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CONTRIBUTED PAPER

T Falls Apart: On the Status of Classical Temperature in Relativity

Eugene Yew Siang Chua

Department of Philosophy, University of California San Diego, La Jolla, CA, USA
Email: eychua@ucsd.edu

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Abstract

I argue that the classical temperature concept falls apart in special relativity by examining four consilient procedures for establishing classical temperature: Carnot processes, thermometers, kinetic theory, and black-body radiation. I show that their relativistic counterparts demonstrate no such consilience. I suggest two interpretations for this situation: eliminativism akin to *simultaneity*, or pluralism akin to *rotation*.

> There is actually no compelling method in the sense that one view would simply be "correct" and another "false." One can only try to undertake the transition as naturally as possible.
>
> Einstein (1953) on extending thermodynamics to relativity. (Liu 1992, 200)

1. Introduction

Do the laws and concepts of classical thermodynamics (CT) hold universally? Einstein wrote: "[CT] is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, it will never be overthrown" (Einstein 1946/1979, 33). Given such proclamations, and how research in black hole thermodynamics—birthed from formal analogies with CT—continues to this day, one naturally assumes CT *can* be extended into the relativistic regime and beyond—there's no limit to its "framework of applicability" (Dougherty and Callender 2016; Wallace 2018).

It's therefore interesting that a parallel debate remains without resolution. Although Planck and Einstein pioneered special relativistic extensions of thermodynamical concepts by developing a set of Lorentz transformations, they by no means settled the issue. *Temperature* resists a canonical relativistic treatment: there are different equivocal ways of relativizing temperature. While physicists treat this as an empirical problem (Farías et al. 2017, 5), or a matter of convention (Landsberg and Johns 1967), the issue seems *conceptually* problematic to me.

I argue that the classical non-relativistic temperature concept, $T_{\text{classical}}$, breaks down in special-relativistic regimes. Procedures which jointly provided physical meaning to $T_{\text{classical}}$ do not do so in relativistic settings; there *is* a limit to the framework of applicability of *classical* thermodynamic concepts.

My argument rests *not* on the fact that there's no way of defining temperature in relativistic regimes; rather, there are *many*, *equally valid* procedures for defining relativistic temperature which *disagree* with each other. I focus on four procedures (and their relativistic counterparts): (relativistic) Carnot cycles, (co-moving) thermometer, (relativistic) kinetic theory, and (moving) black-body radiation.

I propose we understand Einstein's notion of "natural"-ness as follows: there's strong *consilience* between classical counterparts of these procedures in determining $T_{\text{classical}}$'s physical meaning, in the operational sense that the temperature established by each procedure agrees with other procedures. Contrariwise, their relativistic counterparts demonstrate no such consilience: different procedures predict starkly different behaviors for relativistic temperature. "Natural" procedures in CT do not appear "natural" in relativistic settings.

I propose two possible interpretations for this situation: *eliminativism*, where we interpret temperature akin to simultaneity, or *pluralism*, where we interpret temperature akin to relativistic rotation.

2. Relativistic Thermodynamics

The pioneers of relativistic thermodynamics (Einstein 1907; Planck 1908) sought Lorentz transformations for CT (Liu 1992, 1994) just as, for example, position and time. For instance, an observer O' with position and time (x', y', z', t') moving along the x-axis away from another observer O at constant velocity v can be understood by O to be at position and time (x, y, z, t) via:

$$t' = \gamma(t - vx/c^2), \qquad x' = \gamma(x - vt), \qquad y' = y, \qquad z' = z, \tag{1}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor, and c the speed of light.

Relativistic thermodynamics hopes to find similar transformations for thermodynamic quantities like temperature, pressure, volume, etc. The assumption is that thermodynamics has physical meaning in relativistic regimes only when we have Lorentz transformations under which thermodynamic quantities transform—like position and time.[1]

Planck and Einstein derived transformations for most thermodynamic quantities like pressure p, volume dV, and entropy S (Liu 1994, 987):

$$dV' = dV/\gamma, \qquad p' = p, \qquad S' = S. \tag{2}$$

[1] That only quantities invariant or covariant under Lorentz transformations are physically meaningful, or that the laws must be form-invariant in all inertial frames, is a common idea (Lange 2002, 202; Maudlin 2011, 32).

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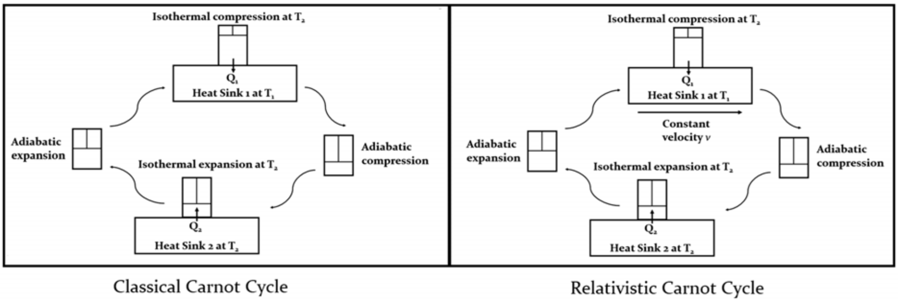

Figure 1. Left: Classical Carnot cycle with $T_2 > T_1$. Right: Relativistic Carnot cycle.

Fixing S might appear to indirectly fix the concepts of heat and temperature, via the standard $dQ = TdS$. Surprisingly, *temperature*'s Lorentz transformation turns out to be highly equivocal.

3. Classical Temperature

In CT, at least four well-known procedures establish the concept of temperature. Notably, there's significant *consilience* between them, suggesting a physically significant quantity: $T_{\text{classical}}$.

3.1. Carnot Cycle

Carnot cycles define absolute temperature in terms of heat (Chang 2004, ch. 4). The typical idealized example is an ideal gas acting on a piston in a cylinder ("engine") while undergoing reversible processes (see the left panel of Figure 1).

A simple relationship between the heat exchange to/from the heat baths and their temperature is definable:

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}. \tag{3}$$

The ratio between the temperatures of the two baths is theoretically derivable from the amount of heat exchanged. This theoretical concept of temperature is then given empirical meaning through operationalizations in terms of actual thermometer measurements, a sign of consilience between the theoretical temperature defined by Carnot cycles and the empirical one established by thermometers (Chang 2004, "Analysis: Operationalization").

3.2. Thermometer

This brings us to thermometers and how they establish the concept of temperature. Fahrenheit's invention of a reliable thermometer allowed independent and repeatable measurements of temperature; this allowed an understanding of temperature as a robustly measured numerical concept rather than one associated with vague bodily sensations (Chang 2004, chs. 1–2; McCaskey 2020).

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However, while thermometers made with the *same* material *were* reliable with respect to each other, thermometers made with *different* materials differed in their rates of expansion and contraction. Importantly, Carnot cycles provide *theoretical* foundations for temperature by providing a material-independent definition of temperature (Thomson 1882, 102). Thus, as another sign of consilience, Carnot cycles in turn provide theoretical foundations to observed temperature measurements provided by actual thermometers.

3.3. Kinetic Theory

Kinetic theory provides another way to understand temperature via the Maxwell–Boltzmann distribution. For a system of ideal gas particles in equilibrium with temperature T, the notion of temperature connects explicitly with the notion of bulk particle velocities via:

$$f(v) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\left[\frac{1}{2}mv^2 + V(x)\right]/kT}, \tag{4}$$

where m is the particle's mass, T is the temperature, k is Boltzmann's constant, $V(x)$ is the system's position-dependent potential energy, and v is an individual molecule's velocity (Brush 1983, §1.11). $f(v)$ tells us, for a system of ideal gas particles in equilibrium, how many particles we expect to find with some range of velocities v to $v + dv$, given some temperature.

This distribution plays a significant conceptual role by allowing us to derive the well-known formula

$$\langle \frac{1}{2} mv^2 \rangle = \frac{3}{2} kT. \tag{5}$$

This provides foundational support for thermodynamics—and the concept of temperature—in terms of particle mechanics, as the concept of temperature is understood in terms of the particles' mean kinetic energy.

3.4. Black-Body Radiation

Finally, black-body radiation connects temperature to electromagnetic radiation. Black-bodies absorb (and emit) all incident thermal radiation without reflecting or transmitting the radiation, for *all* wavelengths and incident angles. Notably, since black-bodies don't distinguish directionality, they emit *isotropic* radiation.

There are simple laws relating radiation to black-body temperature (Brush 1983, §3.1). The Stefan–Boltzmann law states:

$$j^* = \sigma T^4, \tag{6}$$

where j^* is the black-body's radiant emittance, T is its temperature, and σ is the Stefan–Boltzmann constant.

Wien's displacement law,

$$\rho(f, T) = f^3 g(f/T), \tag{7}$$

states that the energy density ρ of radiation from systems with temperature T, at frequency f, is proportional to $f^3 g(f/T)$ for some function g (Brush 1983, ch. 3).

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Integrating over all f amounts to computing the total energy density of radiation, and entails the Stefan–Boltzmann law regardless of choice of g. Furthermore, if $\rho(f, T)$ achieves its maximum for some value of f, $f_{\max}$:

$$f_{\max} \propto T, \tag{8}$$

or, in terms of peak wavelength λ_{peak}:

$$\lambda_{\text{peak}} \propto \frac{1}{T}. \tag{9}$$

This connects radiation to $T_{\text{classical}}$ by capturing the familiar observation that things which are heated first turn red and then other colors associated with higher frequencies—and hence shorter wavelengths—as their temperature increases.

4. Relativistic Temperature

In CT, the above procedures show *remarkable consilience*; $T_{\text{classical}}$ can be determined or understood in terms of any of these procedures without much issue. This motivates why we might find the temperature concept—and its application in these contexts—"natural," to borrow Einstein's words. However, there's no such consilience for these procedures' *relativistic counterparts.*

4.1. Relativistic Carnot Cycle

The relativistic Carnot cycle assumes that the same heat–temperature relations (3) hold when one heat bath is *moving* with respect to the other with velocity v (see Figure 1, right). In completing this cycle, we adiabatically accelerate or decelerate the engine from one inertial frame to another (von Mosengeil 1907; Liu 1992, 1994; Farías et al. 2017; Haddad 2017, 39–42; see also Earman 1978, 177–178).

For a heat bath at rest with temperature T_0 and another moving with respect to it with "moving temperature" T', with the engine *co-moving* with the respective heat baths during the isothermal processes:

$$\frac{T'}{T_0} = \frac{Q'}{Q_0},$$

$$T' = \frac{Q'}{Q_0} T_0. \tag{10}$$

What remains is to define the appropriate heat exchange relations. However, there are two ways to understand the heat exchange between the engine and the moving heat bath, and there doesn't seem to be a fact of the matter which is appropriate (Liu 1992).

Firstly, one may, like Planck and early Einstein, understand the heat transfer from the perspective of the stationary bath's rest frame. In exchanging heat with the moving bath, the engine also exchanges energy. By relativistic mass–energy equivalence, this causes the bath to lose/gain momentum by changing its mass. However, without further work, the bath cannot stay in inertial motion—it will decelerate or accelerate. To keep it moving inertially, extra work must be performed on it. Einstein thus proposes:

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$$dW = pdV - \mathbf{u} \cdot d\mathbf{G}, \tag{11}$$

where pdV is simply the usual compressional work done by the piston due to heat gain/loss from the bath. However, there's a crucial inclusion of the $\mathbf{u} \cdot d\mathbf{G}$ term, where $\mathbf{u}$ is the moving bath's relativistic velocity (more specifically, $\mathbf{v}/c$) and $d\mathbf{G}$ is the change of momentum due to heat exchange. Einstein dubbed this the "translational work." When $\mathbf{u}$ is 0, the work done reduces to the usual definition. The first law generalizes from

$$dU = dQ - pdV \tag{12}$$

to

$$dU = dQ - pdV + \mathbf{u} \cdot d\mathbf{G} \tag{13}$$

for moving systems. One then obtains a relationship between the quantities of heat exchanged [Liu(1994), 984–987]:

$$\frac{dQ'}{dQ_0} = \frac{1}{\gamma}, \tag{14}$$

and hence:

$$T' = \frac{1}{\gamma} T_0. \tag{15}$$

We thus arrive at a Lorentz transformation for temperature, according to which moving systems have lower temperature and appear cooler than systems at rest. This is the *Planck–Einstein formulation* of relativistic thermodynamics.

Secondly, one may, like later Einstein (in private correspondence to von Laue) or Ott (1963), doubt the need for translational work. Later Einstein wrote:

> When a heat exchange takes place between a reservoir and a "machine," both of them are at rest with each other and acceleration-free, it does not require work in this process. This holds independently whether both of them are at rest with respect to the employed coordinate system or in a uniform motion relative to it. (Einstein 1952, in Liu 1994)

In the *moving heat bath*'s rest frame, heat exchange is assumed to occur isothermally (as per the classical Carnot cycle) when both the engine and the heat bath are *at rest* with respect to each other. From this perspective, everything should be as it is classically. There should thus be no additional work required other than that resulting from heat exchange. What was thought of as work done to the system in the Planck–Einstein formulation should instead be understood as part of heat exchange in the Einstein–Ott proposal. Without the translational work term in the equation for work, the moving temperature transformation is instead given by (Liu 1992, 197–198):

$$T' = \gamma T_0. \tag{16}$$

Contra (15), a moving body's temperature appears *hotter*.

I won't pretend to resolve the debate. However, note that this procedure is *equivocal* about relativistic temperature: it either appears lower (on the Planck–Einstein formulation) or higher (on the Einstein–Ott formulation) than the rest-frame temperature. Importantly, both proposals reduce to $T_{\text{classical}}$ in the rest frame; translational work vanishes in this case on both proposals. $T_{\text{classical}}$ seems safe, though its relativistic counterpart's fate remains undecided.

I end by briefly raising skepticism about relativistic Carnot cycles, by asking whether there's a principled answer to whether energy flow should be understood as "heat" or "work" here. As Haddad observes:

> In relativistic thermodynamics this decomposition is not covariant since heat exchange is accompanied by momentum flow ... there exist nonunique ways in defining heat and work leading to an ambiguity in the Lorentz transformation of thermal energy and temperature. (Haddad 2017, 39)

Since heat flow is accompanied with momentum flow, heat exchange can always be reinterpreted as work done (i.e., as the translational work term). This raises doubt about the very applicability of thermodynamics beyond the rest frame (i.e., CT in quotidian settings), given the fundamentality of heat and work relations in thermodynamics.

4.2. Co-Moving Thermometer

That relativistic thermodynamics is essentially "just" quotidian CT is echoed by Landsberg (1970), who employs the procedure of using thermometers (and the temperature concept they establish) via *co-moving* thermometers:

> One has a box of electronics in both [the relatively moving frame] and [the rest frame] and one arranges, by the operation of buttons and dials, to note in [the relatively moving frame] the rest temperature T_0 of the system. This makes temperature invariant. (259)

A system's co-moving temperature is stipulated to be its relativistic temperature. But this is *simply its rest-frame temperature*. On this proposal, the Lorentz transformation is:

$$T' = T_0. \tag{17}$$

This proposal can be seen as an extension of $T_{\text{classical}}$, in the sense that there's *some* proposed Lorentz transformation. In practice, though, nothing is different from the classical application of thermometers: we are measuring the rest-frame temperature of the system, as in CT. Landsberg partly justifies this with the claim that "nobody in his senses will do a thermodynamic calculation in anything but the rest frame of the system" Landsberg (1970, 260). *Contrary to the relativistic Carnot cycle*, relativistic temperature transforms *as a scalar*, as per many relativistic thermodynamics textbooks (e.g., Tolman 1934).

Landsberg (1970, 259) argues that this procedure doesn't also trivially define alternative Lorentz transformations for other mechanical quantities, e.g., position or time, in terms of rest-frame quantities:

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> Measurements in a general [reference frame] can be made of mechanical quantities, but in my view not of temperature, [so] our prescription for T' – namely "measure T_0" – is quite unsuitable for extension to mechanical quantities.

Prima facie, Landsberg is proposing a novel Lorentz transformation for temperature. However, this argument simply suggests that *there's no relativistic temperature to speak of*; we insist on the classical—rest frame—temperature concept. His comparison with mechanical quantities makes this clear: the concept of temperature understood via Landsberg's proposal is *not* relativistic the way other quantities are. This suggests that $T_{\text{classical}}$ cannot be extended past the rest frame, i.e., into the relativistic domain.[2]

4.3. Relativistic Kinetic Theory

A classical ideal gas can be understood in terms of particles whose velocities are distributed according to the gas's temperature (Section 3.3). How does *that* notion of temperature extend to relativistic regimes?

Cubero et al. (2007) analyzes the Maxwell–Jüttner distribution, a Maxwell–Boltzmann-type distribution for ideal gases moving at relativistic speeds. They conclude that temperature should transform as a scalar (Landsberg's proposal). Interestingly, they explicitly choose a reference frame where the system is stationary and in equilibrium. But that's just the system's rest frame! It's unsurprising that there's no transformation required for temperature.[3]

Elsewhere, Pathria (1966, 794) proposes yet another construction (Liu 1994), considering a distribution F for an ideal gas in a moving frame with relativistic velocity $\mathbf{u}$:

$$F(\mathbf{p}) = [e^{(E-\mathbf{u}\cdot\mathbf{p}-\mu)/kT} + a]^{-1}, \tag{18}$$

where $\mathbf{p}$ is a molecule's momentum, E its energy, μ the chemical potential, k the Boltzmann constant, T the system's temperature in that moving frame, and a is 1 or -1 for bosonic and fermionic gases respectively. This describes, as with the classical case, how many particles we expect to see with momentum $\mathbf{p}$. F is shown to be Lorentz invariant, and we can compare them as such:

$$\frac{E-\mathbf{u}\cdot\mathbf{p}-\mu}{kT} = \frac{E_0-\mu_0}{kT_0}. \tag{19}$$

With the (known) Lorentz transformations for energy and momentum, $T = (1/\gamma)T_0$ follows, i.e., the *Planck–Einstein* formulation.

One might think that this suggests some consilience between kinetic theory and the relativistic Carnot cycle *for* the Planck–Einstein formulation. However, one would

[2] This is what physicists do when considering the temperature of distant astrophysical bodies. They extrapolate and observe other properties of a body—like luminosity—associated with its *rest-frame* temperature. Moving temperature is not considered. See also Anderson (1964, 179–180).

[3] Cubero et al. (2007, 3) admits: "Any (relativistic or nonrelativistic) Boltzmann-type equation that gives rise to a universal stationary velocity PDF implicitly assumes the presence of a spatial confinement, thus *singling out a preferred frame of reference*" (my italics).

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be disappointed. Balescu (1968) showed that Pathria's proposed distrbution can be generalized:

$$F^*(\mathbf{p}) = [e^{\alpha(\mathbf{u})\left(E - \mathbf{u}\cdot\mathbf{p} - \left(\mu/\beta(\mathbf{u})\right)\right)/kT} + a]^{-1}, \tag{20}$$

with the only constraint that $\alpha(\mathbf{0})$ and $\beta(\mathbf{0}) = 1$ for arbitrary even functions α and β. F^* tells us the particle number (or, in quantum mechanical terms, occupation number) associated with some $\mathbf{p}$ or E over an interval of time. Balescu showed that any such distribution recovers the usual Maxwell–Boltzmann-type statistics: distributions with arbitrary choices of these functions all *agree* on the internal energy and momenta in the rest frame when $\mathbf{u} = \mathbf{0}$: $T_{\text{classical}}$ is safe from these concerns.

Choosing these functions amounts to choosing some velocity-dependent scaling for temperature via α and chemical potential via β. The question of how temperature scales when moving relativistically is precisely what we want to decide on, yet it's also the quantity rendered arbitrary by this generalization! Balescu showed that:

- Choosing $\alpha = 1$ amounts to choosing the Planck–Einstein formulation $T = \left(1/\gamma\right)T_0$.
- Choosing $\alpha = \gamma^2$ amounts to choosing the Einstein–Ott formulation $T = \gamma T_0$.
- Choosing $\alpha = \gamma$ amounts to choosing Landsberg's formulation $T = T_0$.

As Balescu notes: "Within strict equilibrium thermodynamics, there remains an arbitrariness in comparing the systems of units used by different Lorentz observers in measuring free energy and temperature," and "equilibrium statistical mechanics cannot by itself give a unique answer in the present state of development" (Balescu 1968, 331). Any such choice will be a *postulate*, not something to be assured by statistical considerations. In other words, contrary to the classical case, there appears to be no fact of the matter as to how temperature will behave relativistically, given particle mechanics.

Contrary to classical kinetic theory, there's again no univocality about relativistic temperature.

4.4. Black-Body Radiation

Finally, when we consider *moving* black-bodies, there's again no clear verdict on the Lorentz transformation for relativistic temperature. The concept of a black-body appears to be restricted to the rest frame.

McDonald (2020) provides a simple example: consider some observed Planckian (thermal) spectrum of wavelengths from some distant astrophysical object with peak wavelength λ_{peak}. We want to find its temperature directly. In our rest frame, using Wien's law (9):

$$\lambda_{\text{peak}} = \frac{b}{T}, \tag{21}$$

where b is Wien's displacement constant. Supposing we know the velocity $\mathbf{v}$ of the distant astrophysical object, we can compare wavelengths over distances in relativity

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using the relativistic Doppler effect to find the peak wavelength of the object λ'_{peak} at the source:

$$\lambda'_{\text{peak}} = \frac{\lambda_{\text{peak}}}{\gamma\left(1 + \left(v\ \cos\theta/c\right)\right)}, \tag{22}$$

where θ is the angle in the rest frame of the observer between the direction of $\mathbf{v}$ and the line of sight between the observer and the object. Given this, we can compare temperatures:

$$T' = \frac{\lambda_{\text{peak}}}{\lambda'_{\text{peak}}} T = \gamma\left(1 + \frac{v\ \cos\theta}{c}\right)T. \tag{23}$$

The predicted temperature thus depends on the *direction* of the moving black-body to the inertial observer.

Landsberg and Matsas (1996) show similar results and demonstrate how a relatively moving black-body generally doesn't have a black-body spectrum from the perspective of an inertial observer. Crucially, they emphasize just how problematic this is for the notion of *black-body radiation* which is defined as *isotropic*:

> [the equation for a moving black-body] cannot be associated with a legitimate thermal bath (which is necessarily isotropic) ... the temperature concept of a black body is unavoidably associated with the Planckian thermal spectrum, and because a bath which is thermal in an inertial frame S is non-thermal in [a relatively moving] inertial frame S', which moves with some velocity $v \neq 0$ with respect to S, a universal relativistic temperature transformation ... cannot exist. (Landsberg and Matsas 1996, 402–403)

The general lesson is simple. A black-body was defined in the rest frame, i.e., in the non-relativistic setting: it has isotropic radiation with a spectrum, and can be understood to be in equilibrium with other objects and measured with thermometers. However, there was no guarantee that a *moving* black-body would still be observed as possessing some *black-body* spectrum with which to ascribe temperature. And it turns out that it generally does *not*. Without this assurance, we cannot reliably use the classical theory of black-body radiation to find a relativistic generalization of temperature.

5. $T_{\text{classical}}$ Falls Apart

Examining four relativistic counterparts to classical procedures reveals a *discordant* concept: a moving body may appear to be cooler, or hotter, the same, or may not even appear to be thermal at all. Despite how well these procedures worked classically, they do *not* work together to establish an unequivocal concept of relativistic temperature. Furthermore, *within* each procedure, various conceptual difficulties suggest that the concept of relativistic temperature doesn't find firm footing *either*. Returning to Einstein's quote, it appears that there's no "natural" way to extend $T_{\text{classical}}$.

$T_{\text{classical}}$ thus fails to be extended to relativity: well-understood procedures that unequivocally establish its physical meaning in classical settings fail to do so in

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relativistic settings. These procedures appear to work *just fine* in classical settings, i.e., in the rest frame. However, attempting to extend them to relativistic settings immediately led to conceptual difficulties. This all suggests that the concept of temperature–and, correspondingly, heat—is inherently a concept restricted to the rest frame.

More generally, any relativistic extension of CT violates some classical intuitions and will appear "unnatural." No matter our choice of temperature transformation, something from CT must go. Broadening Balescu's point (Section 4.3), Landsberg (1970, 263–265) generalizes the thermodynamic relations in terms of arbitrary functions $\theta(\gamma)$ and $f(\gamma)$:

$$TdS = \theta dQ, \qquad dQ = fdQ_0, \tag{24}$$

where f is the force function:

$$f = \frac{1}{\gamma} + r\left(1 - \frac{1}{\gamma^2}\right). \tag{25}$$

Here, $r = 0$ if we demand the Planck–Einstein translational work, or $r = \gamma$ for the Einstein–Ott view without such work. Again, we only require $\theta(1) = f(1) = 1$ so that in the rest frame everything reduces to CT. Different choices of θ, f, and r entail different concepts of relativistic temperature, but also other thermodynamic relations (and hence the thermodynamic laws), since $dQ/TdS = 1/\theta$, $dQ/dQ_0 = f$, and $T/T_0 = \theta f$. For some choices, we find that the moving temperature is:

- lower ($T/T_0 = 1/\gamma$) if $\theta = 1$, $f = 1/\gamma$, and $r = 0$;
- higher ($T/T_0 = \gamma$) if $\theta = 1$, $f = \gamma$, and $r = \gamma$;
- invariant ($T/T_0 = 1$) if $\theta = \gamma$, $f = 1/\gamma$, and $r = 0$.

No choice preserves all intuitions about $T_{\text{classical}}$ and CT. Demanding a lower (higher) moving temperature leads to non-classical behavior. Landsberg (1970, 260–262) considers two thermally interacting bodies *A* and *B* moving relative to one another. *A* (in its rest frame) sees the other as cooler (warmer) and hence heat flows from (to) *B*. But the same analysis occurs in *B*'s rest frame to opposite effect! So heat flow becomes frame-dependent and indeterminate, contrary to our classical intuitions.

However, demanding temperature invariance entails that the classical laws of thermodynamics are no longer form-invariant in all inertial frames. Notably, we must revise their form by including some variations of functions f, γ, and θ (Landsberg 1970, 264). We preserve some intuitions about heat flow but give up the cherished form of classical thermodynamical laws. Interestingly, it's this classical form that Bekenstein (1973) appealed to when making the formal analogies between thermodynamics and black holes.

I end with two possible interpretations of my analysis: an *eliminativist* viewpoint, and a *pluralist* viewpoint (Taylor and Vickers 2017). On the former, one might interpret temperature akin to *simultaneity*: both concepts are well-defined within some rest frame, but there's no absolute fact of the matter as to how they apply *beyond* for relatively moving observers. If one believes that the only physically significant quantities are those which are frame-invariant or co-variant (recall fn. 2), temperature's frame-dependence might lead one to abandon talk of temperature as physically significant, just as we have for simultaneity (Janis 2018).

On the latter, one might *instead* interpret temperature akin to relativistic *rotation*. Analogously, Malament (2000) identifies two equally plausible criteria for defining rotation which agree in classical settings, yet disagree in general-relativistic settings. Importantly, both violate some classical intuitions. Nevertheless:

> There is no suggestion here that [this] poses a deep interpretive problem ... The point is just that ... we may have to disambiguate different criteria of rotation, and ... that they all leave our classical intuitions far behind. (Malament 2000, 28)

On this view, we accept that $T_{\text{classical}}$ breaks down: (relativistic extensions of) classical procedures fail to unequivocally define a relativistic temperature. However, we need not abandon *temperature* altogether; we need only disambiguate and generalize the concept of temperature (and thermodynamical laws).

In any case, I hope to highlight how *messy* the situation is in relativistic thermodynamics. While physicists continue to chime in (McDonald 2020), not much has been said by contemporary philosophers, despite "how rich a mine this area is for philosophy of science" (Earman 1978, 157). Besides Earman, the only other philosopher to have discussed this topic in detail appears to be his student, Liu (1992, 1994). Through this paper, I hope to have at least re-ignited some interest in this topic.

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