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ELECTRICAL IMPEDANCE TOMOGRAPHY USING NONCONFORMING MESH AND POSTERIOR APPROXIMATED REGRESSION

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Electrical impedance tomography (EIT) is an imaging technique where the internal electrical conductivity distribution is reconstructed using current and voltage measurements on the boundary. The idea of EIT is that if parts of an object can be distinguished by their electrical conductivity, then we are able to identify the location of these parts using EIT. EIT is noninvasive, portable and cheap, so it has been widely applied across a number of different applications, for example, medical, environmental and nondestructive testing. The contributions of this thesis are largely in three parts.

In the first part of the thesis, we present the formulation of the EIT inverse problem using a nonconforming mesh. This is in contrast to the traditional approach which uses a conforming mesh. The benefit of employing a nonconforming mesh is that finer discretisation can be employed on a localised region, which can be computationally more efficient. While extensive research has solved forward problems using nonconforming meshes, this is the first time EIT inversion has been studied using a nonconforming mesh. The state-of-the-art nonconforming finite element method today is the mortar element method and is the numerical scheme used here. In this thesis, we address the inverse problem in the statistical setting, which is natural when there are errors in the measurements or modelling uncertainties. In particular, we consider Bayesian statistical inference, which is the standard approach for statistically solving inverse problems today.

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In the second part of the thesis, we focus on a situation where computational resources are severely limited yet the solutions to inverse problems need to be obtained quickly. Such is the case in process tomography where images need to be reconstructed on a scale of milliseconds, using industrial computers comparable to somewhat outdated personal computers. Given the unknown $x \in \mathbb{R}^n$ and its related data measurement $y \in \mathbb{R}^m$, the goal in Bayesian statistical inference is to extract all information about x from the conditional distribution $\pi(x|y)$, called the posterior distribution. However, if x is large dimensional, that is, if n is large, then considering standard methods in Bayesian inference such as the Markov chain Monte Carlo (MCMC) and the computation of the maximum *a posteriori* (MAP) point estimate can be practically infeasible unless, for example, heavy model reduction is implemented. The idea presented in the thesis is to, instead, pose the Bayesian inverse problem as a statistical forward problem via the construction of a regression model. This means that an image can be reconstructed by solving of a single statistical-based forward problem. A benefit of such a formulation is that it allows us to fit highly nonlinear operators in the regression model, since a single evaluation of a relatively complicated operator can still be achieved at a cheap price. Highly nonlinear operators can be needed to accurately describe the statistical mapping from y to x. We construct the regression using joint samples $(x, y) \sim \pi(x, y) = \pi(y|x)\pi(x)$ drawn from the related likelihood function $\pi(y|x)$ and prior distribution $\pi(x)$. Therefore, the regression model naturally provides us with an approximation of the posterior distribution $\pi(x|y)$. In this connection, we call such a regression model the *posterior approximated regression model*.

In the third part of the thesis, we are interested in the prior modelling of discretised non-Gaussian random fields. By far the most used type of prior distribution in Bayesian inverse problems is the Gaussian distribution. But Gaussian priors tend to produce smoothing effects on the MAP estimate which can lead to underestimates of the unknowns. At the same time, however, constructing the prior model is often the most challenging stage in implementing Bayesian inference. This is because the *a priori* information we have of the unknown is typically *qualitative*, yet what is required is a *quantitative* representation of such information. Hence, considering a wide range of different non-Gaussian prior distributions when the unknown x is large dimensional is, in general, a nontrivial task. A commonly used technique to simulate non-Gaussian discretised random fields is via the transformation of Gaussian discretised random fields. Depending on the technique used, it is possible to simulate numerically a large family of different nondiscretised random fields. Such a technique, however, has not yet been used to construct non-Gaussian discretised random fields as a means for prior modelling. We investigate whether the method can be used to partially resolve the smoothing problem in employing discretised Gaussian random fields, for example, by fitting non-Gaussian prior distributions that are longer tailed than the Gaussian.

The proposed formulation, method and strategy are verified via various numerically simulated EIT problems on two-dimensional domains.

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