do not arise out of ordinary problems, and should be classed as non-essential for elementary examinations.

"*Algebraic factors* may arise in the first instance in the extension of the corresponding part of arithmetic. They may also arise

$$[e.g.\ a^2 - b^2 = (a + b)(a - b)]$$

in the adaptation of formulae to computation. But, as a rule, they are taken up and studied systematically with a view to dealing with quadratic equations. Being included in the course on this ground, they should not go beyond the quadratic type, $a^2 - b^2$, $a^2 \pm 2ab + b^2$, $ax^2 + bx + c$. Such forms as $a^3 \pm b^3$ and higher forms belong to formal algebra, and should not be essential for elementary examinations. . . .

"*The idea of functionality* is so important, socially as well as scientifically, that it should be fundamental even in an elementary course of mathematics. Work with graphs is now an accepted part of such a course ; we recommend that the study of variation should be made correlative with it."

Soon after this report was issued many examining bodies revised their syllabuses and algebra papers more or less conformed to the recommendations of the report. But it is fatally easy for an examiner to set questions that merely test manipulative skill; and, in my opinion, algebra papers in recent years have shown a tendency to require a skill that cannot be useful except to the specialist.

After the School Certificate stage, those who require mathematics for its own sake or for science or engineering will need much drill at manipulation ; but that is no reason for giving this drill before that stage to all pupils alike, whether they will ever need manipulative skill or not. It is mere waste of time to polish a tool for all pupils when the majority of them will never use it. Experience shows that the future mathematician, scientist or engineer acquires manipulative skill much more quickly in the post-certificate stage than he does in the earlier stage, so there is no loss for him in postponing the drill.

A. W S.

CORRESPONDENCE.

School Certificate Mathematics.

To the Editor of the *Mathematical Gazette*.

Dear Sir,—The alternative syllabus in geometry and trigonometry issued by the Cambridge Local Syndicate is welcome because it will stimulate teachers to experiment with new teaching methods and with alterations in the content of the elementary course. It deserves extensive and intensive discussion.

There is only one satisfactory method of assessing the value of a method of teaching, and that is by trying it in class or in a succession of classes. Discussion suggests possible plans to be tried, but an opinion based on general reasoning, without backing from trial in the classroom, has little if any value.

In the matter of the Cambridge Syllabus, there are two lines of enquiry, not wholly independent of one another, to be followed : the use and abuse of theorem work ; and the extent to which geometry and trigonometry can be profitably fused.

The object of this note is to set out some of the considerations which will arise out of such an enquiry, with a view to provoking discussion among teachers. It may be hoped that the Teaching Committee of the M.A. will

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eventually issue some report on the subject, but the value of such a report will depend greatly upon the extent to which it has been possible to establish contact with the opinions and class-room experience of a large number of teachers, expressed at meetings of Branches of the Association, in the various educational journals, and elsewhere.

There are advantages to be gained from the writing out of theorems, and these may be summarised as follows :

(i) The pupil learns a clear-cut form in which to express a logical argument. In learning to make a correct presentation of a proof, he has to realise amongst other things, the importance of a clear statement of what is given and of what is to be proved.

(ii) For many pupils, theorems provide the only opportunity of practice in setting out a sustained argument. Solution of riders requires accurate thought and clear expression, but most pupils can only tackle successfully one-step and two-step riders, and it is more profitable to use the time in tackling large numbers of riders than in continually writing them out.

(iii) Special emphasis on a small number of carefully selected theorems will familiarise the pupil with types of construction and of arguments which are needed to strengthen his powers of application.

(iv) A firm grasp of a few theorems will supply the necessary cohesion in the geometry course without which it may degenerate into a series of unrelated, though possibly interesting, snippets.

Roughly it may be said that, while memorising of the sense or outline of a proof is necessary, any memorising of the wording must be harmful. Also we must be careful that any attention to special theorems leaves ample time for numerical and formal rider-work in which admittedly the principal interest and value of the subject lie. We believe that these requirements can be met satisfactorily, and further we hope that geometry may make a valuable contribution towards remedying what the writers of the Norwood Report appear to regard as a very serious weakness in secondary school education. On p. 13, they say :

“ . The complaint briefly is that too many pupils show marked inability to present ideas clearly to themselves, to arrange them and to express them clearly on paper or in speech, they are too often at a loss in communicating what they want to communicate in clear and simple sentences. .”

Perhaps the fault often arises from mental laziness : the pupil is reluctant to force himself to express on paper a line of argument which is present in outline in his mind but which can only be formulated by a definite effort.

The reforms that have already been carried out in the teaching of geometry are partly due to the frank recognition that much of main-school geometry is physics rather than pure mathematics. This recognition accounts for the general acceptance of a broad group of assumptions upon which stage B geometry is built : angles at a point, angle properties associated with parallels, tests for congruence, and also, in view of the early introduction of scale-drawing and numerical trigonometry, the tests for similarity which may be introduced along with the tests for congruence by analogous methods.

The policy of building up a course of geometry round a small selection of theorems will be harmful if the enunciations and proofs of the theorems do not fall into their proper places in the general scheme.

It is suggested that in the class-discussion of certain of the selected theorems, three stages should be distinguished :

(i) The careful detailed preparation for the theorem by appeals to experience, by general illustrations of the geometrical ideas involved, and by one-step riders. These will be designed to make the pupil understand exactly what the theorem means, to clear up any technical difficulties due to

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unfamiliar mathematical words and phrases, and to make him familiar with the types of arguments needed for the proof of the theorem.

(ii) The statement and proof of the theorem itself.

(iii) The harvesting of the results, viz. the dependent theorems taken as riders, numerical examples, and one-step or two-step riders which arise naturally from the facts established.

Of these stages the first is perhaps the most important and the most neglected. It well repays the time required for a complete treatment. The second stage is the shortest: for if the first stage has been carried out successfully, most pupils will " know " the theorem already, *i.e.* will be in a position to make the attempt at setting out the proof. The length of the third stage will depend largely on the time at the teacher's disposal. Thus although a concentrated and intensive attack has been made upon an individual theorem, most of the time will have been spent in strengthening the pupils' appreciation of geometrical principles and applying them to numerical and other riders.

For example, suppose that the selected theorem is " parallelograms on the same base and between the same parallels are equal in area ". Stage (i) would contain (*a*) illustrations of figures and surfaces of solids which are seen to be of equal area, although they are of different shapes, without appeal to numerical measurement by actual deformation; (*b*) illustrations to clear up the meanings of such phrases, as " between the same parallels ", " distance between parallel lines ", and " height and corresponding base "; (*c*) one-step riders to rehearse the arguments required in the theorem, namely congruence and the subtraction of areas. Since the latter is the main feature in the proof of this theorem, and since the pupil tends to add rather than subtract, a few one-step riders are necessary to emphasise this idea. Some of these can be of the discovery type. This first stage may easily spread over a couple of school periods, but the result is that when stage (ii) is reached, it occupies very little time and the work passes quickly into stage (iii) where the dependent theorems wanted for future rider-work are taken as riders and further applications are made.

In selecting the theorems it seems desirable to pay regard to the following considerations:

(i) Emphasis may suitably be laid on theorems which introduce a new theme or serve as fundamental theorems in a new chapter.

(ii) The theorems selected should illustrate diversity of method, both in construction and in proof, so as to fortify the pupils' power of application.

(iii) To help in the practice of teaching, it is desirable to include some theorems which lend themselves to valuable preparatory work, and from which a number of useful dependent results flow, thus supplying an element of cohesion in the pupils' equipment.

(iv) Some theorems should be chosen which give the opportunity for a sustained argument.

(v) Difficulty should not be a reason for exclusion, nor easiness for inclusion.

To compose a short list of these key-theorems round which the course is to be arranged is a thorny task, but it may at least provoke discussion if the following ten theorems are set out in order to give point to what has been said above:

1. The exterior angle of a triangle is equal to the sum of the opposite interior angles.
2. Opposite sides of a parallelogram are equal.
3. The " equal intercept " theorem.

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4. The locus of a point equidistant from two given points (*i.e.* a theore and its converse).
5. Parallelograms on the same base and between the same parallels a equal in area.
6. Pythagoras' Theorem.
7. In the triangle ABC, if $AB > AC$, then $\angle C > \angle B$.
8. The angle at the centre is twice that at the circumference standi on the same arc.
9. The perpendicular to a radius of a circle at its extremity is a tangent
10. Intersecting chords of a circle have the products of their segments equ

To these may be added a curtailed list of key-constructions. These a additional to the usual stage A constructions.

I. The reduction of a quadrilateral to an equivalent triangle.
II. The construction of tangents to a circle from a given external point
III. The construction on a given line of a segment of a circle containi an angle equal to a given angle.
IV. The construction of a mean proportional.

The list given in the new Cambridge Local Syllabus does not entirely me the requirements stated above. Most teachers will probably agree that is our No. 1 that is fundamental rather than the "angle-sum" propert The "equal intercept" theorem can be regarded as an instructive examp of congruence which is the basis of an important section of elementa geometry. The inclusion in the Cambridge list of a group of depende theorems on the circle can be criticised on the ground that these are use and interesting facts which the pupil should be able to tackle as riders. O No. 9 is omitted, but this is probably because it is a limit treatment of tanger that is contemplated; and if so it is rightly omitted. Nevertheless it is t essential result about tangents, though the "alternate segment" theore is also extremely useful and is no doubt included for that reason. Apa from the reference to Pythagoras, there is no application included of t properties of similarity, though such applications have their origin in stage work. An example of a theorem and its converse should be given; and t is a reason for the inclusion of No. 4 in the above list. The locus section the Cambridge syllabus seems unsatisfactory. Lastly, the final theore ($a^2 + b^2 + c^2 = d^2$) is a strange choice.

However, we are in complete agreement with the general policy of maki cuts in the old-fashioned long syllabus of theorems and with the inclusi of trigonometrical and practical work. It will be all to the good if this h the effect of discouraging unintelligent memorisation of proofs of theore for examination purposes.

In addition to the key-theorems, there will be a number of derived resu important for their use in rider-work. Their results must be known to t pupil. The Cambridge syllabus, we think rightly, leaves it to the teacher decide which theorems belong to this category.

Probably no three people will agree about all the details, but we find ou selves holding the same general views on matters of principle.

C. V. Durell, C. W. Parkes, A. Robsc

August 1943.

1437. Leofric's face . . was full of the contemplative innocence seldom be seen except on the faces of apes and noted mathematicians.—E. Ferra *Don't Monkey with Murder* (1942), p. 62. [Per Prof. E. H. Neville.]