CORRECTIONS TO SOME TERMS OF NUTATION DEDUCED FROM THE PARIS ASTROLABE OBSERVATIONS

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Introduction

Any error in the position of the Earth's axis of rotation in space gives an error in the equatorial coordinates of stars used in the reduction of latitude and time observations and consequently gives an error in the results of these observations. This may be due to an error in the classical terms of nutation or to the neglected nutation in space produced by the theoretically predicted nearly-diurnal wobble (Hough, 1895; Poincaré, 1910; Jeffreys, 1949; Molodensky, 1961; Toomre, 1974). A preliminary study of these errors in latitude and time observations was reported in a preceding paper (Capitaine, 1975), which was subsequently corrected for a minor sign error (Capitaine, 1977).

We use here the latitude and time group-differences of the Paris astrolabe observations from 1956.6 to 1976.8 in order to evaluate the errors in the largest terms of nutation and to give a limit for the amplitude of a possible nearly-diurnal wobble.

I - Formulation of the effect of an error in any nutation in space on latitude and time observations

Let Γ_i and τ_i be respectively the frequency and the phase of a nutation, i, of the Earth's axis of rotation in space. This nutation can be expressed by its corresponding variations in longitude ψ (directed positively in the trigonometric sense) and obliquity ε.

If N_i and O_i are the corrections that must be added to the coefficients $\mathcal{N}_i$ and $\mathcal{O}_i$ used in the computations for this nutation, i, the corrections to be added to the corresponding variations are then:

$$\sin\varepsilon \cdot \Delta_i\psi = N_i \sin[\Gamma_i(t-t_o) - \tau_i] \qquad (1)$$
$$\Delta_i\varepsilon = O_i \cos[\Gamma_i(t-t_o) - \tau_i] \quad ,$$

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where t_o is the origin of time t. The corresponding corrections to be added to the computed values ϕ_H and C_H of latitude and time given by the observation of the group H of stars are (Capitaine, 1975;1977):

$$\Delta_i \phi = -N_i \sin[\Gamma_i(t-t_o)-\tau_i]\cos T_H + O_i \cos[\Gamma_i(t-t_o)-\tau_i]\sin T_H \tag{2}$$

$$\Delta_i C = -[N_i \operatorname{cotg} \varepsilon + N_i \operatorname{tg} \phi_o \sin T_H]\sin[\Gamma_i(t-t_o)-\tau_i] - O_i \operatorname{tg} \phi_o \cos[\Gamma_i(t-t_o)-\tau_i]\cos T_H \quad .$$

Here ϕ_o is the latitude of the observing instrument and T_H the local sidereal time of the group H.

The group-differences between the values given by two consecutive groups H and H+1 are, if we consider only the errors in the nutation i in space:

$$\phi_H - \phi_{H+1} = N_1(\cos T_H - \cos T_{H+1})\sin[\Gamma_i(t-t_o) - \tau_i] - O_i(\sin T_H - \sin T_{H+1})\cos[\Gamma_i(t-t_o) - \tau_i]$$

$$(3)$$

$$(C_H - C_{H+1})/\operatorname{tg} \phi_o = N_i(\sin T_H - \sin T_{H+1})\sin[\Gamma_i(t-t_o) - \tau_i] + O_i(\cos T_H - \cos T_{H+1})\cos[\Gamma_i(t-t_o) - \tau_i] \quad .$$

If we put $A_i = (N_i+O_i)/2$, $A_{-i} = (N_i-O_i)/2$, $T_{H,H+1} = (T_H+T_{H+1})/2$, the above group-differences can also be written:

$$\phi_H - \phi_{H+1} = \sin[(T_{H+1} - T_H)/2]\{A_i \cos[\Gamma_i(t-t_o) - T_{H,H+1} - \tau_i] + A_{-i} \cos[\Gamma_i(t-t_o) + T_{H,H+1} - \tau_i]\}$$

$$(4)$$

$$(C_H - C_{H+1})/\operatorname{tg} \phi_o = \sin[(T_{H+1} - T_H)/2]\{-A_i \sin[\Gamma_i(t-t_o) - T_{H,H+1} - \tau_i] + A_{-i} \sin[\Gamma_i(t-t_o) + T_{H,H+1} - \tau_i]\}$$

$T_{H+1} - T_H = 2$ hours, so $\sin[(T_{H+1} - T_H)/2] = 0.26 = \alpha$.

We see that an error in the coefficients of the nutation i appears in latitude and time group-differences as two terms of respective amplitudes αA_i and αA_{-i} and arguments $[\Gamma_i(t-t_o) - T_{H,H+1} - \tau_i]$ and $[\Gamma_i(t-t_o) + T_{H,H+1} - \tau_i]$. As the observations are always made in the middle of the night, the periods of these two terms are respectively:

$$P_{3,i} = (P'_{2,i} \times 365.25)/(365.25 - P'_{2,i})\text{m.d.}$$

$$P_{3,-i} = (P'_{2,i} \times 365.25)/(365.25 + P'_{2,i})\text{m.d.} \tag{5}$$

if $P'_{2,i} = 2\Pi/\Gamma_i$ is the period in mean days of the nutation i.

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Each elliptic nutation i in space, of period $P_{2,i}$ sidereal days and coefficients N_i and O_i, is the sum of two circular nutations of respective periods $P_{2,i}$ s.d. and $P_{2,-i}$ s.d. = $-P_{2,i}$ s.d. and respective amplitudes A_i and A_{-i}. To each circular nutation in space of period $P_{2,i}$ corresponds a wobble of the Earth of period $P_{o,i} = [P_{2,i}/(1-P_{2,i})]$ s.d. For the particular case of the nutation i_o in space corresponding to a retrograde nearly-diurnal wobble of amplitude γi_o and period $P_{o,i_o} = (1/k_{i_o})$ s.d. (with k_{i_o} nearly equal to -1), we have (Capitaine, 1975):

$$P_{2,i_o} = [1/(1+k_{i_o})] \text{ s.d.}, \; N_{i_o} = O_{i_o} = k_{i_o}\gamma_{i_o}/(1+k_{i_o}), \; A_{-i_o} = 0 \; . \qquad (7)$$

We give in Table 1 the periods $P'_{2,i}$, $P_{3,i}$, $P_{3,-i}$ corresponding to several periods $P_{o,i}$ in order to see at which period the effects of an error in the largest terms of nutation or of a neglected nutation in space corresponding to the nearly-diurnal wobble can appear by different methods of analysis.

II - Periods and amplitudes of the greatest observed effects appearing in the Paris astrolabe group-differences

We have used latitude and time group-differences obtained from all the Paris astrolabe observations of consecutive groups of stars from 1956.6 to 1975.8, that is, 3 300 values of each kind. These differences are corrected by group-difference corrections and their secular variations computed for the considered interval.

We have looked for possible effect of any nutation in space in these group-differences in two different ways. We have first computed corrections N_i and O_i for each period $P'_{2,i}$ between 100 days and 20 years by a least-squares fit of the expressions (3) to the data, t being the mean date of observation of the two groups.

Table 2 gives the greatest coefficients found with the corresponding period $P'_{2,i}$. We notice the terms at periods $P'_{2,i}$ = 106d, 182d, 220d, 284d, 365d, 520d, 3.2y, 19y appearing in the two kinds of group-differences. All the coefficients found are lower than 0".030 and consequently we can say that there is no error in the representation of the nutation in space larger than 0".030. We can thus deduce from (7) that the amplitude of any possible nearly-diurnal wobble is lower than 0".030/100.

We have secondly looked for all possible periods $P_{3,i}$, $P_{3,-i}$ appearing in spectral analysis of the data by aliasing of periods $P_{2,i}$. These data were first smoothed by Vondrak's method (Vondrak, 1969) and then interpolated to obtain a point every five days. These points were analysed by Blackmann and Tukey's method (Blackmann and Tukey, 1958). The corresponding spectra smoothed by the "hanning" spectral window are given in Figures 1 and 2.

Table 1. Values of periods $P_{o,i}$, $P'_{2,i}$, $P_{3,i}$, $P_{3,-i}$ for various values of i.

	$P_{o,i}$ in sidereal hours	$P'_{2,i}$ in mean days	$P_{3,i}$ in mean days	$P_{3,-i}$ in mean days
	-24.2650	91.31	121.75	73.05
	-24.2328	103.79	145.00	80.83
	-24.2294	105.32	148.00	81.75
	-24.2200	109.80	157.00	84.42
	-24.2180	110.78	159.01	85.00
	-24.2098	115.07	168.00	87.50
	-24.2040	118.31	175.00	89.36
	-24.2021	119.43	177.45	90.00
	-24.2018	119.61	177.85	90.10
π_i	-24.1982	121.75	182.63	91.31
	-24.1898	127.13	195.00	94.30
	-24.1879	128.40	198.00	95.00
	-24.1866	129.23	200.00	95.46
	-24.1771	136.12	217.00	99.17
	-24.1727	139.60	225.98	101.00
	-24.1703	141.50	231.00	101.99
	-24.1635	147.36	247.04	105.00
	-24.1504	160.09	285.00	111.30
	-24.1491	161.53	289.62	112.00
	-24.1476	163.19	295.00	112.79
	-24.1434	167.85	310.58	115.00
	-24.1389	173.31	329.80	117.54
	-24.1353	177.84	346.62	119.61
P_1	-24.1318	182.62	365.24	121.75
	-24.1318	182.63	365.28	121.75
	-24.1314	183.19	367.53	122.00
	-24.1282	187.67	386.00	123.97
	-24.1181	203.59	460.00	130.72
	-24.1170	205.53	470.00	131.52
	-24.1168	205.89	471.92	131.67
	-24.1136	211.65	503.30	134.00
	-24.1133	212.32	507.12	134.27
	-24.1097	219.23	548.41	137.00
	-24.1016	236.51	671.00	143.55
	-24.0657	365.22	∞	182.62
S_1	-24.0657	365.26	∞	182.63
	-24.0622	386.00	-6792.31	187.67
	-24.0183	1305.47	-507.12	285.40
	-24.0070	3399.19	-409.21	329.80
N_p	-24.0035	6798.37	-385.98	346.62
N_p	-23.9965	-6798.34	-346.62	385.98
	-23.9930	-3399.18	-329.80	409.21
ψ_1	-23.9347	-365.26	-182.63	∞
	-23.9312	-346.64	-177.85	-6804.38
	-23.8843	-205.89	-131.67	-471.92
	-23.8808	-199.84	-129.17	-441.29
	-23.8697	-182.63	-121.75	-365.28
ϕ_1	-23.8697	-182.62	-121.75	-365.24
	-23.8662	-177.84	-119.61	-346.62
	-23.8627	-173.31	-117.54	-329.80
Σ_1	-23.8050	-121.75	-91.31	-182.63
	-23.7853	-110.48	-84.82	-158.39
	-23.7743	-105.03	-81.57	-147.43
	-23.7624	-99.73	-78.34	-137.18
	-23.7400	-91.06	-72.89	-121.30

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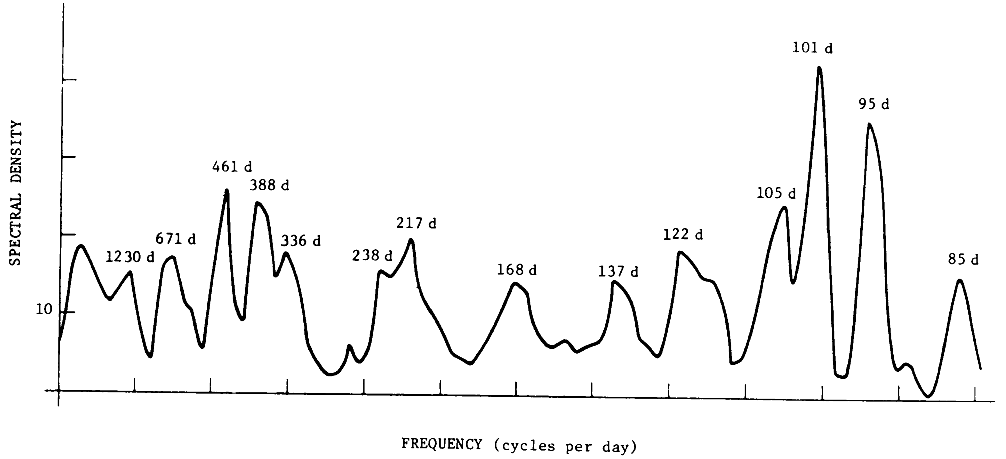

Figure 1. Smoothed spectrum of latitude group-differences.

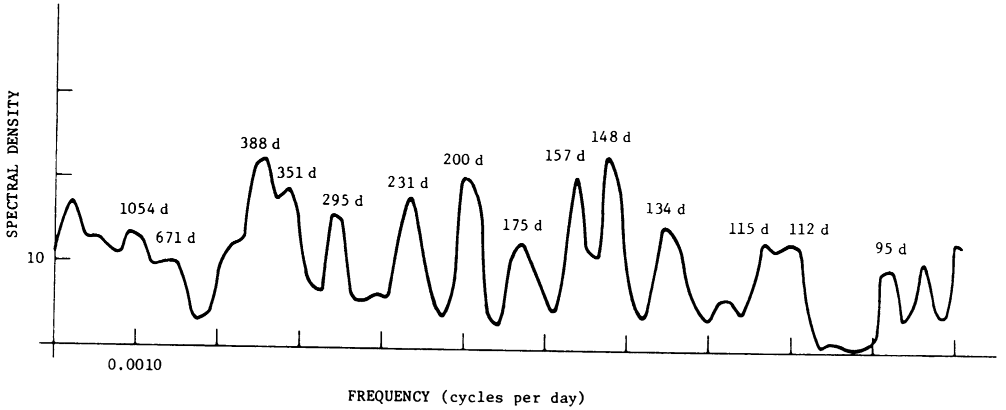

Figure 2. Smoothed spectrum of time group-differences.

We notice the peaks at 95d, 101d, 123d, 137d, 167d, 217d, 238d, 336d, 388d, 461d, 671d in latitude and 91d, 95d, 111d, 114d, 133d, 145d, 178d, 195d, 228d, 285d, 351d, 388d, 470d, 666d in time. Table 1 shows that some of them (95d, 123d, 137d, 145d, 220d, 285d, 340d, 390d, 470d, 666d) confirm the effects found by the preceding method.

Table 2. Periods and coefficients of the largest terms of type (3) appearing in latitude and time group-differences.

Latitude			Time		
$P'_{2,i}$ in mean days	N_i in 10^{-3}"	O_i in 10^{-3}"	$P'_{2,i}$ in mean days	N_i in 10^{-3}"	O_i in 10^{-3}"
106	9	15	105	16	12
112	10	8	109	13	13
148	12	7	116	13	12
181	10	14	118	18	12
190	8	11	138	17	11
210	11	10	152	17	5
220	15	4	160	18	15
268	11	8	168	12	8
284	12	11	178	18	16
320	2	20	182	14	16
338	7	10	191	13	21
365	20	18	200	9	24
430	10	12	216	17	1
515	10	7	224	16	8
577	9	13	248	12	6
796	1	17	284	21	18
1169	10	6	338	9	14
6797	2	16	365	15	14
			396	15	18
			433	14	6
			475	10	13
			522	7	25
			573	19	2
			829	8	15
			1169	10	7
			2082	11	17
			3653	7	20
			6797	5	28

III - Determination of the corrections to some terms of nutation

The theoretical representation of the nutation in space for terms larger than 0".005 is given by:

$$\begin{aligned} \sin\varepsilon\, D\psi &= N_1 \sin\Omega + N_2 \sin 2\Omega + N_3 \sin 2L + N_4 \sin(L-\pi) \\ &\quad + N_5 \sin(L+\pi) + N_6 \sin(3L-\pi) + N_7 \sin(2L-\Omega) \\ D\varepsilon &= O_1 \cos\Omega + O_2 \cos 2\Omega + O_3 \cos 2L + O_4 \cos(L-\pi) \\ &\quad + O_5 \cos(L+\pi) + O_6 \cos(3L-\pi) + O_7 \cos(2L-\Omega) \end{aligned} \tag{8}$$

where Ω is the longitude of the ascending node of the moon, L and π are the mean tropic longitude respectively of the sun and of the perigee of the sun and $(\mathcal{N}_i, \mathcal{O}_i)_{i=1,7}$ the conventional values for the corresponding coefficients of nutation. The expressions (3) were fitted by the least-squares method in the 3 300 latitude group-differences and 3 300 time group-differences for each argument $(\Gamma_i)_{i=1,7}$ such that:

$$\Gamma_1(t-t_o) = \Omega,\ \Gamma_2(t-t_o) = 2\Omega,\ \Gamma_3(t-t_o) = 2L,\ \Gamma_4(t-t_o) = L-\pi,$$

$$\Gamma_5(t-t_o) = L+\pi,\ \Gamma_6(t-t_o) = 3L-\pi,\ \Gamma_7(t-t_o) = 2L-\Omega.$$

The corrections of type (1) to be added to the variations in longitude and obliquity due to the nutation in space are obtained in the form:

$$\sin \varepsilon\ \Delta\psi = \sum_{i=1,7} \{N_i \sin \Gamma_i(t-t_o) + M_i \cos \Gamma_i(t-t_o)\}$$

$$\Delta\varepsilon = \sum_{i=1,7} \{O_i \cos \Gamma_i(t-t_o) + P_i \sin \Gamma_i(t-t_o)\} \quad . \qquad (9)$$

Table 3 gives the corrections $(N_i, O_i)_{i=1,7}$ to be added to the conventional coefficients $(\mathcal{N}_i, \mathcal{O}_i)_{i=1,7}$ and the terms $(M_i, P_i)_{i=1,7}$ (due to a possible phase-lag) obtained for latitude and time separately. Fitting the sum of the seven terms of arguments $(\Gamma_i)_{i=1,7}$ gave nearly the same results.

We see that significant corrections are obtained for the 18.6 yearly and semi-annual nutations. The final values obtained for these two nutations are respectively, from latitude and time group-differences:

$\mathcal{N}_1 = 6''842 \pm 0''006$, $\mathcal{N}_1 = 6''831 \pm 0''008$

$\mathcal{O}_1 = 9''211 \pm 0''006$, $\mathcal{O}_1 = 9''205 \pm 0''008$

$\mathcal{N}_3 = 0''522 \pm 0''006$, $\mathcal{N}_3 = 0''522 \pm 0''008$

$\mathcal{O}_3 = 0''558 \pm 0''006$, $\mathcal{O}_3 = 0''542 \pm 0''008$

Conclusion

This work is based upon 3 300 latitude group-differences and 3 300 time group-differences of Paris astrolabe observations during 20 years. It shows that there is no error in the theoretical representation of the nutation in space larger than 0''030 and consequently that the amplitude of a possible nearly-diurnal wobble of the Earth is lower than 0''0003. It also gives computed corrections to the largest conventional coefficients of nutation.

Table 3. Corrections to some conventional coefficients of the nutation in space deduced from latitude group-differences and from time group-differences

Argument	$P'_{2,i}$ in years	P_i in 10^{-3}"	O_i in 10^{-3}"	M_i in 10^{-3}"	N_i in 10^{-3}"
		from latitude group-differences			
Ω	-18.6129	-2±6	1±6	2±6	-16±6
2Ω	-9.3065	-5±5	-3±6	4±6	-0±6
2L	0.5000	8±6	7±6	-5±6	16±6
L-π	1.0000	-16±9	-13±7	-13±8	-13±7
L+π	0.9999	-17±9	12±7	-16±8	9±7
3L-π	0.3333	-1±6	-6±6	-5±6	9±6
2L-Ω	0.4869	-3±6	1±6	12±6	-5±6
		from time group-differences			
Ω	-18.6129	-0±8	-5±8	-8±8	-27±8
2Ω	-9.3065	5±8	1±9	14±9	14±7
2L	0.5000	10±8	-8±8	2±7	16±8
L-π	1.0000	11±10	-11±11	-12±10	8±11
L+π	0.9999	-7±9	-13±11	4±9	14±12
3L-π	0.3333	-0±8	3±8	2±8	-3±8
2L-Ω	0.4869	4±8	-17±8	-10±8	12±8

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