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A REMARK ON THE PROOF OF A THEOREM OF LAUFER AND TOMBER

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In this note we give a correction to the proof of the following theorem [**1**, Theorem 2].

THEOREM. *Let* $\mathfrak{A}$ *be a flexible, power-associative algebra, over an arbitrary algebraically closed field* Ω *of characteristic* 0. *If* $\mathfrak{A}^{(-)}$ *is a simple Lie algebra, then* $\mathfrak{A}$ *is a simple Lie algebra isomorphic to* $\mathfrak{A}^{(-)}$.

Step (i) of the proof, which proves that the Cartan subalgebra $\mathfrak{H}$ of $\mathfrak{A}^{(-)}$ is a nil subalgebra of $\mathfrak{A}$, is incomplete. Assuming that $\mathfrak{H}$ is not a nil subalgebra of $\mathfrak{A}$, there exists an idempotent $e \neq 0$ in $\mathfrak{H}$. Setting $ex - xe = \alpha x$, $ex = \beta x$, and $ey - ye = -\alpha y$, $ey = \beta' y$, where $\alpha \neq 0$ in Ω and $x \neq 0$ and $y \neq 0$ belong to the roots $\alpha(\mathfrak{H})$ and $-\alpha(\mathfrak{H})$, respectively, Laufer and Tomber conclude that $\beta + \beta' = 0$ by claiming that xy or yx belongs to the root $\beta + \beta'$ relative to e. But since the left multiplication L_e in $\mathfrak{A}$ is not in general a derivation of $\mathfrak{A}$, xy may not belong to the root $\beta + \beta'$ of L_e.

In fact, from the flexible law $L_{xy} - L_y L_x = R_{yx} - R_y R_x$ we obtain $R_e - R_e^2 = L_e - L_e^2$ and this applied to x and y in turn implies that $\beta = \frac{1}{2}(1 + \alpha)$ and $\beta' = \frac{1}{2}(1 - \alpha)$, thus $\beta + \beta' = 1$. However, it follows from [**2**] that $\mathfrak{A}$ is a nil algebra. Indeed, if $\mathfrak{A}$ is not nil, then since $\mathfrak{A}$ is a simple, flexible, power-associative algebra, it is shown in [**2**] that $\mathfrak{A}$ has an identity element 1. Hence 1 belongs to the centre of $\mathfrak{A}^{(-)}$, but since $\mathfrak{A}^{(-)}$ is a simple Lie algebra, this is impossible and therefore $\mathfrak{A}$ is nil and so is $\mathfrak{H}$.

REFERENCES

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