$\therefore$ BP : PC = BG : GC,
= AD : DC, (Construction)
= FP : PC; (Eucl. VI. 4)

$\therefore$ BP = FP, and BEFP is a rhombus.

But FC = FP, since AC = AD;

$\therefore$ BE = EF = FC.

Cor. 1. When triangle ABC is isosceles, EF is parallel to BC.

Cor. 2. When P moves up to D, F moves up to A. In this case, which is the limiting one for the point P within the triangle, BD = DA = AC. The limiting case therefore occurs when one of the sides is double of the other.

Cor. 3. When AB is greater than twice AC, the point P is outside the triangle, F is on CA produced, and, as before, BE = EF = FC.

Fourth Meeting, February 8th, 1884.

A. J. G. Barclay, Esq., M.A., Vice-President, in the Chair.

The Promotion of Research—A Presidential Address.

By Thomas Muir, M.A., F.R.S.E.

This paper has been printed by Mr Muir for distribution among the Members of the Society.

Illustrations of Harmonic Section.

By Hugh Hamilton Browning, M.A.

[*Abstract.*]

The object of the paper was to draw attention to a few important and well known cases of the harmonic section of a straight line, and to show their application to one or two problems of interest, more especially the method of drawing tangents to a conic by the ruler only. The effort throughout was to secure clearness, brevity, and freshness of proof, coupled with purely geometrical treatment.

Among other propositions were the following:

(*a*) O, P, V, W, X, are points in a straight line such that PV : PX = $OV^2 : OX^2$, and OP = PW; show that OV, OW, OX are in

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harmonic progression, and apply the proof to show that any secant from a point outside a parabola is cut harmonically by the chord of contact of tangents from the point.

(*b*) O, P, V, W, C, X, P_1, are points in a straight line such that $CP^2 = CW \cdot CO$, $CP = CP_1$, and $OV^2 : OX^2 = CP^2 - CV^2 : CP^2 - CX^2$; show that OV, OW, OX are in harmonic progression, and apply the proof to the ellipse.

(*c*) A proof, by the reduction of the proportion when CP is less than CV and CX, suited to the hyperbola.

The importance of the fact that in an harmonic pencil any ray is the locus of the middle points of straight lines intercepted by the rays on either side of it and parallel to the fourth ray was illustrated by showing that as it holds when the rays are produced backwards, it immediately leads to such theorems as:

1. The intersection of the diagonals of a quadrilateral inscribed in a circle is upon the polar of the intersection of the opposite sides.

2. The theorem proposed by Mr James Taylor for simple proof at the Society's meeting on 14th December 1883.

3. The intersections of the tangents at the extremities of all chords of a conic which pass through one point lie on a straight line.

Among other proofs offered was the following, which shows that, with no further aid than that afforded by Euclid I.–III., tangents can be drawn to a circle with the ruler only.

Let BCED be a quadrilateral inscribed in a circle whose centre is K; let CB and ED meet at A, and CD, EB at O. Join OK, AK, and from O draw OR perpendicular to AK. Through the points B, C, O describe a circle, meeting AO again at X, and join CX.

Then $\angle CXO = \angle ABO = \angle ADC$;

therefore the points A, D, X, C are concyclic.

Hence $AB \cdot AC = AO \cdot AX = AO^2 + AO \cdot OX = AO^2 + CO \cdot OD$,

$= AK^2 + OK^2 - 2AK \cdot KR + CO \cdot OD$,

$= r^2 + AB \cdot AC + r^2 - CO \cdot OD - 2AK \cdot KR + CO \cdot OD$;

therefore $r^2 = AK \cdot KR$, a result which proves that O lies on the chord of contact of the tangents from A.

In a similar way another point may be found situated on the chord of contact; and thus the chord of contact is determined.

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