

# THE JOURNAL OF SYMBOLIC LOGIC

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The JOURNAL invites submission of original scholarly work in mathematical logic and its applications.

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## TABLE OF CONTENTS

A marriage of Brouwer’s intuitionism and Hilbert’s finitism I: arithmetic TAKAKO NEMOTO and SATO KENTARO .....	437
Completely separable MAD families and the modal logic of $\beta\omega$ TOMÁŠ LÁVIČKA and JONATHAN L. VERNER .....	498
The consistency strength of the perfect set property for universally Baire sets of reals RALF SCHINDLER and TREVOR M. WILSON.....	508
A structural dichotomy in the enumeration degrees HRISTO A. GANCHEV, ISKANDER SH. KALIMULLIN, JOSEPH S. MILLER, and MARIYA I. SOSKOVA.....	527
On categoricity in successive cardinals SEBASTIEN VASEY .....	545
Closed and unbounded classes and the Häftig quantifier model PHILIP D. WELCH .....	564
Infinite strings and their large scale properties BAKH KHOUSSAINOV and TORU TAKISAKA.....	585
Identity crisis between supercompactness and Vöpenka’s principle YAIR HAYUT, MENACHEM MAGIDOR, and ALEJANDRO POVEDA .....	626
Continuous sentences preserved under reduced products ISAAC GOLDBRING and H. JEROME KEISLER .....	649
Epsilon theorems in intermediate logics MATTHIAS BAAZ and RICHARD ZACH.....	682
Variations on $\Delta_1^1$ determinacy and $\aleph_{\omega_1}$ RAMEZ L. SAMI.....	721
A Q-Wadge hierarchy in quasi-Polish spaces VICTOR SELIVANOV .....	732
Asymptotic analysis of Skolem’s exponential functions ALESSANDRO BERARDUCCI and MARCELLO MAMINO.....	758
The $\Sigma_1$ -definable universal finite sequence JOEL DAVID HAMKINS and KAMERYN J. WILLIAMS.....	783
Luzin’s (N) and randomness reflection ARNO PAULY, LINDA WESTRICK, and LIANG YU .....	802
More on Fréchet–Urysohn ideals SALVADOR GARCÍA FERREIRA and OSVALDO GUZMÁN .....	829
Information in propositional proofs and algorithmic proof search JAN KRAJÍČEK .....	852
Retraction – Measuring club-sequences together with the continuum large DAVID ASPERÓ and MIGUEL ANGEL MOTA .....	870
Surreal ordered exponential fields – Erratum PHILIP EHRLICH and ELLIOT KAPLAN .....	871