Fermat, Descartes, Wallis, Barrow, Gregory and others. Following the Chapter on "Newton and Leibniz" are one on the 18th century ("Period of Indecision") and one on the 19th century ("Rigorous Formulation"), which together contain comments on an equally impressive list of names. The conclusion summarizes both the ideas which have proven fruitful and those which have hampered the development. But, as the author emphasizes: "Each [notation] is to be considered in the light of the mathematical and scientific milieu of the period in which it appeared."

To prevent misunderstandings let it be said again that the book under review is not a history of all aspects and implications of the calculus. Within its restrictions to the fundamental concepts of derivative and integral it presents a clear and painstaking examination of the development basic to all further extensions and additions. It can be highly recommended to every student of mathematics, for he will find in it what is rarely combined to such a degree in a book on the history of mathematics: historical information and clarification of important basic concepts.

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<u>Modern Probability Theory and its Applications</u>, by E. Parzen. John Wiley and Sons, Inc., New York and London. 464 pages. $10.75.

This pleasing book contains a large variety of theoretical and applied results in probability. Nevertheless it is so readable that one is surprised, when one has finished it, to realize how much has been covered. The book starts with elementary set theory and occupancy problems, and ends with inversion theorems for characteristic functions, and Lyapunov's condition for the central limit theorem. Obviously many students could read the beginning who could not get through to the end, but the plan of the book is that elementary calculus should be all that one needs until near the end, and more advanced techniques are only used in the last two chapters.

The plan is good but there may be argument about the consistency and success with which it is carried out. The author introduces probability in a continuum as early as possible. This greatly enriches the collection of applied problems, but it means that he must face well-known difficulties in developing the theory, and he has tackled these with ingenuity: In Chapter 4 he introduces "Numerical-Valued Random Phenomena" but apart from a brief reference and a provisional definition he defers the notion of a random variable for another hundred pages. From Chapter 4 onwards, however, he presents many interesting results and problems on continuous distributions and this is where his difficulties begin. His development of the theory involves mentioning

Borel sets, and phrases like "it may be shown that ...." appear several times in this and later chapters. The reviewer fears that some people may be put off by these remarks about topics which are not fully treated, and by the fact that random variables are not defined until quite late in the book. However, most of these remarks can be ignored by a reader who is prepared to accept some results as intuitive, and also much basic work on probability laws is done early in the book in spite of the late introduction of the name "random variable".

The reviewer has some minor criticisms. In the proof of the normal approximation to the binomial law the author considers the number X of successes in n independent repeated Bernoulli trials with probability p of success at each trial. He works in terms of the normalized sums h given by $h = (X - np)(npq)^{-1/2}$, without stating whether or not h is bounded, as $n \to \infty$. In fact one can allow $h \to \infty$ provided that $h^3 n^{-1/2} \to 0$, without affecting this proof, while for larger h one could use the known "probabilities of large deviations" or "tail" estimates.

The numbering of exercises and formulae is somewhat repetitive. For example it is convenient to use equation (4.5) of page 62 to do "theoretical exercise" 4.5 on page 65; and then on page 66, one finds "exercise" 4.5 still belonging to the same section. The conditions for the maximal term of the binomial distribution in "theoretical exercise" 3.4 of page 109 are rather awkwardly stated. On page 230 the author indicates that the law of large numbers justifies the use of the observed relative frequency f_n of successes, as an estimate of p, in a system of Bernoulli trials. He then remarks rather boldly that "this estimate becomes perfectly correct as the number of trials becomes infinitely large".

In conclusion the reviewer would like to repeat that this is an entertaining and stimulating book which can be used with selection by a class of limited background, and which should be very rewarding for more advanced students. For the advanced students, the early introduction of simple versions of topics like Markov chains, and the limit theorems of the last two chapters, are especially attractive.

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