

THE APPLICATION OF HIGH ANGULAR RESOLUTION STELLAR INTERFEROMETRY TO THE  
STUDY OF SINGLE OBJECTS IN THE VISUAL REGION OF THE SPECTRUM

John Davis

Astronomy Department, School of Physics, University of Sydney, Australia.

ABSTRACT

A brief review of existing angular diameter measurements of single objects is given. Potential astronomical programmes for high angular resolution stellar interferometry in the visual region of the spectrum are discussed with particular regard to the sensitivity, resolution, and accuracy required for significant astrophysical results to be obtained. This discussion is used as a basis for suggesting the minimum requirements for future high angular resolution stellar interferometers.

1. INTRODUCTION

The application of high angular resolution measurement techniques to astronomy has enormous potential for the determination of fundamental data and it is the purpose of the first three papers of this Colloquium to review the possibilities with the exception of astrometry. The two following papers will deal with the potential of observations in the infrared region of the spectrum and of binary systems. As the title of this review indicates, the discussion here will be restricted to the astronomical potential of high angular resolution observations of single objects in the visual region of the spectrum.

2. THE ACHIEVEMENTS OF HIGH ANGULAR RESOLUTION STELLAR INTERFEROMETRY  
FOR SINGLE STARS

The achievements of high angular resolution stellar interferometry, while remarkable, have hardly scratched the surface of its astronomical potential. Nevertheless, they have given a clear indication of that potential, and it is appropriate to review briefly the history and current

status of angular diameter measurements of single stars.

## 2.1 Angular Diameter Measurements

The first determination of the angular diameter of a star was made by Michelson and Pease<sup>1</sup> when they measured  $\alpha$  Ori using a 20 ft Michelson interferometer mounted on the 100 inch telescope. Subsequently the angular diameters of several stars were obtained with the 20 ft instrument and the results for 7 stars were published by Pease<sup>2</sup>. The result for  $\alpha$  Her was preliminary and was omitted from a later list<sup>3</sup>. The measurement of  $\alpha$  Ori has an estimated uncertainty of  $\pm 10\%$ <sup>1</sup> but the remaining results are less certain and probably have a standard error in the range 10 to 20%<sup>4</sup>.

During the 1920's a 50 ft Michelson interferometer was constructed at Mt. Wilson<sup>2</sup> but it proved hard to operate and no final results were published.

Stellar interferometry came to a standstill with the failure of the 50 ft Michelson instrument and remained in that state until the intensity interferometer was developed in the 1950's. The measurement of  $\alpha$  CMa (Sirius) by Hanbury Brown and Twiss<sup>5</sup> was the first angular diameter determination for a main-sequence star and led to the design and construction of the stellar intensity interferometer at Narrabri Observatory<sup>6,7</sup>. This instrument, which was used to carry out an extensive observational programme from 1963 to 1972, has given a clear indication of the astrophysical potential of high angular resolution stellar interferometry. The achievements of this programme for single stars were:

- (i) The measurement of the angular diameters of 32 stars in the spectral range O5f to F8<sup>8</sup>.
- (ii) The carrying out of a number of exploratory experiments to establish the potential of the technique. The Narrabri instrument lacked the sensitivity to push these experiments to the level of astrophysical significance but they included:
  - (a) a measurement of the angular size of the envelope of gas surrounding the Wolf-Rayet component of  $\gamma^2$  Vel in the light of an emission line<sup>9</sup>

- (b) a study of the effects of limb darkening on angular diameter measurements with observations of  $\alpha$  CMa<sup>10</sup>
- (c) an attempt to detect an extended corona around  $\beta$  Ori by observations in two orthogonal planes of polarization with a view to setting an upper limit to mass loss<sup>11</sup>
- (d) a study of the effects of rapid rotation on the shape and brightness distribution across a star with observations of  $\alpha$  Aql<sup>7,12</sup>

The Narrabri intensity interferometer was limited to stars brighter than  $B = +2.5$  and of spectral types O-F inclusive. The accuracy of the angular diameter determinations depended primarily on brightness and averaged  $\pm 6.5\%$  for the 32 stars measured. Experience showed that there were a number of small systematic effects (errors in baseline orientation, changes in optical bandwidth with elevation angle, etc.) which individually gave rise to uncertainties of the order of 1% and which, taken together, limited the accuracy attainable to the order of  $\pm 2\%$ .

The technique of speckle interferometry developed by Labeyrie and his colleagues<sup>13</sup> is restricted in resolving power by available telescope apertures and has limited potential for single stars. Only 6 stars have been measured to better than  $\pm 15\%$ <sup>13-15</sup> although the equivalent uniform disc angular diameter for  $\alpha$  Ori has been determined to  $\pm 2\%$  by Lynds et al<sup>15</sup> The speckle technique also permits image reconstruction within the limited resolution available<sup>15</sup> and this aspect will be reviewed by Nisenson later in this meeting.

Currie and his colleagues<sup>16</sup> have reported a modern version of Michelson stellar interferometry which has produced repeatable angular diameter measurements. Results for 12 stars measured with their "amplitude" interferometer attached to the 100 inch and 200 inch telescopes have been reported<sup>17</sup>. The estimated total uncertainty of these measurements averages  $\pm 14.9\%$  with 9 better than  $\pm 15\%$  although the result for  $\alpha$  Her A<sup>18</sup> differs significantly from measures by other workers<sup>2,13</sup> Currie<sup>19</sup> reports that measurements have now been obtained for 19 stars with significant improvements in accuracy compared with the earlier list<sup>17</sup> Accuracies of the order of

$\pm 5\%$  have been achieved in some cases with  $\pm 3.4\%$  for  $\alpha$  Ori. So far this technique has been restricted by available telescope apertures and, like the speckle technique, has only resolved cool giant and supergiant stars.

It is the extension of the University of Maryland "amplitude" interferometer to a two telescope instrument and the work of Labeyrie<sup>20,21</sup> in developing a large aperture two-telescope Michelson interferometer that promise to contribute to high angular resolution stellar interferometry.

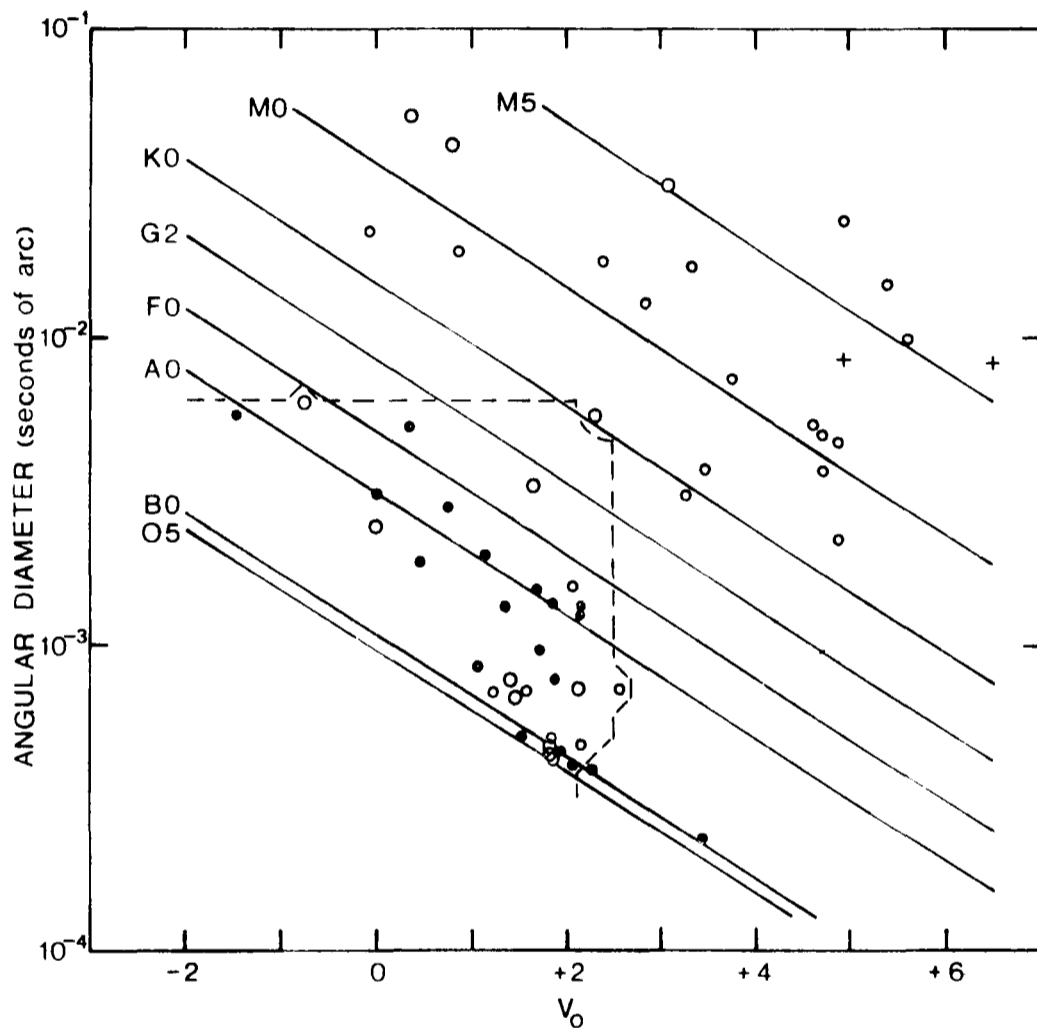


Figure 1. Measured stellar angular diameters (accuracy better than  $\pm 15\%$ ) as a function of unreddened magnitude  $V_0$ . Luminosity classes are represented by: ● = IV and V; ○ = III; ⊙ = I and II; + = type C stars. The diagonal lines represent predicted mean relationships for the spectral types indicated on the diagram. Stars below and to the left of the broken line were measured with the Narrabri intensity interferometer<sup>7</sup>.

Figure 1 shows stars whose angular diameters have been determined with an accuracy better than  $\pm 15\%$  and includes the 32 early-type stars measured at Narrabri<sup>8</sup>, 2 early-type stars measured during an occultation by Jupiter<sup>22</sup> and 22 cool giant or supergiant stars measured with Michelson's interferometer<sup>17</sup>, by speckle interferometry<sup>13-15</sup> or by lunar occultations. In the case of lunar occultations the original works were consulted using the recent compilation by Barnes et al<sup>23</sup> as a guide. Figure 1 illustrates the distribution of stars by luminosity class and spectral type - luminosity classes IV and V have been represented by a single symbol, as have classes I and II, for the sole purpose of keeping the diagram simple.

The stars in Figure 1 have angular diameters determined to better than  $\pm 15\%$  and it is worth noting that some 86% have been measured to better than  $\pm 10\%$  but less than 40% to better than  $\pm 5\%$ . These figures appear to apply throughout the diagram. For example, considering only angular diameters measured to better than  $\pm 15\%$ , the relative percentages of values with published accuracies better than  $\pm 10\%$  and  $\pm 5\%$  are much the same for early-type stars and for late-type stars.

## 2.2 Applications of Angular Diameter Measurements

The applications of angular diameter measurements will be discussed in some detail in Section 3 in the context of their astronomical potential. However, some examples of important applications made with existing measurements will be described here.

The most direct application is in the determination of the absolute emergent flux at a stellar surface  $F_{\nu}$  by combining an angular diameter measurement with the absolute flux received from a star  $f_{\nu}$ . The comparison of absolute surface flux distributions determined in this way with the predicted flux distributions for theoretical model stellar atmospheres<sup>24,25</sup> has revealed deficiencies in the predictions and led to improvements in the theoretical models.

Code et al<sup>26</sup> have combined the angular diameters of the 32 early-type stars measured at Narrabri<sup>8</sup> with observations of ultraviolet flux from the Orbiting Astronomical Observatory (OAO-2) and ground-based photometry to determine absolute emergent fluxes at the stellar surfaces and, by integration of these

fluxes, empirical effective temperatures for these stars. The effective temperature scale for stars hotter than the Sun is based on these data. For 13 of the Narrabri stars Code et al<sup>26</sup> also determined radii and luminosities with the aid of the known parallaxes, thus allowing a fundamental H-R diagram of  $\log L - \log T_e$  to be plotted.

Some applications of angular diameter measurements have been highlighted by several workers<sup>23,27-29</sup> who have developed the idea of Wesselink<sup>30</sup> of establishing the relationship between a surface brightness parameter and a photometric index using measured angular diameters and photometry. The relationship is then used to predict angular diameters solely from photometric observations. The recent comprehensive study by Barnes et al<sup>23</sup> improved the relationship between their visual surface brightness parameter  $F_v$  and the  $(V-R)_o$  colour index by using a total of 76 angular diameters. These include those shown in Figure 1 plus an additional 20, which are mainly lunar occultation measurements with uncertainties  $> \pm 15\%$ . Barnes et al<sup>23</sup> assert that their  $F_v - (V-R)_o$  relation is independent of luminosity class and applicable to all spectral types O4-M8, S and C. This type of relation has been used in a number of astrophysical applications. For example, it has been used to determine absolute magnitudes and linear dimensions of main-sequence stars<sup>30,31</sup> to determine linear radii of white dwarfs<sup>32,33</sup> and to establish independent distance scales for novae<sup>34</sup>, eclipsing binaries<sup>35</sup>, and classical Cepheids<sup>36</sup>.

In the absence of directly measured angular diameters the surface brightness method has been shown to be of great value but it must be remembered that it is only as good as the angular diameter measurements on which it is based.

It is with this background of angular diameter measurements and their applications that the astronomical potential of high angular resolution stellar interferometry is to be considered.

### 3. THE ASTRONOMICAL POTENTIAL OF HIGH ANGULAR RESOLUTION STELLAR INTERFEROMETRY FOR SINGLE OBJECTS.

The astronomical potential of stellar interferometry clearly depends on the sensitivity and resolution that can be achieved. However, it is perhaps more stimulating to approach this from the astronomical viewpoint and, with

this in mind, potential programmes given in the following list will be discussed and their sensitivity and resolution requirements estimated.

### List of Potential Astronomical Programmes for Single Objects

- (i) Emergent fluxes and effective temperatures for single stars
- (ii) Stellar radii and luminosities
- (iii) Limb darkening
- (iv) Stellar rotation
- (v) Extended atmospheres of early-type stars
- (vi) Emission-line stars
- (vii) Interstellar extinction
- (viii) Cepheid variables
- (ix) Galactic nuclei and quasars

This programme list is not intended to suggest an order of importance nor to be exhaustive but rather to indicate the wide range and potential of stellar interferometry

### 3.1 Introductory Remarks to Discussion of Potential Astronomical Programmes

In order to determine the brightness distribution across an object uniquely it is necessary to measure both the amplitude and phase of the complex fringe visibility over a wide range of spacings. However, for most astronomical applications of stellar interferometry, it is reasonable to assume symmetry in the brightness distributions of objects under study, and measurements of the modulus of the fringe visibility alone suffice. In fact all interferometric determinations of angular diameters have been made by measurements of the modulus of fringe visibility or its equivalent whether by Michelson, intensity, speckle or "amplitude" interferometers.

Several of the programmes to be discussed have been studied with intensity interferometry specifically in mind and it is worth noting that an intensity interferometer measures a quantity proportional to the correlation factor  $\Gamma_{\lambda}^2(d)$ .  $\Gamma_{\lambda}^2(d)$  is proportional to the square of the modulus of the Fourier transform of the intensity distribution across the star reduced to an equivalent strip distribution parallel to the interferometer baseline. It is thus

equivalent to the square of the modulus of the fringe visibility. In the following discussions it will be assumed that the modulus of the fringe visibility (or its square) alone is to be measured unless the phase is explicitly mentioned.

For the purposes of the discussions the limiting sensitivity of an interferometer will be taken to be the visual magnitude  $V(\text{limit})$  for which a signal/noise ratio of 3 is achieved in 1 hour of integration. This limit is arbitrary but is adopted on the basis of experience with the Narrabri intensity interferometer<sup>7</sup>.

The stellar statistics quoted in the discussions are generally those applicable to an instrument sited at latitude  $30^\circ$  south (for observations at zenith angles less than  $60^\circ$ ) unless explicitly stated otherwise. The stellar statistics for an instrument sited at latitude  $30^\circ$  north do not differ significantly from those quoted.

### 3.2 The Potential Astronomical Programmes

#### (i) Emergent fluxes and effective temperatures for single stars

Our empirical knowledge of stellar emergent fluxes  $\mathcal{F}_v$  and effective temperatures  $T_e$  is limited to those stars for which angular diameters have been determined. Figure 1 shows the distribution by luminosity class and spectral type of stars whose angular diameters have been measured to better than  $\pm 15\%$  and it can be seen that the sample for any given spectral type is small. For main-sequence stars the situation is worse. Inspection of the data shown in Figure 1 reveals that there are no angular diameter determinations for main-sequence stars of spectral type later than F5 (exceptions which lie outside the diagram are the sun (G2V) and the eclipsing binaries YY Gem<sup>23,37</sup> (M0.5V) and CM Dra<sup>23,38</sup> (M4V). At the other end of the range of spectral types only one main-sequence O star has been measured ( $\zeta$  Oph, O9.5V)<sup>8</sup>. If the discussion was limited to data with accuracies better than say  $\pm 5\%$  the sample would be reduced by more than 60% and the coverage even further restricted. Thus a major programme would be to extend measurements to cover a large sample of stars so that  $\mathcal{F}_v$  and  $T_e$  may be found over as wide a range in spectral type as possible



and also as a function of luminosity, composition, spin, etc.

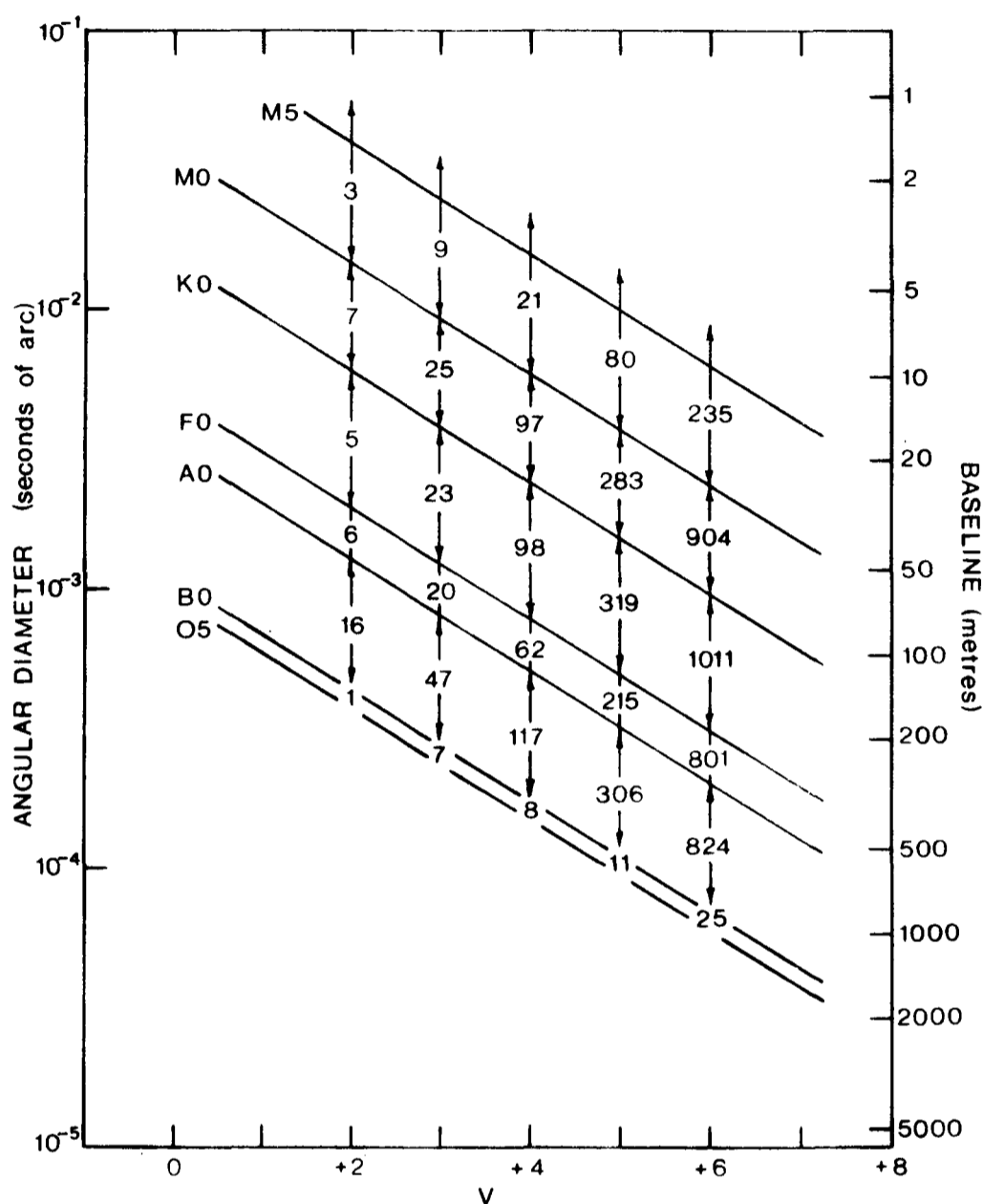


Figure 2. Distribution of stars with respect to apparent brightness and angular size. The diagonal lines represent mean relationships for the spectral types shown and the figures give the total number of stars in the Catalogue of Bright Stars<sup>39</sup> brighter than a given V magnitude with declination  $< +30^\circ$  for each spectral group (e.g. there are 215 A-type stars with  $V < +5$ ). The ordinate on the right-hand side of the diagram gives the minimum baseline required for resolution ( $\Gamma_\lambda^2(d) = 0.5$ ) at a wavelength of 550nm.

Inspection of Figure 2 suggests that an instrument capable of measuring stars with  $V = +4$  would enable a large sample of stars to be observed and this is true for the middle range of the spectral distribution. Closer

examination of the statistics reveals that for early O and for M main-sequence stars a greater sensitivity is required. For example, to measure hot O stars it is necessary to reach at least  $V = +6$  and, in order to obtain a reasonable sample,  $V(\text{limit})$  should exceed  $+7$ . For main-sequence stars of spectral type M0 and later it is also necessary to have  $V(\text{limit}) > +7$ . In fact, the  $V(\text{limit})$  needed to measure main-sequence stars increases towards later spectral types and at M5  $V(\text{limit})$  must exceed  $+9.5$ .

Figure 2 shows that an aperture separation, or baseline, of 200–300 m would be adequate to resolve all except the hottest stars, but for these, baselines in excess of 1 km are required.

An accuracy of  $\pm 5\%$  has already been achieved in angular diameter measurements and this should be regarded as the minimum acceptable in a future instrument. Astrophysicists would like  $\pm 1\%$ <sup>40</sup> and this should be the target.

#### (ii) Stellar radii and luminosities

Apart from interferometry, our direct knowledge of stellar radii comes from eclipsing double-lined spectroscopic binaries and the sun. If only "reliable" systems<sup>41</sup> are considered results are currently available for some 35 binary systems<sup>42</sup>.

An interferometer can provide measurements of stellar radii in two ways. First, measurements of the angular diameter of a star of known parallax allows the radius of the star to be calculated. Second, for suitable double-lined spectroscopic binaries both the distance to the binary and the angular size of at least the primary can be found and hence the radius of the primary can be calculated. The case of binary stars will be discussed by McAlister. As an example of the first case, Code et al<sup>26</sup> have determined radii for 12 stars of reliably known trigonometric parallaxes whose angular diameters had been measured with the Narrabri intensity interferometer<sup>8</sup>. Luminosities were also determined by combining these radii with empirical effective temperatures. A  $\log L - \log T_e$  diagram based directly on observations could then be plotted. This fundamental H-R diagram can be compared directly with the predictions of theoretical model stellar interiors.

The spectral types of the 12 stars of Code et al<sup>26</sup> range from B3 to F5 but only for 8 stars of spectral types A0 and later were the trigonometric parallaxes known to better than  $\pm 20\%$ . In fact, it is generally true that radii for stars of known trigonometric parallax can only be obtained with an accuracy better than  $\pm 20\%$  for spectral types A0 and later.

Table 1 has been derived from the list of stars with trigonometric parallaxes greater than  $0''.050$  compiled by Stoy<sup>43</sup>.

TABLE 1

Statistics of stars brighter than  $V = +10$  with  
trigonometric parallaxes greater than  $0''.050$

R.M.S. Accuracy of Trigonometric Parallax	Number of Stars	
	For Declinations South of $+30^\circ$	For Declinations North of $-30^\circ$
$\sigma < \pm 5\%$	11	13
$\sigma < \pm 10\%$	46	52
$\sigma < \pm 20\%$	118	111

All the stars in Table 1 are of spectral type A0 and later. From these data it is concluded that fundamental radius determinations for a significant sample of stars of spectral type A0 and later are possible by means of high angular resolution stellar interferometry. The instrumental requirements are similar to those outlined in (i).

In addition to radii for stars of known trigonometric parallax, radii can be determined via interferometry by measuring the angular sizes of stars in clusters such as the Hyades, Sco-Cen association, etc and combining them with the cluster parallaxes. Although not as precise as the best values from trigonometric parallaxes and binaries such measurements would provide valuable data on stellar radii.

(iii) Limb darkening

The response of an interferometer to a star at any given baseline depends not only on the angular diameter but also on the limb darkening of the star. Hanbury Brown et al<sup>10</sup> have investigated the effects of limb darkening on measurements of angular size and have shown that for a whole family of limb-darkening laws the curves of the correlation factor  $\Gamma_{\lambda}^2(d)$  (Section 3.1) are, for all practical purposes, indistinguishable in shape. The principal effect of limb darkening is to change the scale, but not the shape, of the  $\Gamma_{\lambda}^2(d)$  curve out to its first minimum. The degree of limb darkening does affect the height of the first secondary maximum of the  $\Gamma_{\lambda}^2(d)$  curve but, as shown by Hanbury Brown et al<sup>10</sup>, any given curve can be matched at all baselines to better than 1 part in  $10^3$  of  $\Gamma_{\lambda}^2(0)$  over a wide range of limb-darkening parameters. Thus any interferometer measuring  $\Gamma_{\lambda}^2(d)$ , which is proportional to the square of Michelson's fringe visibility, will not be capable of distinguishing between different limb-darkening laws unless it can achieve an absolute accuracy significantly better than  $\pm 0.1\%$  of  $\Gamma_{\lambda}^2(0)$ .

The best that can be done with lower accuracy is to test whether the height of the first secondary maximum of the  $\Gamma_{\lambda}^2(d)$  curve is correctly predicted by model atmospheres. As shown by observations of  $\alpha$  CMa (Sirius) with the Narrabri intensity interferometer<sup>10</sup>, an accuracy of the order of  $\pm 0.1\%$  or better is required even for this. The Narrabri experience indicates that this type of measurement would only be feasible for stars at least 5 magnitudes brighter than the sensitivity limit of the instrument.

(iv) Stellar rotation

Rotation results in distortion of the shape of a star, increasing the equatorial diameter relative to the polar diameter, and also variation in gravity over the stellar surface which is predicted to lead to polar brightening relative to the equatorial region in the surface brightness distribution. The result of these two effects on the Fourier transform of the intensity distribution of the equivalent strip source will depend on the orientation of the axis of rotation of the star with respect to the direction along which the equivalent strip intensity distribution is taken. It follows that

observation of a rotating star with an interferometer will result in an apparent variation in angular diameter with position angle unless the star is seen pole-on.

The extent and nature of the variation of the response of an interferometer with position angle depends on the mass and rotational velocity of the star, the orientation of the rotational axis with respect to the interferometer baseline, and on the degree of differential rotation in the star. As an example, Johnstone and Wareing<sup>44</sup> have predicted, using simple models, maximum apparent changes with aspect of about 6% for  $\alpha$  Leo and 4% for  $\alpha$  Aql. Lake<sup>12</sup> has taken this work further and computed the correlation factor  $\Gamma_{\lambda}^2(d)$  for a number of models of uniformly and differentially rotating stars. For stars rotating near their critical velocity Lake has shown that the effects of rotation on the correlation factor may be quite large, especially near the first secondary maximum of  $\Gamma_{\lambda}^2(d)$ , and in extreme cases it may vary between  $\sim 0$  and  $\sim 0.45$  with position angle.

The rapidly rotating star  $\alpha$  Aql has been observed with the Narrabri intensity interferometer<sup>7,12</sup> and although the observed variation with position angle could be explained in terms of a model of a rotating star, the signal/noise ratio was inadequate to allow a critical comparison of results with theory. However, the observations did show that observations with an interferometer could contribute to the study of rapidly rotating stars and might settle the question of the existence of stars rotating at the critical velocity<sup>45</sup>.

The prime instrumental requirements for this programme are accuracy and a large range in position angle coverage (ideally  $180^{\circ}$ ). An accuracy of  $\pm 1\%$  should be the target. The sensitivity required for programme (i) would also be adequate for this programme.

(v) Extended atmospheres of early-type stars

Many early-type stars have extended atmospheres and are losing mass through radiatively driven winds. The primary evidence comes from the large negative velocities found in absorption lines in the far-ultraviolet spectra of hot stars<sup>46</sup>, but the detection of infrared excesses and radio emission<sup>48</sup> have been interpreted in terms of the mass-loss model<sup>49</sup> Efforts

to explain the large differences in temperature derived from line spectra and continuous spectra for a number of Wolf-Rayet stars and, in particular, the low effective temperature of the O5f star  $\zeta$  Pup as determined from angular diameter measurements<sup>50</sup> have also invoked extended atmospheres. Several calculations of model atmospheres for stars which have spatially extended atmospheres<sup>51-55</sup> have been made.

The extensions of the atmospheres of hot, early-type stars will be almost completely ionised and Thomson scattering from free electrons will lead to polarization of the scattered radiation so that its electric vector is preferentially in the tangential direction referred to the centre of the star. If a star with an electron-scattering atmosphere is viewed through a plane polarizing analyzer the contours of constant intensity will be approximately elliptical with their major axis perpendicular to the polarizing direction of the analyzer<sup>55,57</sup>. Observations with a stellar interferometer should reveal, at least in principle, variations in apparent angular diameter as analyzers are rotated in front of the interferometer and the possibility of detecting this effect has been investigated by a number of workers<sup>11,55-57</sup>.

Cassinelli and Hoffman<sup>57</sup> have determined interferometer response functions for four extended electron-scattering atmospheres and shown that the equivalent uniform disc diameter may vary by as much as 10% when measured in orthogonal senses of polarization. In particular they point out the potential usefulness of a sensitive interferometer in differentiating between models and in determining the structure of early-type stellar atmospheres. Sams<sup>55</sup> has carried out an extensive theoretical investigation for a wide range of models with extended electron-scattering atmospheres and concludes that Wolf-Rayet stars, early-type stars of classes Of and O, and O and B giants and supergiants including P Cygni and "P-Cygni-like" stars could be successfully investigated with a sensitive interferometer. Sams also concludes that measurements of  $\Gamma_{\lambda}^2(d)$  with an accuracy better than  $\pm 1\%$  of  $\Gamma_{\lambda}^2(0)$  are required for this purpose.

Hanbury Brown et al<sup>11</sup> have measured the angular diameter of  $\beta$  Ori in two orthogonal polarization planes in an attempt to detect and to measure the density of an extended corona so that an estimate could be made of the rate

of mass loss. The spectral type of  $\beta$  Ori (B8Ia) and sensitivity of the Narrabri interferometer made detection of a corona unlikely and no significant variation of apparent angular diameter with polarization was detected. Limits were set to the density of any corona around  $\beta$  Ori and to the rate of mass loss but they are too coarse to be of much value. Nevertheless, these measurements show that, given adequate sensitivity, mass-loss rates can be determined with an interferometer although it appears unlikely that they would compete in accuracy with determinations by other techniques<sup>48</sup>.

Observations of early-type stars with a stellar interferometer capable of measurements in orthogonal planes of polarization would enable the structure of their extended atmospheres to be investigated and would allow estimates of mass-loss rates to be made. Resolution requirements for a given star are essentially the same as for programme (i). High accuracy in the difference between measurements in orthogonal planes of polarization is the major interest and a minimum requirement for the difference would be  $\pm 1\%$  of  $\Gamma_{\lambda}^2(0)$ <sup>55</sup> with a target of  $\pm 0.1\%$ . It would not be necessary to determine the absolute values of  $\Gamma_{\lambda}^2(d)$  to this order of accuracy. The implication of the desired accuracy is that programme stars would need to be some 5 magnitudes brighter than  $V(\text{limit})$  as defined in Section 3.1.

(vi) Emission-line stars

(a) Be stars and shell stars

A Be emission-line star appears to combine the effects discussed in the last two programmes, namely, rotation and extension of the atmosphere. Be stars are pictured as being in rapid rotation with ejection of material into a circumstellar envelope<sup>58</sup>. Parts of this envelope which are projected against the stellar disc will produce absorption lines while the remaining parts will contribute emission features to the spectrum. Models of Be, Ae and shell stars are based on theory and the indirect evidence of photometry and spectroscopy. Spatial interferometry could contribute to our understanding of these objects by measurements of apparent angular size in both the continuum and emission lines as a function of position angle. Although a detailed study of interferometric possibilities has not been made it would appear that differences in  $\Gamma_{\lambda}^2(d)$  as a function of position angle would be at least as

great as those for ordinary rapidly-rotating stars<sup>12</sup>.

Some 20% of B stars exhibit hydrogen emission lines and are classified as Be<sup>58</sup> so inspection of Figure 1 gives at least an indication of their distribution with brightness and the resolution required for their study. The instrumental sensitivity and desired accuracy of measurement will follow that outlined in Section 3.2(iv) for rotating stars.

(b) Wolf-Rayet stars

Wolf-Rayet stars exhibit broad emission lines superposed on a relatively weak continuum. These prominent spectral features come from different parts of the apparently complex atmospheres<sup>58</sup> and measurements of angular size in the continuum and in the emission lines would contribute to our understanding of the structure of Wolf-Rayet stars. A measurement of this kind was attempted with the Narrabri interferometer<sup>9</sup> with observations of  $\gamma^2$  Vel in the continuum ( $\lambda 4430$ ) and in the CIII-IV emission features at  $\lambda 4650$ . The results showed that the emission region had an angular size some five times greater than that of the brighter of the two stars in the  $\gamma^2$  Vel system. It has been shown<sup>59</sup> since that the O9I star is brighter than the Wolf-Rayet component but the measurements did reveal the potential of applying high angular resolution interferometry to the study of emission-line stars.

Wolf-Rayet stars are comparatively rare objects with only about 100 brighter than 11th magnitude known.  $\gamma^2$  Vel ( $V = +1.8$ ) is the only one brighter than  $V = +5$  and a  $V(\text{limit})$  of the order of  $+8$  is necessary to obtain a sample of  $\sim 10$  stars for declinations south of  $+30^\circ$  or north of  $-30^\circ$ . The resolution requirements are essentially the same as for O5 stars in Figure 1 and, in the first instance, accuracies of the order of  $\pm 5\%$  would be useful.

(vii) Interstellar extinction

One of the problems in the study of interstellar extinction is the determination of the total extinction from observations of its colour dependence. If it is assumed that there is no neutral component, that is, extinction independent of wavelength, the ratio of total to selective extinction can be obtained from reddening curves.



In principle an interferometer can be used to give direct measurements of total interstellar extinction. Gray<sup>60</sup> has proposed calibrating the expected flux from a star by means of its measured angular diameter and theoretical model atmosphere predictions of the emergent flux at the stellar surface. Comparison of the expected fluxes with observed fluxes would give the interstellar extinction. As pointed out by Gray the dependence on theory is a weakness and this can be eliminated if measured angular diameters and fluxes are used to establish the emergent flux at the stellar surface  $F_{\lambda}$  for a selected class of stars. Combination of measurements of angular size and flux for more distant stars of the same class would enable apparent values of  $F_{\lambda}$  to be calculated and hence the neutral and selective components of interstellar extinction to be found.

Ideally luminous supergiants with absolute magnitudes as high as -8 would be used for this programme. In the absence of interstellar extinction they could be measured at distances exceeding 10 kpc with an interferometer having a limiting magnitude of  $V(\text{limit}) = +7.5$  and to approximately 40 kpc with  $V(\text{limit}) = +10$ . With an extinction of 2.0 mag/kpc the corresponding distances are approximately 2.0 and 2.9 kpc respectively. Stars of such high luminosity are comparatively rare and in practice relatively common early main-sequence stars might be used. As an example, stars of type B0 V with  $M_V = -4.4$  would allow measurements out to about 2.4 kpc ( $V(\text{limit}) = +7.5$ ) or 7.6 kpc ( $V(\text{limit}) = +10$ ) in the absence of extinction and to 1.0 kpc and 1.7 kpc for an extinction of 2.0 mag/kpc.

An interferometric programme to study interstellar extinction might measure the ratio of total to selective extinction, establish whether there is a component of neutral extinction, and study variations with direction in the galaxy. Given a class of stars to be used as extinction "probes" the instrumental requirements for measuring the angular diameters of the stars are the same as those outlined for programme (i).

(viii) Cepheid variables

The study of Cepheid variables with an interferometer is potentially a very important programme. The uncertainty in Cepheid temperatures and the problem of reconciling their masses derived from pulsation and evolutionary

theories<sup>61</sup> can both be resolved by an interferometric study since it is capable of specifying both temperatures and luminosities for Cepheids. In addition to contributing to the physical understanding of the pulsation mechanism interferometry is capable of providing an independent calibration of Cepheid luminosities which are fundamental to the calibration of the astronomical distance scale.

Observations of a Cepheid variable with an interferometer would yield both its mean angular diameter and the apparent variation in angular diameter as the star pulsates. Combination of the variation in angular size with the radial displacement of the surface of the star obtained by integration of spectroscopic radial velocity observations would enable the distance of the Cepheid to be found. In practice, it would be necessary to apply some corrections to the observational results before the distance could be obtained. Firstly, the observed radial velocity would have to be corrected for the fact that it represents the average over the visible surface of the star and is therefore less than the actual radial velocity. Secondly, if the interferometric observations were made in the continuum, which arises from deeper in the stellar atmosphere than the line radiation from which the radial velocities are measured, it would be necessary to apply a correction if the separation between the levels alters during the pulsation. Finally, corrections for the effects of limb darkening must be applied to the angular diameter measurements.

Thompson<sup>62</sup> has made a detailed study of the potential of a programme of interferometric observations of Cepheids and has evaluated the potential accuracy of the programme taking into account the uncertainties in the corrections listed above. Thompson finds that the ultimate accuracy in Cepheid distance determinations is limited by uncertainties in the corrections to  $\pm 3\%$  ( $\pm 0.07$  magnitudes in distance modulus). To this must be added the uncertainties in the apparent radial displacement and angular diameter determinations. From an examination of radial velocity curves for some bright Cepheids Thompson took the uncertainty in distance due to the radial displacement as  $\pm 3\%$ . While not many Cepheids have been measured to this accuracy it is indicative of what can be achieved. Thus, if the variation in angular diameter can be determined to  $\pm 10\%$  the uncertainty in distance would be  $\pm 11\%$  ( $\pm 0.24$  magnitude in distance modulus) and if it can be determined to  $\pm 5\%$  the uncertainty in distance would be  $< \pm 7\%$  ( $\pm 0.14$  magnitudes).

In determining absolute magnitudes the uncertainty in Cepheid colour excesses must be taken into consideration and as Thompson points out this will be the major source of uncertainty. Nevertheless, the accuracy achievable would be sufficient to provide a valuable check on Cepheid luminosities which are uncertain to 0.15 - 0.25 magnitudes.

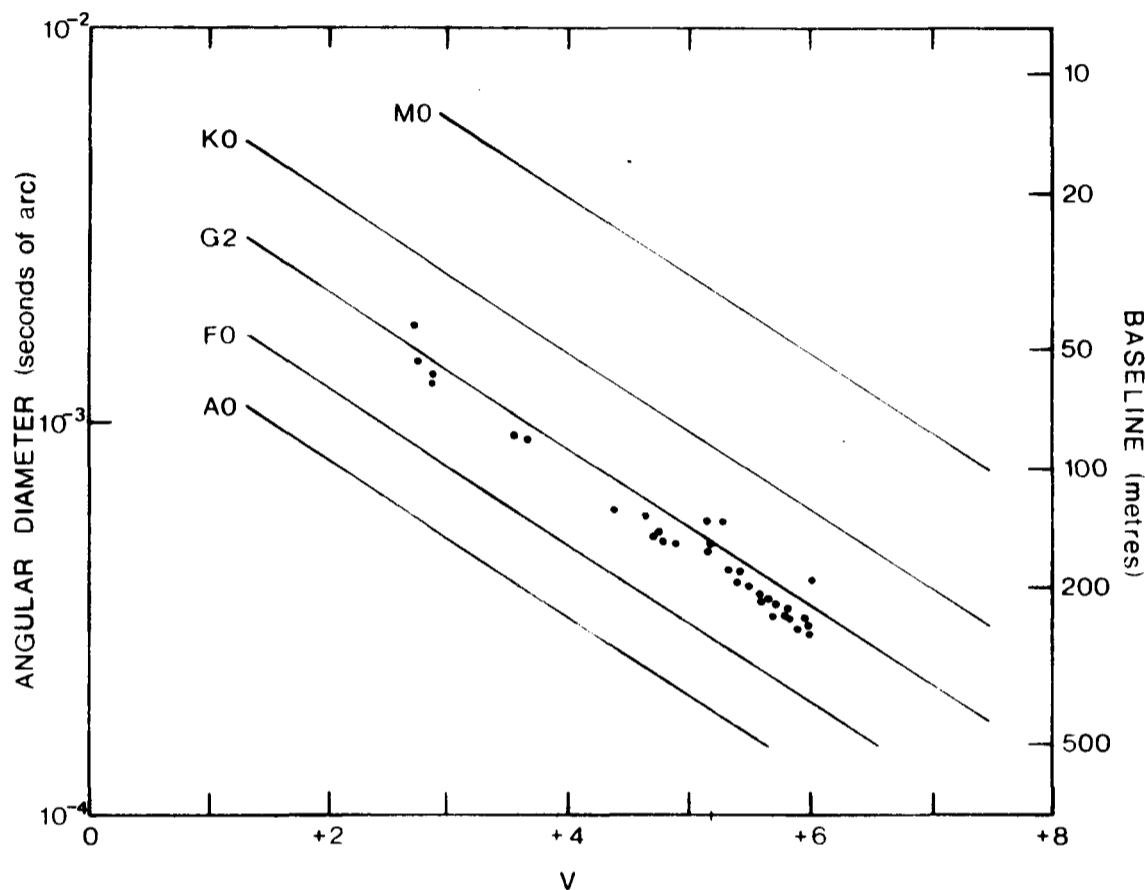


Figure 3. Predicted mean angular diameter as a function of mean visual magnitude for Cepheid variables with mean visual magnitude brighter than +7.0 and south of declination  $+30^{\circ}$ . Diagonal lines represent predicted mean relationships for the spectral types shown. The baselines shown as the ordinate on the right-hand side of the diagram are optimum for observing Cepheids at a wavelength of  $550\text{nm}$ <sup>62</sup>

Figure 3 shows Cepheid variables classed as population type I with mean visual magnitude brighter than +7.0 and declination south of  $+30^{\circ}$ . For the purposes of the diagram mean angular diameters were obtained by estimating the mean spectral types from the periods using the spectral type-period diagram of Code<sup>63</sup> and then interpolating in a plot of measured angular diameters against spectral type constructed by reducing all the angular diameters shown in Figure 1 to  $V = 0.00$ . Thompson<sup>62</sup> has shown that the mean brightness of a Cepheid must exceed the sensitivity limit of an interferometer by about 2.5

magnitudes, the actual amount depending on the range in brightness, if its variation in angular size is to be measured with an accuracy of  $\pm 10\%$ . For an accuracy of  $\pm 5\%$  it must exceed the sensitivity limit by some 3.5 magnitudes. Inspection of Figure 3 shows that to measure the group of 6 Cepheids brighter than  $V = +5$  to  $\pm 10\%$  a limiting sensitivity of  $V(\text{limit}) > +7.5$  is required and for  $\pm 5\%$   $V(\text{limit}) > +8.5$ . To measure all the Cepheids in Figure 3 would require  $V(\text{limit}) > +9.5$  for  $\pm 10\%$  and  $V(\text{limit}) > +10.5$  for  $\pm 5\%$  accuracy but this would include RS Pup, S Nor and U Sgr, three of Sandage and Tammann's<sup>64</sup> calibration Cepheids. The group of 6 bright Cepheids could be resolved with a maximum baseline  $< 100\text{m}$  but to resolve all the Cepheids in Figure 3 requires a maximum baseline of  $\sim 250\text{m}$ .

A detailed analysis for Cepheids north of declination  $-30^\circ$  has not been carried out but it appears that the distribution is similar to that shown in Figure 3.

The method proposed by Barnes et al<sup>29</sup> for determining the distance of a Cepheid by using a surface brightness-colour relationship to obtain its angular diameter has been shown to be capable of an accuracy of the order of  $\pm 20\%$ <sup>36</sup>. Barnes et al<sup>36</sup> estimate that with improved photometry and a substantial increase in the number of measured angular diameters for F and G spectral types, the method is capable of distance measures to  $\pm 15\%$  as originally suggested<sup>29</sup>. The value of the surface-brightness approach is that it can be applied to Cepheids too faint for direct angular diameter measurements and, as pointed out by Barnes et al<sup>36</sup>, it should prove to be a valuable tool in galactic structure studies and, if freed from the necessity for radial velocity curves as proposed by Fernie<sup>65</sup>, it could possibly be applied to even the brighter extragalactic Cepheids. Interferometry can contribute to the surface-brightness method in two ways. Firstly, it can provide the angular diameters of stars of spectral types F and G that will enable the surface brightness-colour relationship to be improved. Secondly, the measurements of the brighter Cepheids discussed earlier would provide direct calibration of the method.

#### (ix) Galactic nuclei and quasars

The possibility of resolving the nuclei of the brighter Seyfert galaxies, quasars and other extragalactic species would be an exciting prospect.

The uncertainty in the nature of the energy sources and in the structure and physical size of these objects would justify a considerable effort to provide high angular resolution measurements of them. The general increase in continuum radiation towards longer wavelengths perhaps makes observations in the infrared a better proposition, and Dr. Low will discuss this in the next paper, but, nevertheless, observations in the visual, if feasible, are also desirable.

Rees<sup>66</sup> has described a 'two component' model to explain the spectra of quasars (and objects such as Seyfert galaxies). The model is composed of an extensive low-density region, which emits narrow forbidden and permitted lines, and a compact high density region emitting only broad permitted lines. Spectroscopic studies and, in some cases, indications of variability, suggest that the second region is only a few light days across - in line with theoretical estimates of minimum size based on surface brightness considerations. The optical continuum is envisaged as synchrotron-type emission, possibly from an even smaller region at the centre. In principle, at least, it should be possible to test this type of model observationally with an interferometer. Since the continuum and emission spectra originate in different regions, observations in the continuum and in emission lines with an interferometer, providing it has adequate sensitivity and resolving power, would allow both the relative sizes of the regions to be determined and also their physical sizes by combining the angular data with distances determined from their redshifts.

A significantly greater sensitivity is required for extragalactic studies than for the stellar programmes described earlier. The nuclei of Seyfert galaxies are generally fainter than 15th magnitude - in the list of Khachikian and Weedman<sup>67</sup> the only Seyfert nuclei with brighter apparent blue magnitudes are NGC 4151 (+13.0), NGC 7469 (+14.3), NGC 3783 (+14.5) and NGC 5548 (+14.8). The situation is similar for quasars. In the optical catalogue of QSO's by Burbidge et al<sup>68</sup> only 4 have V magnitudes brighter than +15 with 3C273 the brightest at +12.9. Fainter than V = +15 the numbers increase rapidly with some 30 quasars between V = +15 and +16. For a reasonable sample of Seyferts and quasars a limiting sensitivity in the vicinity of V = +16 is required. A limiting magnitude of at least +13 is necessary to measure only one example of each of these types of object.

Resolution requirements are difficult to predict. The largest Seyfert nucleus (NGC 1068) shows emission lines from a region 8" in diameter but the continuum source is unresolved in a telescope<sup>69</sup>. Balloon borne photographic observations of NGC 4151<sup>70</sup> give an upper limit of 0".08 to the diameter of the continuum source and speckle observations of the quasar 3C273<sup>71</sup> have indicated the presence of an unresolved component at the scale of 0".02. While any improvement in resolution in the visual would be an advance it must be noted that radio VLBI results for the Seyfert galaxy 3C120<sup>72</sup> show structure smaller than 0".0004 and for BL Lacertae<sup>73</sup> less than 0".0003. Presumably the radio sources are no smaller than the optical sources. Thus it would appear from radio results, from limits set by light-travel-time arguments applied to the time scale of brightness variations, and from theoretical predictions that a resolution significantly better than 0".001 is needed to resolve any continuum sources. The angular extent of emission regions should be measurable with much lower resolving power.

Because of the great uncertainties in the sizes of these extragalactic objects a much lower accuracy is acceptable than for the other programmes that have been discussed - even lowering of upper limits would represent progress in most instances.

### 3.3 Concluding Remarks on the Discussion of Potential Astronomical Programmes

Although a wide range of potential programmes has been discussed in some detail it is not feasible to cover, nor even anticipate, all the possibilities. Those which have not been discussed include the measurement of Mira variables through their cycles, the resolution of Magellanic Cloud planetary nebulae with a view to determining their masses and the resolution of white dwarfs which would require baselines of the order of kilometres.

Nevertheless, the programmes that have been discussed show the enormous potential of high angular resolution stellar interferometry and enable targets of resolution, sensitivity and accuracy to be set for the designers and builders of future stellar interferometers.

#### 4. SUMMARY

It has not been the purpose of this paper to judge which particular type of interferometer would be most suited for a given programme. Rather the intention has been to outline the instrumental requirements for a range of astronomical programmes so that it is possible, given the  $V(\text{limit})$ , maximum baseline, and limiting accuracy of an interferometer to assess its astronomical potential. Alternatively, the information may be used to decide what instrumental characteristics are required to carry out a given astronomical programme.

Some of the data presented in this review were originally prepared with the second approach in mind. The data were used to assess the sensitivity and resolution that should be the target for a next generation intensity interferometer and formed the basis for the proposal by Hanbury Brown and Davis<sup>7,74</sup> for a large intensity interferometer. Professor Hanbury Brown will discuss this assessment in his review of intensity interferometry later in this meeting.

To summarise, it can be seen that an instrument with  $V(\text{limit}) = +7.5$  would be capable of making a significant contribution to astrophysics through several of the programmes which have been discussed. For this limiting sensitivity a baseline of about 300m would be adequate to resolve all but the hottest stars and for them, baselines up to 2 km would be needed. For late main-sequence stars  $V(\text{limit}) > +10$  is needed and this sensitivity would enable a much larger sample of Cepheids, including some examples used to calibrate the distance scale, to be measured. It would also enable a better sample of O-type and Wolf-Rayet stars to be measured but, to resolve these, baselines of several kilometres would be necessary. For these stellar programmes an accuracy of  $\pm 5\%$  of  $\Gamma_{\lambda}^2(0)$  should be regarded as the minimum acceptable and ideally the aim should be  $\pm 1\%$ . For programmes involving the measurement of the variation of apparent angular diameter with position angle, such as observations of rotating stars, an accuracy of  $\pm 0.1\%$  of  $\Gamma_{\lambda}^2(0)$  in the difference between  $\Gamma_{\lambda}^2(d)$  as a function of position angle is the target. For limb-darkening investigations an absolute accuracy better

than  $\pm 0.1\%$  of  $\Gamma_{\lambda}^2(0)$  is required in measurements of  $\Gamma_{\lambda}^2(d)$ .

In order to measure extragalactic objects  $V(\text{limit})$  must be at least +13 and to obtain a number of results for Seyfert galaxies and quasars  $V(\text{limit}) = +16$  is required. Baselines of a few metres may be sufficient to resolve the emission regions in galactic nuclei and quasars but to resolve the continuum sources may require baselines of the order of kilometres.

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