ROTATIONAL AND TIDAL PERTURBATIONS OF NONRADIAL OSCILLATIONS IN A POLYTROPIC STAR

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The degeneracy of frequencies of nonradial oscillations is resolved in the presence of rotation. As observed in an inertial frame the frequency of a nonradial oscillation in a uniformly rotating star is given as

$$\sigma_{k\ell}^{(m)} = \sigma_{o,k\ell} - m\Omega(1-C_{1,k\ell}) + C_{2,k\ell m}\,\sigma_o \left(\frac{\Omega}{\sigma_o}\right)^2 + O(\Omega^3) \quad , \qquad (1)$$

where Ω is the angular frequency of rotation and σ is the pulsation frequency. The integers k and (ℓ,m) designate modes of radial and angular dependences for the oscillation, respectively. The subscript o indicates a quantity in a non-rotating spherical star, and $C_{1,k\ell}$ and $C_{2,k\ell m}$ are dimensionless quantities. As shown in equation (1), this resolution of the degeneracy produces a few slightly different frequencies for a given (k,ℓ). The quantity $C_{1,k\ell}$ is easily obtained using eigenfunctions in a non-rotating single star (e.g., Ledoux and Walraven 1958). For a uniformly rotating polytropic star, $C_{2,k\ell m}$ can be expressed as

$$C_{2,k\ell m} = X_{1,k\ell} + m^2 Y_{1,k\ell} + Z_{k\ell} + \left(1 + \frac{3}{2}\frac{M_2}{M+M_2}\right)(X_{2,k\ell} + m^2 Y_{2,k\ell}) \quad , \qquad (2)$$

where $X_{i,k\ell}$, $Y_{i,k\ell}$ and $Z_{i,k\ell}$ $(i = 1,2)$ are quantities which are obtained by numerical computations, and M and M_2 are total masses of a pulsating star and of its companion star, respectively. In deriving equation (2), we employed the assumption of rotation synchronous with the orbital motion. The term $(X_{1,k\ell} + m^2 Y_{1,k\ell})$ is due to the second order effects of the Coriolis force and the other terms in equation (2) are caused by the deformation of the equilibrium structure due to centrifugal and tidal forces. Numerical results are listed in Saio (1981) for the polytrope n = 3 and $\gamma = 5/3$.

The results for $\ell = 2$ are compared with four closely separated frequencies of 12 Lac (Jerzykiewicz 1978, Jarzebowski _et al._ 1979), which

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are reproduced in the lower part of Figure 1. We consider here two sets of mode assignments:

	Σ_1	Σ_2	Σ_3	Σ_4	
Case I	$m = 0$	radial	$m = -2$	$m = -1$	(Smith 1980)
Case II	radial	$m = 0$	$m = -2$	$m = -1$	.

Note that the mode with the largest amplitude is chosen as a radial mode in Case II (see Fig. 1). We examine here the relative frequencies of nonradial modes only, because the relation between frequencies of radial and nonradial modes may be model-dependent. An appropriate expression of the rotationally modulated frequency for 12 Lac is

$$\sigma^{(m)} = \sigma_o - 0.870\, m\Omega - (4.704 + 1.560\, m^2)\left(\frac{\Omega}{\sigma_o}\right)^2 \sigma_o \quad . \tag{3}$$

From this equation we obtain

$$[\sigma^{(-2)} - \sigma^{(0)}]/\langle\sigma\rangle = 1.740\left(\frac{\Omega}{\sigma_o}\right) - 7.754\left(\frac{\Omega}{\sigma_o}\right)^2 \tag{4}$$

$$[\sigma^{(-1)} - \sigma^{(0)}]/\langle\sigma\rangle = 0.870\left(\frac{\Omega}{\sigma_o}\right) - 2.317\left(\frac{\Omega}{\sigma_o}\right)^2 \tag{5}$$

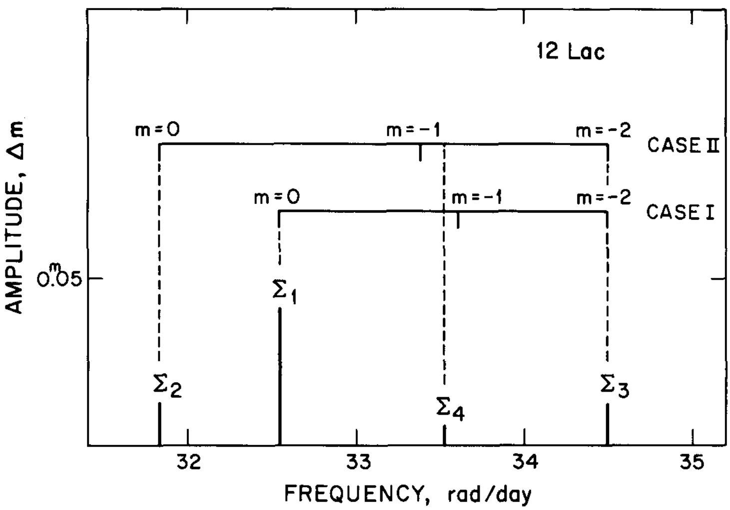

Fig. 1. The yellow light amplitudes versus pulsation frequencies for 12 Lac (Jerzykiewicz 1978). The two horizontal lines with short vertical lines express the theoretical predictions for Cases I and II.

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where $\langle\sigma\rangle \equiv [\sigma^{(-2)} + \sigma^{(-1)} + \sigma^{(0)}]/3$. Replacing the left-hand side of equation (4) by the observed quantity for each case, we obtain (Ω/σ_0), which is introduced into equation (5) to get the relative frequency for $\sigma^{(-1)}$. The results of this comparison are shown by short vertical lines attached to the two horizontal lines in Figure 1.

The equatorial rotational velocities obtained are ~94 km s^{-1} for Case I and ~147 km s^{-1} for Case II. In Case I, $\sigma^{(-1)}$ is closer to Σ_4 than in Case II. However, $|\Sigma_4 - \sigma^{(-1)}|$ in Case I is about 0.09 rad/day, which is more than 30 times the observational uncertainty listed in Jerzykiewicz (1978). Since $\Sigma_3 - \Sigma_4 = \Sigma_4 - \Sigma_1$ within the observational uncertainty, the difference between Σ_4 and $\sigma^{(-1)}$ has been caused only by the second-order (Ω^2) terms. To reduce $|\Sigma_4 - \sigma^{(-1)}|$ to below the observational uncertainty, the second-order terms must be reduced by more than a factor of 30.

In Case II, the discrepancy $|\Sigma_4 - \sigma^{(-1)}|$ is about 0.14 rad/day, which is about 40% of the second-order effects. This will be removed if the second-order terms in equation (3) are increased by about 15%. Such an amount of change could be possible because of the difference in physical conditions between a polytrope and an actual star. Therefore, if our analysis does not have a significant error, Case II is more likely than Case I to explain the observed frequencies of 12 Lac.

More detailed description for this investigation is available in Saio (1981).

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DISCUSSION

JERZYKIEWICZ: Do you consider the $\ell = 3$ case?

SAIO: No, the results of line profile variations by Myron Smith suggest that the $\ell = 2$ case is better.

JERZYKIEWICZ: Do you consider only g-modes?

SAIO: No, I consider all of the modes, including p-modes.

GOOSSENS: Is there any reason why modes with negative m-values are excited and modes with positive m-values are not excited?

SAIO: It is based on the results of line profile variations. Myron Smith has analyzed the line profile variations and compared the models

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with observed line profiles. He suggests that negative m-values are better.

A. COX: I believe that is how the line shapes are phased in time. I think that the question was, theoretically do you expect negative m-values rather than positive m-values?

SAIO: Theoretically, negative m-values are slightly more unstable.

ROBINSON: Do your results depend on the structure of the star?

SAIO: Perhaps, I don't think it strongly depends on structure, because our results are in reasonable agreement with the results for white dwarf stars.

A. COX: It seems to me that there might be other stars that you can apply this to. How about 16 Lac?

SAIO: I did a thorough analysis and in this star, we cannot assume strict corotation. We need rotation slightly greater than that given by corotation. In that case, the positive m-mode should be considered as an excited mode.

A. COX: Is that identified by the line profiles?

SAIO: No.

JERZYKIEWICA: No, that star has zero v·sin i. It is a very slow rotator. There is no reason to assume that the strongest frequency in 16 Lac is radial.

FITCH: There is no reason not to either.