CORRESPONDENCE.

To the Editor of the *Mathematical Gazette*.

DEAR SIR,—Mr. Siddons' plaint about the treatment of his book by Professor Carslaw in the May issue of the *Mathematical Gazette* involves a number of problems in connection with the teaching of mathematics which, in my view, are not restricted in their importance to the school period. It raises, in fact, the whole question of the purpose and function of mathematical teaching. I am not, and except for an almost negligible period have never been, a school teacher, but although a layman in this respect I think I do appreciate the real difficulty that oppresses Mr. Siddons, and that he is challenging us to face.

To the question—what is to be the function and purpose of mathematical teaching at the school stage—there is perhaps no definite concise answer, but, broadly speaking, it might be agreed that these functions fall under three heads:

(1) to impart a knowledge of mathematical methods, their power and their limitations;

(2) to convince the pupil that in mathematics there are embodied truths in some sense which I need not define; and

(3) to accustom the pupil to appreciate such fine distinctions in logical argument *as his physical make-up will allow.*

The first is perhaps mainly utilitarian in its object, the second purely philosophic, and the third purely pedagogic. Although it does not require much experience to recognise that all three headings are interlocked, it has always appeared to me clear that nothing but confusion can arise if these three objects are not kept strictly in mind. The real difficulty of course arises from the fact that all three hares have to be chased simultaneously—no one hare can be caught without at least knowing where the other two are. In practice the stress that has to be laid on either of these objects will depend very much on the age of the pupil and his future intentions.

It might be argued that (3) and the limitations referred to in (1) cannot possibly be undertaken at all unless they are undertaken thoroughly on an absolutely unimpeachable logical basis. Whether such an "absolute" has yet been attained is a question we need not discuss here. For, after all, teachers must be realists, they have to deal with the brains that are actually in the heads of their pupils and they are therefore limited in a very definite way to logical discussion below a certain definite level; to work outside this limit is bad teaching. It suffices, I think, to remember that most of the modern exponents of rigour were themselves brought up in a less mathematically ascetic school. Euclid may now have toppled from his exalted pedestal but in my school-days he supplied just the right kind of "punch." To carry through (3) effectively it appears merely necessary then that the argument, the proofs, etc., should be as rigorous as is consistent with the brain capacity of the pupil; and to satisfy the limitations in (1) it is important that these should be accurately stated, provided the statement actually conveys something to the pupil; but the latter need not necessarily have been led through the logical proof. There is no half-way house, it seems to me, between this attitude and that of the "whole-hogger," who maintains that every proof which is presented must be fundamentally unimpeachable. If this is to mean anything it implies that until the logical basis of the "theory of number" has been laid on permanently secure foundations the teaching of arithmetic cannot be begun; that before convincing the boy that this is a house and that it may be useful for some purposes he must be led microscopically over every inch of the foundations so that he may realise that if a house *were* to be built on them there are some purposes for which it may *not* be used. This may be important to the adult, who may desire to produce a complete comprehensive logical presentation of a subject, but it is not in the least important to the school-boy.

Do not let us confuse text-books with original memoirs. A memoir extends or demarks the bounds of knowledge, it is an adjunct to research and plays its part in the development of the mature brain; a text-book is an adjunct to teaching and presents its case to a less mature biological specimen. They have different ends in view.

If this point of view is acceptable we recognise at once how difficult is the function of a reviewer. For, in order to see the mathematical material presented in true perspective, he must be an adult in the subject, but in order to be a true critic of the presentation he must possess the mental acumen of a brilliant boy in the form for which the book is intended.—Yours faithfully,

Imperial College of Science and Technology. H. LEVY.

HIGHER TRIGONOMETRY FOR SCHOOLS.

DEAR SIR,—There will be little disagreement, I think, with the principles to which Mr. Siddons expresses his adherence in the May *Gazette*; the immediate issue, which is whether he and his collaborator have succeeded in applying these principles in their work on trigonometry, is one on which readers must judge for themselves. The point I wish to raise is impersonal.

We can admit that tentative work is sometimes indispensable and often valuable and still maintain that an easy rigorous method, when one does exist, is intrinsically preferable to one dependent on delicate assumptions, however frankly these assumptions are disclosed. This is specially clear if the assumptions, or the results to which they lead, are not plausible, and here is where in the matter of the power series for the sine and cosine the case against compromise is very strong. For it is one thing to suggest that because x^n tends to zero for fractional values of x, there is some likelihood of being able to find a power series that will fit such a function as the sine over some unspecified range of small values of the argument. It is quite another thing to suggest—or as is more usual tacitly to assume—that the range over which the series fits the function has something to do with the range over which the series when discovered is itself convergent. When we consider how the relative importance of the terms of a power series changes as the variable increases indefinitely, it seems fantastically improbable that the sum of such a series can be a periodic function, and when we find that the series which, on quite reasonable assumptions, fits the sine for small values, is in fact convergent for all values, the natural conclusion surely is that the correspondence between the series and the function breaks down somewhere. That the correspondence does not break down is one of the delightful surprises of mathematics, of which the learner should not be cheated by the teacher's familiarity with the result.

The questions of the infinite products and the series of partial fractions are at present on a different footing from that of the power series. As far as I know, no proofs of these expressions have been put forward that are comparable in simplicity with the proofs of the power series by inequalities, and I agree whole-heartedly with Mr. Siddons that the substance of Prof. Carslaw's paper in the March *Gazette* is quite unsuitable for a first course. Also the morphology of the expressions reproduces so precisely that of the trigonometrical functions to which the expressions are related that the formal assumptions to which attention has to be called are really plausible.
—Yours, etc. E. H. NEVILLE.

ERRATUM.

Vol. xv, p. 129, ninth item. *For* 'commenced the building of a' *read* 'opened the'.

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