THE LOCATION OF THE ZZ CETI STARS ON THE HERTZSPRUNG-RUSSELL DIAGRAM

John P. Cox

and

Carl J. Hansen

Joint Institute for Laboratory Astrophysics*
and Department of Physics and Astrophysics
University of Colorado
Boulder, Colorado 80309

Abstract

By means of a simple analysis, it is shown that the combination of long periods and relatively high effective temperatures in ZZ Ceti variables may be made consistent with each other if the driving region for instability occurs at a temperature of (1-2) x 10^5K. Since this temperature does not coincide with the zone of He^+ ionization, the driving mechanism is then not the same as for the Cepheid variables and the linear extension of the Cepheid strip down to the ZZ Ceti white dwarfs may be regarded as a coincidence. A Stellingwerf "bump" type mechanism may be more appropriate for the latter. However, it is also shown that the short period modes computed by Dziembowski and found to be unstable, are consistent with the Cepheid mechanism. A special plea is made for further opacity calculations at temperatures of ~10^5 K and densities within a few orders of magnitude of unity -- a ρ-T region of considerable interest for the envelopes of ZZ Ceti variables.

It has been noted by Van Horn (1978), Nather (1978), and Hansen (1979) that, if the Cepheid instability strip is extrapolated linearly downward on a log (luminosity) - log (effective temperature) (or H-R) diagram, the strip will intersect the observed ZZ Ceti variables (see Fig. 3 of Van Horn 1978). This fact has prompted the conjecture (Robinson and McGraw 1976; McGraw 1977; Van Horn 1978) that the same physical mechanism that is responsible for the instability of the Cepheid variables, may also be the basic cause of the variability in the ZZ Ceti variables (namely, the kappa and gamma mechanisms in the He^+ ionization zone).

*Operated jointly by the National Bureau of Standards and the University of Colorado.

https://doi.org/10.1017/S0252921100076259 Published online by Cambridge University Press

While this conjecture is appealing, and the above coincidence is remarkable, we wonder if this coincidence is not purely fortuitous, i.e., a "red herring." After all, the long periods of the ZZ Ceti variables (200-1000 sec) are strongly suggestive of high-order non-radial g modes, whereas the periods of Cepheids and Cepheid-like stars presumably correspond to a radial fundamental (or possibly to a radial first overtone). If this change in modal character of the oscillations is taken into account, then (assuming the same physical mechanism causes the instability of the ZZ Ceti variables as causes the instability of the Cepheid variables) we conclude that the instability strip should curve toward lower effective temperatures at lower luminosities, being almost vertical on an H-R diagram, and thus should fall coolward of the observed ZZ Ceti variables. Physically, for a star of given mass and luminosity, a period in the range 200-1000 sec would imply considerable heat content, and hence mass, above the level ($\sim$[40-50] $\times$ 10^3 K) of He^+ ionization. Since the mass above a given temperature level increases strongly with increasing radius, or decreasing effective temperature, any vibrational instability of the assumed kind could set in at the observed periods only at considerably lower effective temperatures (say around 5000 K) than those (~10,000 K) observed for the ZZ Ceti variables.

More quantitatively, the requirement of "driving" by an envelope ionization mechanism is that, roughly, the internal energy in the layers above the driving region must be of the order of the energy radiated by the star in a pulsation period. In symbols,

$$\frac{\langle c_V T\rangle \Delta m_*}{\Pi L} \sim 1 \quad , \tag{1}$$

where angular brackets denote an appropriate average over the layers lying above the driving region, Δm_* is the mass contained in these layers, Π is the pulsation period, and L is the equilibrium luminosity of the star. To rough order of magnitude, the above condition also applies to nonradial oscillations. We now assume that most of the contribution to $\langle c_V T\rangle$ arises from the layers near the driving region, and we accordingly replace $\langle c_V T\rangle$ by $c_V T_*$, where an asterisk denotes the driving region itself, whatever its physical nature (we have here assumed that c_V is essentially constant in the layers above the driving region).

We assume that Δm_* is related to P_*, the total pressure at the level of the driving region, by the requirement of hydrostatic equilibrium:

$$P_* \sim G\,\frac{M\Delta m_*}{4\pi R^4} \quad , \tag{2}$$

where G is the gravitation constant, M is the stellar mass, and R is the equilibrium radius of the star. We also assume that the period obeys the period-mean density relation:

https://doi.org/10.1017/S0252921100076259 Published online by Cambridge University Press

$$\Pi \propto Q \frac{R^{3/2}}{M^{1/2}} \quad , \tag{3}$$

so that Q may be thought of as a kind of period for a star of given mass and radius. Finally, we express R in terms of L and T_e (equilibrium effective temperature) by use of the relation $L \propto R^2 T_e^4$. Equation (1) then becomes (taking c_V as a constant)

$$\frac{P_* T_* L^{1/4}}{Q T_e^5 M^{1/2}} \approx \text{const.} \tag{4}$$

We now assume that the structure of the envelope is that of a radiative zero envelope obeying, on the average, a Kramers-like opacity (in particular, opacity $\propto$ density). We therefore have

$$P_* \propto \left(\frac{M}{L}\right)^{1/2} T_*^4 \quad . \tag{5}$$

Combining this result with equation (4) we obtain, finally,

$$\frac{T_*^5}{Q T_e^5 L^{1/4}} \approx \text{const.} \quad , \tag{6}$$

which should hold along an envelope ionization type instability strip, under the above assumptions. If adiabatic convective transfer had been assumed in the envelope, we would have $P_* = K T_*^{2.5}$, and equation (6) would have been replaced by

$$\frac{K T_*^{3.5} L^{1/4}}{Q T_e^5 M^{1/2}} \approx \text{const.} \quad , \tag{7}$$

where there does not exist a simple expression for K.

We note, first, that if both T_* and Q are constants, equation (6) yields $L \propto T_e^{-20}$, which describes very well the usual Cepheid instability strip on the H-R diagram. (This relation also applies, very closely, to the above-mentioned linear extrapolation of the Cepheid strip down to the ZZ Ceti stars.) However, keeping Q constant implies no change in the nondimensional pulsation period. Hence, if the Cepheids are pulsating in, say, the radial fundamental, then so also ought the ZZ Ceti variables, or at any rate in nonradial modes with about the same period as the radial fundamental. But the period of the radial fundamental in a typical white dwarf is only a few (say 5) seconds, roughly 10^2 times shorter than the observed periods of the ZZ Ceti stars.

https://doi.org/10.1017/S0252921100076259 Published online by Cambridge University Press

If only T_* is constant, then, according to equation (6), $QT_e^5 \approx$ const. for a star of given L. Hence an increase in Q by a factor of $\sim 10^2$ would imply a diminution of T_e by a factor of ~2-2 1/2, i.e., $T_e \sim$ 4000K - 5000K. Thus, if the excitation mechanism for the ZZ Ceti variables is the same as for the Cepheids, then we conclude that the ZZ Ceti instability strip should fall well coolward, on the H-R diagram, of the observed ZZ Ceti variables. At such effective temperatures, the assumption of a radiative envelope would no doubt be invalid.

Dziembowski (1977) has found unstable g-modes for white dwarfs with $T_e \sim 10^4$ K only for the case when the multipole order and harmonic number were high ($\ell \sim$ 100-400, k ~ 15-20). These high order modes have periods of ~10 sec and Dziembowski states that the region of destabilization was the He^+ ionization zone. A. N. Cox et al. (1979), incidentally, have also found instability in high-order radial modes (periods ~0.3-3 sec), with destabilization due to a combination of driving by the He^+ ionization zone at $T \sim (40-50) \times 10^3$ K and the Stellingwerf "bump mechanism" at $(1-2) \times 10^5$ K. Dziembowski's conclusion is entirely consistent with our preceding discussion since periods of ~10 sec imply $T_* \sim (40-50) \times 10^3$ K. Hence Dziembowski's unstable modes represent the same effect as seen in the Cepheids and these modes occur at the correct effective temperature. Why, then, are the longer period, low order g-modes (periods ~ few $\times 10^2$ sec) stable in Dziembowski's analysis? We will return to this question shortly.

We note another way by which the relation $L \propto T_e^{-20}$, i.e., a linear downward extrapolation of the Cepheid instability strip to the ZZ Ceti stars, could be consistent with the idea of "driving" by an envelope ionization type mechanism. That is for the quantity T_*^5/Q to be approximately constant. If Q for the ZZ Ceti variables were increased by a factor $\sim 10^2$ over the value appropriate for Cepheids, then T_* for the ZZ Ceti variables would be ~2-2 1/2 times the value appropriate for the Cepheids. Taking this last value as $\sim (40-50) \times 10^3$ K, the value of T_* appropriate for the ZZ Ceti variables would have to be $\sim (1-2) \times 10^5$ K, which is very close to the temperature corresponding to the Stellingwerf (1978,1979) "bump." (Taking $T_*^{3.5}/Q$ to be constant, as in eq. (7) for convective envelopes, would not change this conclusion significantly.)

We now return to the question of why Dziembowski (1977) did not observe unstable low order g-modes where, by our simple analysis, we might expect significant driving in regions of $T_* \sim (1-2) \times 10^5$ K. In fact Dziembowski states that these modes were stabilized by damping in layers having temperatures exceeding the temperature of the He^+ ionization zone. We should like to point out that in white dwarf envelopes of $T_e \sim 10^4$ K local temperatures of $T \sim (1-2) \times 10^5$ K are associated with densities ranging from 10^{-3} to 1 g cm^{-3} (Fontaine and Van Horn 1976; Winget and Van Horn 1979), depending on composition. This may be regarded, in our picture, as the crucial regime. However, an inspection of the opacity tables of Cox and Tabor (1976; and references therein) reveals that, especially for the higher densities, their nearly pure helium mixture tables (we assume gravitational settling to have occurred in the ZZ Ceti variables) are either quite noisy or no entries exist at all. This implies that both the envelope structure and the necessary opacity derivatives (used in a stability analysis) are open to serious question. Thus, for now, we cannot answer the question originally posed, but rather plead for more opacity information in this (perhaps) critical ρ-T regime.

We conclude that the excitation mechanism for the oscillations of the ZZ Ceti stars is not likely to be the same as that for the Cepheids, in spite of the remarkable coincidence that a linear downward extension of the Cepheid instability strip to the ZZ Ceti stars crosses them almost exactly. However, such a downward extension would be consistent with driving by a Stellingwerf "bump" type mechanism. A confirmation of our remarks must await more opacity calculations.

This work was supported in part by NSF Grant No. AST78-24115 through the University of Colorado.

References

Cox, A. N., Hodson, S. W., and Starrfield, S. G. 1979, in Proceedings of the Workshop on Nonradial and Nonlinear Stellar Instabilities, eds. H. Hill and W. Dziembowski (Springer, in press).

Cox, A. N., and Tabor, J. E. 1976, Ap. J. Suppl., 31, 271.

Dziembowski, W. 1977, Acta Astron., 27, 1.

Fontaine, G., and Van Horn, H. M. 1976, Ap. J. Suppl., 31, 467.

Hansen, C. J. 1979, in Proceedings of the Workshop on Nonradial and Nonlinear Stellar Instabilities, eds. H. Hill and W. Dziembowski (Springer, in press).

McGraw, J. T. 1977, Ph.D. Dissertation, University of Texas.

Nather, E. 1978, Publ. A.S.P., 90, 477.

Robinson, E. L., and McGraw, J. T. 1976, in Proc. Solar and Stellar Pulsation Conference, ed. A. N. Cox and R. G. Deupree, LASL Report LA-6544C, p. 98.

Stellingwerf, R. F. 1978, Astron. J., 83, 1184.

Stellingwerf, R. F. 1979, Ap. J., 227, 935.

Van Horn, H. M. 1978, in Current Problems in Stellar Pulsation Instabilities, eds. D. Fischel, J. R. Lesh, and W. M. Sparks (NASA Spec. Publ.).

Winget, D., and Van Horn, H. M. 1979, private communication.