EVOLUTION OF ORBITS IN THE OUTER PART OF THE ASTEROIDAL BELT AND IN THE KIRKWOOD GAPS

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It is well known that the semi-major axes of the asteroids between the orbits of Mars and Jupiter are not uniformly distributed. The depleted regions in the outer part and the Kirkwood gaps in the inner part of the belt represent singularities in the frequency distribution of the asteroidal semi-major axes. The question whether these depleted regions and the Kirkwood gaps are due to gravitational perturbations of Jupiter or due to cosmogonic effects has not yet been solved and the discussion is still continuing.

In a series of numerical experiments (1974, 1975), we have tried to depopulate the Kirkwood gaps at the 2/1, 3/1, 5/2 and 7/3 commensurability by Jupiter's gravitational action on fictitious asteroids starting in a gap. In no case did an asteroid leave any of the investigated gaps and remain outside the gap. All the fictitious asteroids librate through the gaps or mostly within the gaps with quite different amplitudes and frequencies depending on starting values. In addition, no fictitious asteroids approached Jupiter closely and therefore none escaped out of the gap. According to our numerical experiments, the Kirkwood gaps cannot be depopulated by Jupiter's perturbations.

We therefore tested the collisional hypothesis for the origin of the Kirkwood gaps, which assumes originally existing asteroids or planetesimals in the gaps. Because of the especially strong perturbations, these objects vary their eccentricities strongly, and therefore can cover a large portion of the asteroidal belt thus increasing the probability of a collision with a belt asteroid. The larger the variation in eccentricity, the higher the collision probability. In our test for the collisional hypothesis, we used a planar Sun-Jupiter-Asteroid model averaged by Schubart's method (1964). Most of the calculated orbits support the collisional hypothesis. The few problematic cases which do not support the collisional hypothesis are the almost circular orbits, the orbits starting at the edges of the observed gaps and the orbits at the 7/3 commensurability. These problematic cases do not vary their eccentricities strongly and therefore cannot be explained by the collision hypothesis.

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Compared to the depletion of the Kirkwood gaps, the gravitational explanation for the depletion of the outer part of the belt between the 2/1 commensurability and the Hilda family seems to be even more difficult according to Lecar and Franklin's (1973) numerical experiment. Lecar and Franklin calculated orbits of fictitious objects over a few thousand years using the elliptic planar Sun-Jupiter-Asteroid model. The expected mechanism which depletes that region is based on perturbations in the semi-major axis and eccentricity of an asteroidal orbit which results in an Jupiter crossing orbit. After a close approach to Jupiter, the asteroid escapes from the considered region.

In Franklin and Lecar's experiment, the objects with higher eccentric orbits, $e > 0.25$, escaped, while for objects with $e < 0.25$ only a few had close encounters with Jupiter.

The existence of these problematic cases in both experiments, for the Kirkwood gaps and for the outer part of the belt, suggests that the corresponding hypotheses which are mainly based on gravitational affects are false. However, before looking for different hypotheses which would need more sophisticated physics, one has to refine the gravitational models with respect to the number of perturbing bodies and with respect to the periods covered. A calculation including Saturn over much longer time spans might reduce the number of problematic cases considerably.

Our calculations were based on Schubart and Stumpff's N-Body Program (1966). Over 100 000 years, we computed the orbits of fictitious asteroids which represent the problematic cases for the collision hypothesis of the Kirkwood gaps and for the ejection hypothesis of the outer belt. Jupiter and Saturn were included as perturbing bodies.

For the Kirkwood gaps, our new numerical experiment yielded the same negative result as before. The number of problematic cases could not be reduced.

The depopulated region, $3.6 < a < 3.9$ AU, in the outer belt was depleted by our experiment (Froeschlé and Scholl, 1978) to a larger extent than by Lecar and Franklin's experiment. The average time scale for the excitation of an orbit in order that it crosses Jupiter's orbit is somewhat larger than the period covered by these authors. After 2 000 years, 25% of all the escapers were obtained in our experiment, 75% after 15 000 years and 100% after 60 000 years. However, our experiment did not depopulate the region $3.6 < a < 3.9$ AU completely. Most of the objects with starting eccentricities of $e < 0.10$ remained in that region and had no close approach to Jupiter. In addition, another family of objects which minimize their eccentricities if their aphelia are precessing through Jupiter's orbital plane, avoided a catastrophic encounter with Jupiter.

For both types of families, observed asteroids are known in the range under consideration but they exist in a much lower abundance than should be expected from our experiment. The three asteroids (721)

Tabora, (522) Helga, and PL - 4164 show a coupling between the precession of perihelia and the long period in eccentricity. The eccentricity becomes smallest when the perihelion and therefore also the aphelion lies in Jupiter's orbital plane. That yields the largest possible distance to Jupiter when the asteroid is at its aphelion. On the other hand, when the eccentricity reaches its maximum and therefore the aphelion distance is largest, thus yielding the smallest possible distance to Jupiter, the argument of perihelion is close to 90° or 270°. Therefore, the asteroid passes through its aphelion when it is high above or below Jupiter's orbital plane. This mechanism prevents close encounters with Jupiter.

We suppose that more objects of that kind might exist but have not yet been detected as observers ordinarily detect minor planets close to the ecliptic. Objects of the kind described above, however, can be detected best at high ecliptic latitudes when they pass through their perihelia. The discrepancy between expected and observed asteroids might be reduced by observations.

For the first mentioned type of asteroids having $e < 0.1$, more observations with very powerful instruments might reduce that discrepancy. It is an interesting problem why we observe so few objects with almost circular orbits in the region 3.6 - 3.9 AU as well as in the Kirkwood gaps, because gravitational models including collisions do not remove such objects to a large extent. For the gap at the 2/1 resonance, Franklin et al. (1975) list objects with low eccentricities which seem to librate in the gap. However, most of these minor planets have very uncertain orbits. Further observations in order to find these objects might reduce the discrepancy between observation and calculation.

Such long runs over 100 000 years are rather expensive. Therefore, several authors (e.g. Nacozy 1976) propose to increase the masses of the perturbing bodies in order to shorten the time scale and consequently the computing time and in order to obtain perturbations with larger amplitudes. The intrinsic problem for such numerical experiments with larger masses consists in the equivalence of a Sun-Jupiter-Asteroid and a Sun -"Super Jupiter"- Asteroid model. It is not obvious that the latter model on a shorter time scale yields the same orbits as the first one. The perturbing mass may not be increased too greatly.

For our purposes, and especially for the depopulation of the outer belt, it is necessary to keep the Hilda family stable in an experiment with a "Super Jupiter", since the Hilda family is actually observed. In order to determine a limiting Jupiter mass up to which the Hilda family remains stable, we computed orbits around the 3/2 commensurability with different Jupiter masses.

According to our calculations, the model with a Jovian mass of 0.007 solar masses yields no stable orbits. All the objects escape. Therefore, a value of 0.005 solar masses might be used for a Super Jupiter without significantly destroying the topology of the model. We repeated the experiment for the region $3.6 < a < 3.9$ AU with a value of

5 times Jupiter's mass ($\gamma = 5$) in order to compare the depletion curves for the Sun-Jupiter-Asteroid and the Sun-Super Jupiter-Asteroid model. The orbits of 47 fictitious objects were calculated. The following Table 1 shows the total number of escapers after certain time intervals given in years.

Table 1

Sun-Jupiter ($\gamma = 1$)		Sun-Super Jupiter ($\gamma = 5$)	
Time	Total Number of Escapers	Time	Total Number of Escapers
0	-	0	-
1 000	3	100	7
2 000	5	200	12
3 000	6	500	17
10 000	10	1 000	26
15 000	16	2 500	33
60 000	19		
100 000	19		

Obviously, the depletion of both models is different. Most of the escapers in the model with $\gamma = 1$ were found after 15 000 years, while in the model with $\gamma = 5$, the 2 500 years do not seem to be sufficient to obtain all the escapers. The use of a Super Jupiter destroys the protection mechanisms which in the model with $\gamma = 1$ avoid close encounters with Jupiter. Therefore, the model with a Super Jupiter yields too many escapers and is therefore not equivalent to the original model ($\gamma = 1$).

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