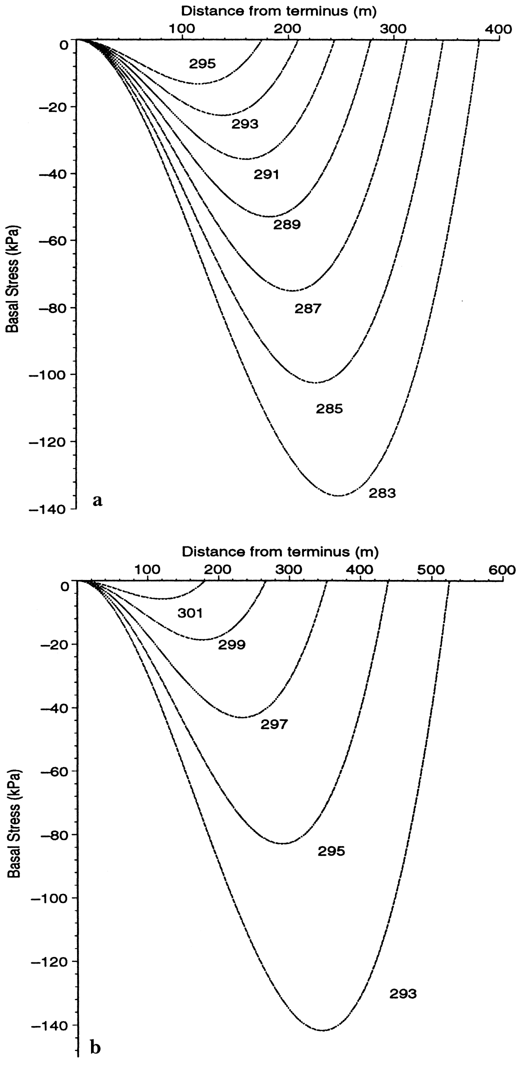

Fig. 1. Maximum basal tensile stress for range of terminal ice thicknesses and surface slopes 5° (a) and 2° (b). In both cases, the water depth $h_{\mathrm{w}} = 275$ m. (Negative stress values indicate tension.)

sile strength on the order of 1 MPa. Using this value, onecan see from the plots in Figure 1 that the basal tensile stress never reaches this magnitude. This would indicate that this model is insufficient for describing the failure of ice under the influence of buoyant stress alone. Values for tensile strength given by Vaughan are on the order of 0.1–0.5 MPa, comparable to the stress maxima in Figure 1, but these data are primarily determined from surface studies where the effects of firn lessen the tensile strength of the ice. They are therefore considered irrelevant in a discussion of buoyant stresses initiating fracture at the base of a glacier, so is ignored.

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Reply to the comments of J. P. Kenneally on "Buoyancy-driven lacustrine calving, Glacier Nef, Chilean Patagonia" by D. Benn and C. R. Warren

We are grateful to James Kenneally for drawing our attention to an error in our modelling work, which was the result of an unfortunate structural flaw in the programming. His recalculations show that the maximum tensile stress at the base of a buoyant glacier tongue (modelled as a rigid beam) is almost an order of magnitude less than that given in our original paper, and is hence unlikely to account for the large tabular icebergs at Glaciar Nef. But if the buoyant calving mechanism that we proposed cannot account for the pattern of calving at the glacier, what can? This question is of more than site-specific importance. Significant numbers of Patagonian outlet glaciers are undergoing rapid calving retreats in deep (>150 m) proglacial lakes, or have done so in recent years (Warren and Aniya, 1999; Harrison and others, 2001; Skvarca and others, 2002), and calving behaviour similar to that observed at Glaciar Nef has also been reported in Alaska, U.S.A. (Lingle and others, 1993). Buoyant forces are the most intuitively obvious explanation for the many large tabular icebergs that are typical of such contexts. Such forces have been identified as triggers for deep-water lacustrine calving at numerous sites around the world (Holdsworth, 1973; Derbyshire, 1974; Theakstone, 1989), and in our original paper we presented several lines of evidence demonstrating the importance of such forces at Glaciar Nef. It seems, however, that buoyancy acting alone exerts insufficient stress to initiate crack propagation at the glacier bed.

Our buoyant calving model neglects the effects of progressive bending of the ice in response to the torque imposed by ice buoyancy. As pointed out in our original paper, bending of the ice will reduce the tensile stress at the ice base by accommodating through creep part of the buoyant force and associated torque. We therefore suggested that the buoyant calving mechanism is likely to require special conditions such as rapid surface ablation and stress build-up. Kenneally's work shows that, even under such conditions, basal tensile stresses are never likely to be large enough to cause the calving of large, coherent sections of the glacier terminus.

Our model also neglects longitudinal stresses in the glacier arising from a down-glacier reduction in basal shear stress and an increase in sliding velocity near the grounding line. We believed this omission to be justified because up-glacier retreat of the grounding line at Glaciar Nef had not been accompanied by a series of calving events. We therefore assumed that longitudinal stresses near the grounding line were too small to initiate calving, and did not consider them. It is, however, interesting to speculate whether calving events at Glaciar Nef (and at other glaciers which approach or reach flotation) could be explained by a *combination* of longitudinal stretching and buoyancy-induced torque. Individually, neither mechanism may be sufficient to initiate fracture, but in some circumstances their combined effect may be large enough to cause high-magnitude/low-frequency calving. Unfortunately, glacier geometry, velocity and subglacial topography are insufficiently well known at Glaciar Nef to test this hypothesis. The calving problem is a multifaceted one, and different combinations of processes may operate in different circumstances. Available evidence from glaciers terminating in deep lakes indicates that, acting in concert with other factors, torque arising from buoyant forces plays a role in calving.

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