Locally finite near-fields

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A near-field is *locally finite* if every finite subset of it generates a finite sub-near-field.

The main aim of this thesis is to give a coherent account of locally finite near-fields, including finite ones. The well known results for finite near-fields are listed and proofs are given where appropriate. The results of Zassenhaus [4] classify finite regular near-fields according to their order, p^{ln} , and the order of their centres, p^{l} , and Lüneburg [3] has determined the number of isomorphism types within each class. A polynomial h is given here which, together with the triple p, l, n , completely determines a finite regular near-field, up to isomorphism. The sub-near-field structure is determined in terms of these invariants and some results concerning near-field embeddings are included.

A classification of locally finite near-fields, similar to that for finite near-fields, is obtained. All locally finite near-fields (with the exception of those belonging to the seven finite irregular isomorphism types) are shown to be regular. Invariants which determine a locally finite near-field up to isomorphism are obtained - these include the invariants for finite near-fields as a special case - and the automorphism group of a locally finite near-field is determined.

Finally, a brief discussion of some possible problems is included and some information is obtained about near-fields whose multiplicative groups satisfy certain conditions.

References

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