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Structure of Dark Halos: Model-independent Information from H I Rotation Curves

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Abstract: Although the approximately flat rotation curves of gas in the outskirts of spirals are generally taken as strong evidence for spherical, isothermal dark matter halos, this conclusion is often incorrect and always model-dependent. A re-examination of the old and (nearly) model-independent inversion technique for determining the surface mass density of galaxies from their kinematics is presented. The method is shown to be relatively insensitive to noise in the kinematics. Due to incomplete kinematical knowledge at large radius, however, the surface mass density is reliable only in the inner half of the galaxy, a result that also applies to traditional rotation curve fitting techniques.

Keywords: galaxies: kinetics, dynamics

1 Fitting Rotation Curves: Mass Models → Kinematics

Long before dark matter in galaxies became the focus of two or three international conferences a year, optical emission-line studies were used to measure the kinematics and, indirectly, the mass distribution in spirals. In those days, spiral galaxies were assumed to be as flat as they looked, and it was natural to devise a method of using the measured kinematics of galaxies to infer the surface mass density of the luminous disk. First attempts assumed that the mass of a galaxy could be reasonably well approximated by either a superposition of oblate spheroidal shells of a given flattening (Burbidge, Burbidge & Prendergast 1959; Brandt 1960) or an infinitely thin disk (Toomre 1963; Nordsieck 1973; Bosma 1981). The rotation curve was then inverted via Poisson's equation to produce the cylindrical mass interior to a given radius.

Later, when more sensitive optical emission spectroscopy and, especially, H I synthesis observations consistently indicated that rotation speeds remained nearly constant to large galactocentric radius, dark matter was invoked and modelled as a spherical, isothermal halo with $\rho \propto r^{-2}$, which produces absolutely flat rotation curves at all radii. In order to account for the slower speeds in the very inner regions of galaxies and the contribution to the rotation curve provided by the luminous mass, the spherical dark halo was then modified to have a finite core. The mass density of the luminous matter was assumed to be proportional to the density of the light, scaled by a radially constant mass-to-light ratio M/L.

Interestingly, much of the original observational motivation for spherical, isothermal dark halos has now been lost, especially over the radial range ($r < 40$ kpc) probed by spiral rotation curves (Sackett 1996). One of the strongest constraints beyond this radius relies on statistical arguments based on the orbits of satellites (Zaritsky & White 1994) that do seem to suggest that halos may be isothermal at >100 kpc. On the other hand, the only known measurement from a cold tracer at these radii, the giant H I ring in Leo ($r \approx 100$ kpc), seems to indicate that the halo has already truncated by ~60 kpc (Schneider 1985). The kinematic advantage of H I rings like that in Leo is the simple orbit structure of the cold gas; unfortunately only one such ring is known. The spatial resolution and sensitivity of the Parkes Multibeam should make it an ideal instrument to search for more.

With this separation of galactic mass into luminous and dark components, the philosophy changed from rotation curve *inversion* to rotation curve *fitting*. Rotation curve fitting generally proceeds by comparing kinematic observations with model rotation curves resulting from multi-component mass models built from at least three of the following components:

- Stellar Bulge: assume M follows L, and fit for M/L_{Bulge}
- Stellar Disk: assume M follows L, and fit for M/L_{Disk}
- Gaseous Disk: assume M follows known 21 cm emission
- Dark Halo: assume a parametrised form, typically fitting two free parameters that control the central dark density and halo core radius.

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Scale lengths of the disk and bulge are typically measured by photometry, with assumptions for the flattening of the bulge and the scale height of the stellar disk. The flattening of the dark halo is also fixed by assumption (generally to be spherical). Such a procedure produces 'best-fit' halo parameters, or—if the mass-to-light ratios are fixed by the maximum disk hypothesis (van Albada & Sancisi 1986)—'maximum-disk' halo parameters.

Given the large number of implicit assumptions, the complications of non-circular motions, the degeneracy of the fitting parameters, and the fact that rotation curves generally exhibit only two or three distinct features (turn-over radius, peak speed and outer slope), one might reasonably be concerned that rotation curve fitting introduces spurious, model-dependent trends in inferred dark matter properties. We are therefore led to begin again, reducing the number of model assumptions and asking, 'In what way is the radial distribution of the total mass (luminous + dark) of a galaxy constrained by its rotation curve?'

2 Back to the Future: Inverted Kinematics → Mass

The simplest case of rotation curve inversion is, of course, obtained under the (extreme) assumption of a spherical potential. In that case, the circular velocity $v_c(R)$ is all that is required to compute the mass interior to R via the usual $M(<r) = v_c^2 r/G$; the density $\rho(r)$ of a shell of radius r is thus given by $4\pi G r^2 \rho = 2 v_c r (dv_c/dr) + v_c^2$. Computation of the mass column within the cylindrical distance R, however, requires an integration along the cylindrical z coordinate, and thus knowledge of the kinematics to very large R. Specifically,

$$\Sigma(R) = \frac{1}{2\pi G} \int_R^\infty \left[\frac{dv_c^2}{dr} + \frac{v_c^2}{r} \right] \frac{dr}{\sqrt{r^2 - R^2}}, \quad (1)$$

(spherical geometry)

so that in general the rotation curve at infinity must be known in order to compute the projected mass density $\Sigma(R)$. The other extreme is to assume that the galaxy can be described by an axisymmetric, infinitely thin disk; in this case the rotation $v_c(R)$ is influenced by mass outside R as well. Using Laplace's equation, Gauss' law and Bessel function identities, it is possible to show that the surface mass density $\Sigma(R)$ of such a disk is related to the rotation curve and its derivative via (Binney & Tremaine 1987),

$$\Sigma(R) = \frac{1}{\pi^2 G} \left[\frac{1}{R} \int_0^R \frac{dv_c^2}{dR'} K\left(\frac{R'}{R}\right) dR' + \int_R^\infty \frac{dv_c^2}{dR'} K\left(\frac{R}{R'}\right) \frac{dR'}{R'} \right], \quad (2)$$

(thin disk geometry)

which again formally requires kinematic information at infinite distance. Integrating $\Sigma(R)$ over the area of the disk within R then gives the enclosed mass.

Effects of Extrapolation and Noise

Note that inverting the rotation curve to obtain the surface mass column is not completely model-independent: assumptions of axisymmetry and the form of the vertical potential must be made. On the other hand, having made these assumptions (which are also made in rotation curve fitting) the radial dependence of $\Sigma(R)$ is then entirely fixed by the kinematics. Two concerns remain about the application of this technique to real data: (1) the effect of missing kinematic information at very large R, and (2) the effects of noise in the rotation curve and its derivative. We now examine each of these in turn.

The effect of extrapolating the rotation curve beyond the measured kinematics is shown in Figure 1. As a test, we begin with the theoretically-derived rotation curve of a thin exponential disk extrapolated beyond 10 disk scale lengths with one of two extreme schemes that can reasonably be expected to bracket reality: *flat extrapolation* [$v_c(R) = \text{constant}$] and *Keplerian decline* [$v_c(R) \propto 1/\sqrt{R}$.] For each extrapolation, the rotation curve is inverted to produce an inferred surface mass density $\Sigma(R)$ for both the spherical (Eq. 1) and flat disk (Eq. 2) geometries. Note that in this case both the Keplerian and flat extrapolations deviate from the true surface mass density (shown as the solid straight line in the log plot) by overestimating $\Sigma(R)$ at large R because the rotation of an exponential disk falls faster than Keplerian, an exact result that can be proved analytically. This departure does not begin at the end of the 'measured' rotation curve, but at half the kinematical radius, in this case at 5 scale lengths.

The incorrect (in this example) assumption that the mass is spherically distributed leads to inferred surface mass densities that are too high in the inner regions, and thus must be compensated for by outer shells with unphysical negative $\Sigma(R)$; this results in an enclosed mass that actually declines with radius in the outer regions. The flat extrapolation scheme produces an overestimate of the enclosed mass interior to the last kinematical point where the departure has already reached ~30%. This then has a very large effect on the *local* estimate for the mass-to-light ratio, which, although taken to be constant in this example, is overestimated by more than a factor of 150 at the end of the rotation curve.

The inversion procedure depends not only on the measured rotation curve, but also on its derivative. Binney & Tremaine (1987) have suggested that the noise in real data will make the technique

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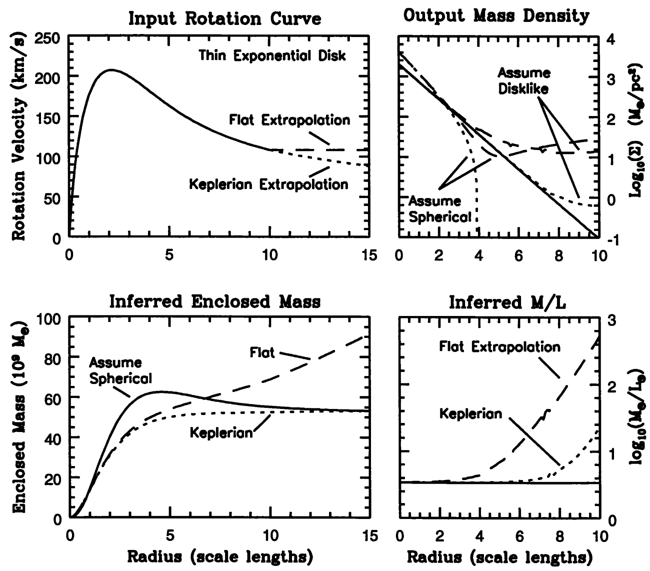

Figure 1—The input rotation curve $v_c(R)$ for a thin exponential disk (upper left) is inverted to produce a surface mass column $\Sigma(R)$ (upper right), enclosed mass $M(<R)$ (lower left), and local mass-to-light ratio M/L (lower right). Two extrapolation schemes are used for $v_c(R)$ beyond 10 scale lengths: flat (long dash) and Keplerian (dotted).

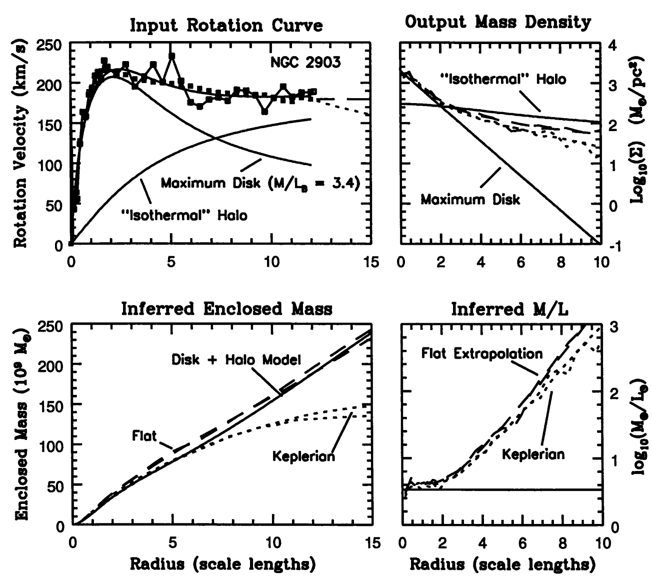

Figure 2—Same as Figure 1, but now inversion is performed under the assumption of disk geometry for the H I rotation curve for NGC 2903 (solid squares) and the same data to which artificial Gaussian noise has been added (open squares). For each case both flat and Keplerian extrapolations are shown. Also shown is a maximum-disk fit using a spherical, isothermal halo and a luminous disk with $M/L_B = 3{\cdot}4$; the $v_c(R)$ and $\Sigma(R)$ of the resulting model halo and disk are shown (solid lines) in the top panels.

unreliable, but as Figure 2 demonstrates, this effect is much smaller than the uncertainties introduced by the extrapolation procedure. For this example, $\sigma = 10\ \mathrm{km\,s^{-1}}$ Gaussian noise has been added to the rotation curve of NGC 2903 (Begeman 1987), with the result that the derived $\Sigma(R)$ is noisier, but suffers no systematic offset. As for the exponential disk, inversion of this more realistic rotation curve demonstrates that the flat extrapolation begins to differ noticeably from a Keplerian one at half the

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kinematic radius; by the end of the rotation curve, the two differ by a factor of 3 in $\Sigma(R)$, and by about 35% in the enclosed mass.

For comparison, a traditional maximum-disk fit has been done for NGC 2903 and is also displayed in Figure 2. Not surprisingly, the mass column of the resulting disk component, which has an $M/L_{\rm B} \approx 3{\cdot}4$, agrees well with that derived from the inversion technique in the inner regions. In the outer half of the galaxy, the model isothermal halo (with a core) has a $\Sigma(R)$ that is indistinguishable from an exponential disk with scale length equal to that inferred from inversion with flat extrapolation. Since the model halo is assumed to be spherical, however, the mass normalisation must be larger by the expected $\sim\pi/2$ to produce the same rotation curve. (Note that because the enclosed mass is defined within a sphere of radius r rather than within a cylinder of radius R, the inverted rotation curve with flat extrapolation and the maximum-disk + isothermal-halo fit produce similar enclosed masses but quite different projected surface mass densities.) The Keplerian extrapolation produces a somewhat shorter outer scale length for the total mass column than either the isothermal-halo fit or the flat extrapolation.

3 What Have We Learned?

The results of the previous section can be summarised as follows:

- For some disk geometries (e.g. an exponential disk), $v_{\rm c}$ can fall faster than $1/\sqrt{R}$ and still be physical. An unambiguous 'faster-than-Keplerian decline' in $v_{\rm c}$ is thus a sufficient, but not necessary, signature of a flattened geometry for the total mass of a galaxy.
- Kinematic noise at the 5–10% level has a very small statistical effect and no systematic effect on mass estimates derived from rotation curve inversion.
- Kinematic extrapolations beginning at R affect mass estimates beginning at about $R/2$. Interior to the last measured point, the enclosed mass is uncertain by $\sim$30% and surface mass density by as much a factor of 3 for flattened geometries; the certainties of both estimates deteriorate very rapidly beyond the measured kinematics.
- Mass estimates from rotation curve fitting are subject to these same uncertainties beginning at half the kinematic radius since, by assuming a functional form for the radial mass distribution, they have implicitly adopted an extrapolation for the kinematics of the galaxy.
- Over the radial range of H I rotation curves, isothermal halos have exponential surface mass densities with scale lengths of a few times the optical scale length. Rotation curve kinematics are thus equally consistent with exponential and isothermal dark matter distributions.

A fuller description of the inversion technique and its advantages, as well as its application to a sample of well-defined rotation curves, will appear elsewhere (Sackett, in preparation).

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