S Cnc AS A CASE STUDY OF CLASSICAL ALGOL EVOLUTION

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ABSTRACT

Basic information on empirical determinables for the system S Cnc are reviewed. Whilst the photometric information can be clearly analysed along well known lines, the spectrographic information is far from clear and requires further study, though a semi-detached configuration, at least, can be confidently deduced.

The potentially important question of some physical relationship of the variable to the open cluster Praesepe is considered, but the result is in the negative.

The evolutionary status of the binary is examined in a general way by making use of some summary formulae, some guidelines for which were taken from previous more general work of Refsdal and Weigert. S Cnc is in good overall agreement with the low mass Case B theoretical mode of binary evolution, and even the absence of a detectable rate of period variation is shown to be not in serious conflict with this picture. Interesting close comparisons with the well known example AS Eri are possible.

INTRODUCTION

S Cnc (= BD 19^{o} 2090, HD 74307, KW (Praesepe List) 552) has been a well known example of what is often loosely called an Algol type variable for the last 135 years. Its relatively long period ($\sim$ 9.5 days) has, however, tended to relegate it from the league of intensively studied close binary systems, particularly in recent years.

The system may, nevertheless, have some puzzles to offer. Could it, for instance, have some connection with the Praesepe cluster (Kholopov, 1958)? Does it contain an "undersized" subgiant (Kopal and Shapley, 1956), or could it be "semi-detached" (Hall, 1974)? If it really is semi-detached, doesn't it mean that the secondary is a really low mass object despite its early K spectral type and $\sim$5.5 R_{o} radius?

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A luminosity excess of some 8 magnitudes over the general mass-luminosity relation is implied. And shouldn't the system be showing a more marked rate of period variation than is apparent (Ziołkowski, 1978; Crawford and Olson, 1980)?

Though the primary is similar to that of AS Eri, a frequently referred to example of classical Algol evolution (e.g., Koch, 1970; Popper, 1973; Plavec, 1973; Refsdal et al., 1974; Plavec and Polidan, 1976; Paczyński and Dearborn, 1980), and the mass ratios may be similar (~ 0.1), the separation between the components in S Cnc is two and a half times greater than that of AS Eri. In the latter case, in fact, the smallness of the present angular momentum seems to require a previous contact or common envelope condition (Paczyński, 1976). Are these two systems simply comparable, or could differences in relation to a possible previous common envelope stage cause any observable consequences?

In what follows we shall explore each of these questions in turn. First we summarise our knowledge of basic parameters characterizing the system as far as we can.

BASIC DETERMINABLES

a. Photometry

S Cnc is an eighth magnitude eclipsing binary system exhibiting deep total eclipses when the early type primary is eclipsed by the cool subgiant companion. Recent out of eclipse magnitudes and colours are given in Table 1.

A particularly good set of photoelectric observations was published by Huffer and Collins in 1962, based on an extensive series of previous observations. This data has been analysed by a number of authors, including ourselves, and some of the various results for the main photometric elements are presented in Table 2. Our optimal curve fit to the λ = 5400Å (yellow) data is given in Figure 1. Some authors have thought that the data is of sufficient quality that a simultaneous solution for the primary limb darkening can be made, but doubts appear to be cast on this by the work of Linnell and Proctor (1971).

A more recent set of light curves with greater coverage in both wavelengths and phase, including for the first time the very shallow secondary minimum, have been published by Crawford and Olson (1980). These authors refer to some possible slight variability of the secondary star, but it is clear that the light curves are relatively stable for a semi-detached system, and it is unlikely that the more recent photometry permits any clearer definition of the informative primary total minimum over the earlier data.

Table 1

Magnitude and Colour of S Cnc

	GCVS	WOFK	Barnes (1974)	Crawford & Olson (1980)	Awadalla & Budding	Hilditch & Hill (1975)
Mag	8.45 (pg)	8.5 (B)		8.34 (V)	8.32 (V)	8.31 (V)
B - V			0.10	0.09	0.09	0.11

Values refer to the combined light of the system between eclipses. This has entailed some small appropriate numerical processing of data from the listed sources in some cases and conversion to the UBV system.

Table 2

Eclipse - Photometric Solution Parameters

a) Geometric elements

Parameter	Sources: Huffer and Collins (1962)		Irwin (1963)		Tabachnik (1969)			Linnell and Proctor (1971) C FIT		Linnell and Proctor (1971) DIFCORT		Caracatsanis (1977)	Present Work (adopted)	
r_1	.07677 ±	.0004	.0827 ±	.0014	.0934 ±	.0013	.0874	.076 ±		.07124 ±		.085	.0838 ±	.0005
r_2	.19293	.00382	.1923	.0012	.01837	.0007	.1872	.196	.011	.2009	.0034	.1937	.1932	.0008
i	$83^\circ.91$	.30	$84^\circ.24$	.15	$85^\circ.342$	.0012	$84^\circ.729$	$83^\circ.55$	.37	$82^\circ.99$	.37	$84^\circ.40$	$84^\circ.05$	.05

b) Physical elements - (Adopted Values only)

	$\lambda = 5400$		$\lambda = 4200$		$\lambda = 3500$	
(i) Fractional Luminosities L_1	0.845 ±	0.001	0.923 ±	0.001	0.968 ±	0.001
L_2	0.155	0.001	0.077	0.001	0.032	0.001
(ii) Limb darkening Coefficient (Linear) u_1	0.47		0.55		0.5	

Remarks: Parameter symbols have their usual designations (c.f. Budding, 1973). Other adopted values come from a simple "spherical model" (8-parameter) fitting to the essentially uncomplicated total eclipse. The solutions are χ^2 consistent ($\chi^2/\nu = 1.1$) for an assumed observational accuracy (individual point) of 0.007. The limb darkening coefficients come from the table of Al Naimiy (1978).

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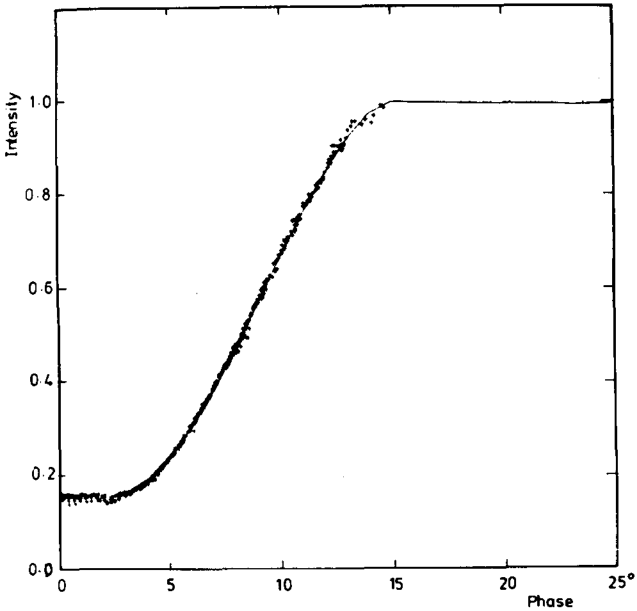

V-Light Curve Fitting for the Primary Minimum of S Cnc.

Figure 1.

b. Spectroscopy

If the photometry of S Cnc is particularly clear, regrettably the same is not true of spectroscopic data. Unpublished results from Joy were used by Kopal and Shapley (1956), but when the original compiled values were released by Abt (1970), and further discussed by Weis (1976) and Batten (1976), it became clear that any radial velocity "curve" amounted to little more than a scatter diagram. In fact, any suggestion of a sinusoidal variation in Joy's data is 180° out of phase with the expected. Closer examination of the spectra at higher resolution by Batten (1976) still seems to suggest an apparent spurious recession of the primary after its total eclipse, though profile tracings show that there are numerous spectroscopic irregularities suggestive of circumstellar material which are known, from other similar cases, to make for considerable complications to radial velocity specification. Popper (1980a), on the other hand, appears to have observed S Cnc when such complications were not so troublesome, though he admits uncertainty on his derived secondary mass. Spectral classification is possibly also subject to uncertainties arising from similar effects. Older references (e.g., Kopal and Shapley, 1956) seem to prefer A0, while some more recent sources (since, for example, Azimov, 1963) allow a B9 designation for the primary. A wider uncertainty attaches to the secondary type, which ranges from mid G to early K - perhaps at least partly due to inherent fluctuations, though the cooler alternative is supported by Crawford and Olson's (1980) colours, and Popper's (1980b) recent summary of spectrographic data.

RELATIONSHIP TO PRAESEPE?

S Cnc is located close to the normally quoted inner region boundary of the star cluster NGC 2632 (Praesepe), at a separation from the centroid of some 70' - about the same as member star 35 Cnc or one and a half times the separation of TX Cnc. In view of the circumstantial consequences which membership of the cluster would give - age and composition then being determinables - the question deserves close attention.

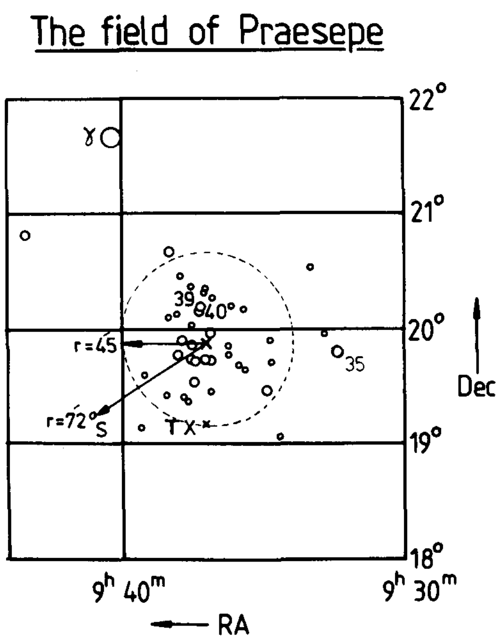

Figure 2. Coordinate Epoch 1950

The position of S Cnc in the colour magnitude diagram of Praesepe though possibly close to the cluster sequence on the strength of older estimates of its brightness and type, throws doubts on any possibility of the primary star being like a normal Main Sequence

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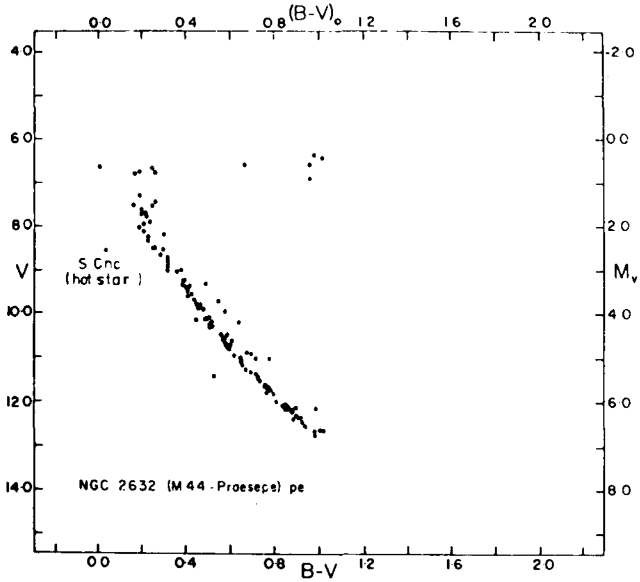
cluster member when more modern values for these quantities are adopted. Thus Crawford and Barnes (1969), found, as we do, the star to be too blue to be on the sequence, and reported it as "probably a non-member". The metallicity index m_1, which from the data of Hilditch and Hill (1975) would be too little at $m_1 = 0.123$ to be in agreement with the Crawford and Barnes value, which at the β value of S Cnc (2.781) corresponds to 0.20 ± 0.02. Low metallicities appear, however, to be a characteristic in Algol systems, (see e.g., Plavec and Polidan's discussion of this subject, 1976), and if the primary has already accepted a considerable amount of material from the evolved component the position is not clear. Thus there is theoretical evidence to suggest that Algol primaries in the latter stages of the semi-detached phase could lie to the left of the appropriate Main Sequence, though observational evidence, as far as can be ascertained from some more well known examples, does not seem to bear this out (Packet, 1980).



The colour and magnitude of S Cnc and the Colour Magnitude diagram of "Praesepe" Cluster (after Hagen, 1970)

Figure 3

Though, in view of spectroscopic complications already mentioned, arguments based on measured radial velocities can be given little weight, Dickens et al's (1968) mean radial velocity of NGC 2632 stars

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as 31 $\pm$ 2 km/sec could not be ruled out on the basis of Batten's (1976) data on S Cnc.

The most serious doubt on S Cnc's credentials as a Praesepe star, however, comes from the measured proper motions, a point already realized by Heckmann (1937). Though when we first examined the scatter of proper motion values of accepted cluster members in the Smithsonian Catalog (1966) ($\sim$ 0."02) the membership of S Cnc appeared to be still plausible, the Smithsonian Catalog has compared a number of independent sources of absolute position determinations to produce its proper motions, while a self contained differential system of measurements of a small area on the same machine, such as that of Heckmann (1925) must have a greater inherent precision, probably an order of magnitude higher than the value just mentioned. (The authors are grateful to Professor van de Kamp for pointing this out). On this basis, the separation in proper motion space between S Cnc and the centroid of Praesepe of some 0".019 (y^{-1}) makes its present membership "nicht in Frage".

"UNDERSIZED" OR "SEMI-DETACHED"?

A number of authors now appear to support Hall's (1974) arguments in favour of a likely semi-detached configuration to the binary. We briefly review the case.

Kopal and Shapley's (1956) deduction of an undersized subgiant for the secondary appears to have been based on radial velocity data which, as has already been mentioned, is unreliable for the intended purpose. Batten's radial velocity data suggest only that any sinusoidal variation passing through the points in the required sense would have low amplitude. On the other hand, Joy's radial velocities for the secondary, obtained during the totality, give K_2 = 140 km s^{-1} a value which is supported by Batten (1976). From this quantity a secondary mass function can be obtained as 2.70. If we use Kopal's (1959) table relating "contact" relative radii to mass ratio the photometric solution yields a mass ratio of 0.09, which then implies a primary mass of 3.3 M_o, which is reasonably close to that expected for a Main Sequence primary.

A slightly higher mass than this is found if we combine the Main Sequence Mass-Radius relation ($R \sim M^n$) with Kepler's third law for the binary, i.e.

$$\log M_1 = \frac{1}{3n-1}(2 \log P + 3 \log r_1 + \log (1+q) + 1.872), \qquad (3.1)$$

where P is the period in days, r_1 the primary relative radius and q the mass ratio. This results in $M_1 \doteq 3.6\ M_o$ when the index n is given a value of 0.71 (corresponding to the slope of the ZAMS M:R relation at A0), though with an obvious sensitivity on the choice of n.

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An undersized secondary would require the primary mass to be higher and more discordant with the apparent Main Sequence character, while putting up the mass ratio significantly should cause a more noticeable genuine primary velocity variation in the radial velocity curve, and the detected interactive effects should also then become less likely in a non-contact situation.

In so far as a near to Main Sequence character can be assigned to the primary star, therefore, we can see that a semi-detached configuration is supported. The argument used by Hall and Neff (1979), was essentially similar to this, but involving the more reliable mass: luminosity rather than mass:radius relationships.

In Table 3 we give our adopted absolute parameters characterizing the system S Cnc.

Table 3

Adopted Physical Parameters of S Cnc

Period	9.48454 days
Primary Mass	3 $M_\odot$
Secondary Mass	0.27 $M_\odot$
Separation	28.0 $R_\odot$
Primary Mean R	2.35 $R_\odot$
Secondary Mean R	5.41 $R_\odot$
Primary Spectral Type	B9.5
Primary Te	10300
Secondary Spectral Type	K0
Secondary Te	4700
ΔM_{bol} (luminosity excess of secondary)	$8^m.1$
P_o } (conservative evolution backwards)	$0^d.264$
A_o	2.57 $R_\odot$

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EVOLUTIONARY STATUS

S Cnc appears to be a natural candidate for rather low mass Case B type binary evolution, as was indeed proposed by Kreiner and Ziołkowski (1978). Calculations for this type of evolutionary scheme are able to demonstrate the observed luminosity excess, and other physical characteristics of the component stars, which are, in this case, somewhat more extreme than the observed mode for similar classical Algol stars. The core of the erstwhile primary has become like that of a low mass giant branch star. A thin shell source above this degenerate core is able to maintain the very tenous envelope with its throughput of relatively excessive luminosity. The basic reason for the luminosity excess is that core or near core processes are virtually uncoupled from the removal of the envelope material due to the Roche lobe overflow (RLOF) mechanism. There is, of course, an ultimate switching off of the shell source when the envelope has been sufficiently depleted. Thereafter the evolved star should sink to a condition like that of a white dwarf (Refsdal and Weigert, 1971) at a separation of up to about an order of magnitude greater than that which existed originally.

One possible discrepancy with the proposed scheme is that there should perhaps be a larger scale of period variation than is actually observed (Ziołkowski, 1976; Kreiner and Ziołkowski, 1978). In what follows we shall try to summarize the situation by reference to some simplified but general formulation.

Four or five basic variables can be recognized in the most essential posing of the RLOF problem, together with one or two special quantities which we may wish to treat as variables, i.e., overall mass and angular momentum of the binary system, though their proper treatment as variables could be technically awkward; and certain other quantities which relate to the structure and rate of surface expansion of the mass losing star, which could be regarded as parameters.

Counting then the mass of either star m_1, m_2, the radius of the mass losing star r_1, the separation of the two mass centres A and orbital period P as the basic variables dependent on time t, we have four fairly clear formulae to interrelate these dependent quantities, namely: an equation for the overall mass

$$m_1 + m_2 = M, \qquad (4.1)$$

one for the orbital angular momentum (rotational momentum is usually neglected)

$$\frac{2\pi A^2}{P}\frac{m_1 m_2}{M} = J, \qquad (4.2)$$

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Kepler's third law

$$\left(\frac{2}{P}\pi\right)^2 A^3 = GM , \qquad (4.3)$$

and some equation for the relative size of the mass losing component (e.g., some "Roche lobe" formula, see e.g., Paczyński, 1971)

$$\frac{r_1}{A} = f\ (m_1, m_2) \quad , \qquad (4.4)$$

Separate formulae are required for the variation of overall mass and angular momentum in the binary, but for the present purposes it is convenient just to regard these as separately specifiable.

A fifth equation for the actual mass transfer is required to formally close the system in terms of time variation. We shall first normalize by writing $x = m_1/M$, the fractional mass of the mass losing star. It can be shown that (using the dot notation for differentiation with respect to time)

$$\dot{x} = -3\eta \frac{x}{r_1} (s - \dot{r}_1) \qquad (4.5)$$

can express the sought transfer equation. This relates the rate of mass transfer x to some source function s, due to inherent expansion of the mass losing star resulting from its own internal structural processes, minus a retention term $\dot{r}_1$, which comes from the expansion of the available volume for m_1. The quantity n expresses the ratio of density of the surface layers of the mass losing star to the mean density of the star as a whole; alternatively η can be defined as

$$\eta = \left(\frac{d \log m_1(r)}{d \log r}\right)_{surface} . \qquad (4.6)$$

Let us consider first the "conservative" case, in which M and J can be taken as constant, at least the former of which may well be applicable in the slow separation phase of classical Algol evolution. Manipulation of the first three equations can easily be shown to yield

$$P = \frac{P_o}{64x^3(1-x)^3} \quad , \qquad (4.7)$$

where P_o is the minimum period, obtaining when $x = 1/2$, and given by $P_o = \dfrac{128\pi J^3}{G^2 M^5}$. Differentiating (4.7) with respect to time we obtain

$$\dot{P} = 3g(x)\ P\ \frac{\dot{x}}{x} \quad . \qquad (4.8)$$

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where we have put $g(x) = {}^{(2x-1)}/(1-x)$. In the normally encountered 'slow phase' of Algol evolution $x < 1/2$, so $g(x) < o$, and, of course, $\dot{x} < 0$ is a necessary condition of mass loss. Under these circumstances, as x becomes small we might expect that $\dot{P}$ will increase in a rather sensitive way to the mass transfer. This will evidently depend also on the behaviour of $\dot{x}$.

The scope of Equation (4.8) can be broadened to include also the possibly significant case where angular momentum of the overflowing material is not immediately transferred back into the orbit. At wider separations the timescale for such angular momentum transfer, even assuming no matter is actually lost from the system, may well become appreciable in comparison to Case B evolution timescales. A phenomenological approach to such effects is to write

$$J/J_{init} = (x/x_{init})^k ,$$

where $k = 0$ corresponds to the conservative case, and we expect $0 \lesssim k \lesssim 1$ in practice. Physically, this expresses the difficulty of putting the increasing proportion of specific angular momentum in the RLOF material at low x back into the orbit. $|g(x)|$ then becomes reduced in size to $(1 - 2x)/(1 - x) - k$.

A combination of (4.5) and the derivative of (4.4) allows us to write

$$\dot{x} \simeq \frac{-3\eta s\, x}{r_1\,(1 - 5.o\eta\, g(x))} , \tag{4.9}$$

where the approximation depends on the form of the Roche lobe approximation (4.4). Hence (4.8) reads on substitution for x

$$\dot{P} = \frac{-9\eta s\, g(x)\, P}{r_1(1 - 5\eta\, g(x))} \simeq \frac{-9\eta s\, g(x)P}{r_1} , \tag{4.10}$$

in view of the expected smallness of η ($\sim 10^{-2}$).

A suitable combination of the first four equations enables (4.10) to be rewritten as

$$\dot{P} = -\left\{\frac{9}{4}\frac{P_o}{A_o}\frac{g(x)}{(1-x)}\right\}\frac{\eta s}{x\, f(x)\,(1 - 5\eta g(x))} , \tag{4.11}$$

where the quantity in curled parentheses tends to a finite constant as $x \to 0$. For small x, $f(x) \to 0.462x^{1/3}$ (Kopal, 1959) so that

$$\dot{P} \to \text{const.}\frac{\eta s}{x^{4/3}} . \tag{4.12}$$

In order to ascertain some suitable values for η and s some detailed calculations have been examined.

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Considering first η ; though a number of calculations of mass losing stars in approximately similar conditions (low mass, Case B) have been calculated, a few authors only have published sufficient numerical details of the subgiant structure to allow appropriate values of η to be directly assessed. Among such calculations, those of Harmanec (1970) are particularly useful for the present purpose, and plots of the subgiant structure in the log m_1 (r), log r plane, based on Harmanec's Figure 9 data , are reproduced in Figure 4.

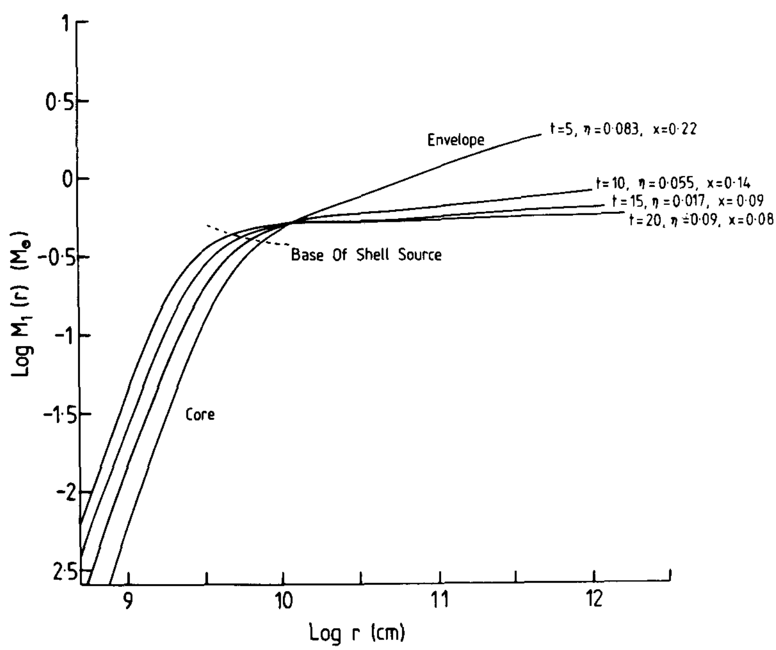

Figure 4. Structure of Semi-Detached Subgiants.

A relatively simple subdivision into core and envelope regions can be seen in such plots. The apparent constancy of η through the envelope region can be justified on the basis of approximate formulae describing the structure of classical Algol shell burning subgiants given by Refsdal and Weigert (1970). Combining together various of the structure equations given by those authors it is possible to reconcile η = constant with a polytrope $n = 3$ like behaviour of a constant opacity, tenuous envelope. If η = constant through the envelope be a reasonable description of the mean behaviour, it is relatively easy to derive a formula such as

$$\eta = \frac{1}{3} \frac{m_{env}}{m_c} \frac{1}{\ln \frac{R}{R_c}} , \qquad (4.13)$$

so that as a result of envelope depletion and expansion η will decrease during the course of the slow phase of mass transfer. Consider, for example, a binary like that of System III of Refsdal and Weigert (1969), which between points i and k of its calculated evolution appears to have some resemblance to S Cnc. We have, for the mid point of this range, (in solar units) $m_c = 0.239$, $m_{env} = 0.094$, $R = 9.59$, $R_c = 0.028$ (from the m_c:R_c relation of Refsdal and Weigert, 1970) so that $\eta = 0.022$, which compares well with values obtained from the Harmanec data at a corresponding stage.

In the early stages of the slow phase $\eta \sim 0.1$ and its initial decrease would appear to be not so rapid, but in the later stages before eventual envelope collapse the decline to $\eta \sim 0.01$ is more steep. Thus the last three values obtained from the Harmanec data show η declining with x according to approximately $\eta \sim 3.5\ 10^3\ x^5$.

The physical justification for our deductions about the behaviour of η is related to the fact, stressed by a number of authors, of the controlling influence of the core. The basic envelope expansion s should also be determined by the behaviour of the core, however it becomes markedly more rapid just after convection in the outer envelope becomes established . The role of core mass increase can be accounted for by differentiating the expression for the mass loss free overall radius given by Refsdal and Weigert (1970), which we could write as

$$s = \frac{\beta}{3} \left(1 - \frac{1}{\alpha} \frac{m_c}{m_{env}} \right) m_c^3 \frac{dm_c}{dt} \qquad (4.14)$$

where α and β are numerical parameters; (typically $\alpha \sim 12$ and $\beta \sim 10^6$ for s in $R_o y^{-1}$, however, this formula does not clearly show the significance of the outer convection zone. Thus in the evolution sequence of a 2.25 M_o star, as published by Iben (1967), the surface expansion proceeds at a mean rate of about $3.6 \times 10^{-7}\ R_o y^{-1}$ during the phase of shell burning while the envelope is still radiative, but increases to $1.4 \times 10^{-6}\ R_o y^{-1}$ during the convective outer envelope phase. These rates, which show a strong dependence on mass ($\sim m^6$ for first and $\sim m^4$ for second) are somewhat greater than those expected for S Cnc.

The fact that η is a small number in the final slow stages of mass transfer, however, does allow us to combine the foregoing equations in an alternative way to the way Equation (4.9) was derived, to show the "feedback" effect of s on $\dot{r}_1$, to which it becomes approximately proportional, thus $s \sim \dot{r}_1 / 5\eta g$. The behaviour of $\dot{r}_1$ from calculated model sequences is relatively easy to study.

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While there are some differences of detail the overall pattern appears to be as that shown in Figure 5. After an initial moderate expansion there is a rapid rise during the early establishing of a relatively thick convective envelope. Depending somewhat on initial orbital parameters and mass ratio, published calculations lead us to expect peak Roche lobe expansion rates during this phase to be around $4 \times 10^{-7}\ m_{1\ \mathrm{init}}^{4}\ R_{o}\,y^{-1}$.

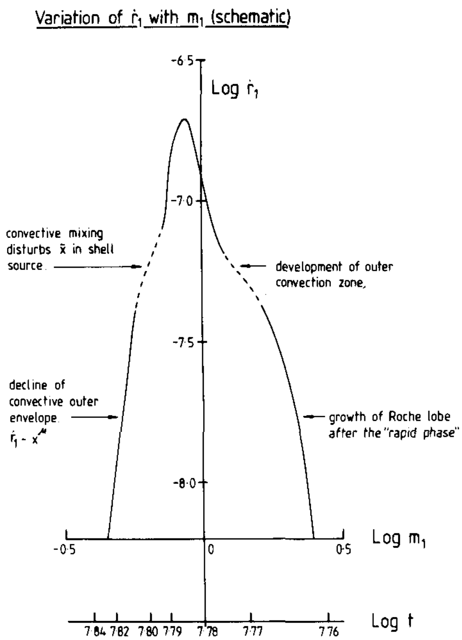

Figure 5. The timescales would be about appropriate for a star of $m_{1\ \mathrm{init}} = 3M_{o}$

What is of significance to the present discussion is, however, that this maximum rate of expansion during the slow phase is reached relatively early on, whereafter there is a relatively long period during which the expansion rate declines as the convective region of the outer envelope narrows down.

The behaviour of $\dot{r}_1$ with m_1 appears to be that of a steep power, i.e., $\dot{r}_1 \sim m_1{}^{\mu}$, where $\mu \sim 8$. This phase of slow decline can last several times as long as the phase leading up to the convective maximum. Hence, to return to formula (4.12), what we should expect in the drawn out phase before eventual collapse, is a rate of period variation which declines with a high power of x, i.e., $\dot{P} \sim x^{\mu - 4/3}$. Such a high power dependency means that if we simply take a mean rate

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of period variation from a pair of well separated interval points on a calculated sequence, the derived value is likely to be representative only towards the initial point of the range, while for most of the interval the actual rate of period variation could be appreciably less. Working against such a weighting among actual observational data would be, of course, the selection effect which favours close pairs for period determination and study.

Consider again the range i to k of the system already referred to; the maximum value of s, according to our foregoing estimates, occurring somewhat before point i, would be $\sim 1.5 \times 10^{-6} R_o{}^{-1}$. Substituting appropriate values into the transfer Equation (4.5) we find,

$$\Delta m_1 \simeq \frac{0.022}{9.595} \left(\frac{1.5 \times 10^{-6}}{\nu} - 7.10^{-8} \right) 7.1 \times 10^7 \ M_o$$

where ν is the index expressing the power law variation of s with m_1 during the slow decline. A value of Δm_1 agreeing with that calculated by Refsdal and Weigert can be found for $\nu \simeq 2$, which gives a tolerable support to the underlying point in view of other approximations and uncertainties. Similar calculations have been performed for other comparable published examples of evolution towards the end of the slow phase which show similar effects.

Actually, the period values in the i - k interval of the System III just referred to are greater than that currently found for S Cnc, while the model is "conservative" throughout, which, even if possible in the drawn out phase under consideration, seems less likely in the range close to minimum period and maximum m_1, where extrapolation back from the present values leads us to expect a stage of contact, with the possibility of appreciable angular momentum loss from the system. Angular momentum loss at this stage of strong interaction would scale down the periods at subsequent stages, which could otherwise then follow a parallel course to conservative model calculations.

Coming to a closer comparison with actual data on S Cnc, we find that even the straight mean period variation during the range i - k is 7.10^{-9} when scaled to the S Cnc period, while Kreiner and Ziołkowski (1978), estimate the accuracy with which such a term can be evaluated from the available data to be $\sim 2.10^{-9}$. From what was said already, such a straight mean would only be representative for a region of order $3/(3\mu-4)$ of the relevant time interval, where η expresses the full power law dependence of P on x in (4.12); and since μ is likely to be quite larger than three, the absence of a detectable present rate of period variation might not be so worrying, if this model could indeed be quite appropriate for S Cnc. The initial mass for this system is however only $1.4M_o$, while we know that the original primary of S Cnc must have been more massive than half the present total mass (i.e., the adopted $m_{1\ init} > 1.6M_o$). In view of the high power dependence of s on $m_{1\ init}$ this could mean that the foregoing period variation should be increased to at least 1.2×10^{-8}. If we want to still adhere to the conservative model, we are forced to suppose that the subgiant is now relatively close to the end of its period in the semi-detached configuration, but a closer examination of the likely core mass (to be carried out in what follows) will lead us to expect this is not quite so, and that the star should be at an intermediate position along the i - k track.

The likeliest way out of this dilemma appears to be to appeal to the previously mentioned possibility of failure of efficient angular momentum transfer. The scaling constant in Equation (4.12) can thereby be reduced by the factor (1 - k), so that the implied reduction in observable period changes can be achieved without great difficulty. In fact, if we consider Lubow and Shu's (1975) more detailed treatment of the hydrodynamics of the RLOF mechanism in an initially wholly conservative regime it would appear that after the mass fraction x drops below about 0.17 it begins to be possible for stable "disk" like orbits to be performed by material accreting towards the detached component, without impinging directly on its surface. (From Lubow and Shu we observe the disk radius ϖ_d satisfies $\varpi_d \simeq 0.075x^{-0.43}$, while the detached star radius satisfies $r_2/A \simeq 8x^2(1 - x)^2$ if it does not grow too much during the slow phase and there is no extensive common envelope stage). If thereafter the transferred angular momentum is stored in a disk of radius $\sim\varpi_d$ rather than put back into the orbit we find

$$\frac{d\ \log J}{d\ \log x} = \frac{0.27y^{1/2}(\ (0.22-g(x))z+x)}{0.27y^{1/2}z\ \ +\ (1-x\ )x^{1.22}} \qquad (4.15)$$

where $y = m_2/M$ and $z = m_{disk}/M$. For values of $x \sim 0.1$, d log J/d log x is slowly varying around 0.5, so that the scaling factor in (4.12) is halved. The absence of detectable period variation, even with a $1.6M_\odot$ initial primary mass, could then be understood. This does lead us to suppose the existence of some kind of accretion disk around the present primary in S Cnc, however; a point which could be tested by further observational study.

A further clarification of the evolutionary status of S Cnc can be made on the basis of Refsdal et al.'s (1974) detailed considerations of AS Eri. Formulae given by those authors allow values of quantities such as core mass, final core mass and maximum radius to be estimated on the basis of an assumed hydrogen profile parameter X for the subgiant. Details of the two systems evaluated in this way are compared in Table 4.

Table 4

Comparison of Evolved States of S Cnc and AS Eri and Some Models

	S Cnc	AS Eri (Aver.Seq. I&II, RRW)
m_c	0.22	0.18
m_{env}	0.05	0.03
R	5.41	2.25
R_c	0.029	0.030
η	0.014	0.013
m_{max}	0.26 (RW, T5)	0.19
R_{max}	10.4 (RRW, E3)	2.54
m_{lf}	0.245 (RRW, E7)	0.203
R_{lf}	6.17 (RRW, E6)	2.31
$\bar{x}$ (assumed)	0.5	0.46, 0.55

	System I of GG Middle of range i - k	System III of R Wa Middle of range i - k
m_c	0.257	0.239
m_{env}	0.066	0.094
R	12.95	9.59
R_c	0.028	0.028
η	0.014	0.022
$m_{1(init)}$	2	1.4
s_{max}	$\sim 6.4.\ 10^{-6}$	$\sim 1.5.10^{-6}$
ν	~ 2.5	~ 2
$\Delta P/P$	2.8×10^{-8}	7×10^{-9}

Abbreviations for references

RW = Refsdal and Weigert (1970)
RWa = Refsdal and Weigert (1969)
RRW = Refsdal, Roth and Weigert (1974)
GG = Giannone and Giannuzzi (1970)
T ≡ Table Number; E ≡ Equation Number

CONCLUSIONS

S Cnc as a well known and relatively bright example of a classical Algol system can provide a good tracer of stellar evolution processes within the circumstance of binarity. The main findings of the present attempt to bring out relevant facts are as follows:

1. A physical connection with NGC 2632 appears very unlikely on purely observational grounds. Moreover, if S Cnc was presently of the same age as the cluster ($\sim 4.10^8$ years) the initial mass of its original primary, on the Case B mass transfer theory, would probably have to be greater than $2M_o$ (c.f., Giannone and Giannuzzi, 1970, 1972), making the currently observed low rate of period variation incompatible with standard Case B theory.

2. The semi-detached configuration following low mass Case B evolution towards the end of the semi-detached phase can provide a good general explanation of the overall system properties.

3. The absence of a noticeable rate of period variation, need not be in such conflict with theory, even on the conservative model, if it is possible to reduce the total mass from the adopted value ($3.27\ M_o$), for example, down to Popper's (1980a) value ($2.8M_o$).

4. The scale of expected period variation could be halved if an "accretion disk" can store angular momentum of the matter currently being lost from the original primary. This situation appears to be more consistent with the expected core mass ($0.22M_o$) required to explain the current overluminosity of the contact component, on the basis of Refsdal and Weigert's (1970) formulae and tables.

5. More observations are required to check on any RW Tau type phenomenon in the system (disk confirmation), or to confirm or otherwise, the apparent disparity between the findings of Batten (1976) and Popper (1980) regarding complications to line profiles.

6. An interesting comparison with the system AS Eri, which was modelled in great detail by Refsdal, Roth and Weigert (1974), is possible. AS Eri probably started with initial separation or mass ratio (m_2/m_1) less than that of S Cnc, bringing it into deep contact during the rapid phase, when non-conservative effects would have been enhanced. Higher proportions of processed material may therefore be present in the outer regions of the stars in AS Eri compared with S Cnc.

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