CHAPTER III. Variability

# A status report on AGN variability

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**Abstract.** Active Galactic Nuclei (AGN) are ubiquitous variable sources. This trademark property allows the study of many aspects of AGN physics which are not possible by other means. In this review I summarize what has been learnt by the close monitoring of AGN flux variations with special emphasis in studies conducted in optical and near-infrared domain. I also highlight what knowledge is still missing from our picture of AGN phenomena, as well as possible developments expected in this new era of time-domain astronomy.

Keywords. galaxies: active, galaxies: nuclei, surveys, methods: statistical

## 1. Introduction

The Preface to the Proceedings of the First Texas Symposium on Relativistic Astrophysics, titled "Quasi-Stellar Sources and Gravitational Collapse", includes the very words that Peter Bergman sent around in June 1963 to motivate the symposium, which include: 'The source 3C273B seems to be a superstar, and according to Harlan Smith, has a diameter of about a light-week. It is the brightest known object in the universe, about a million million times brighter than the sun. According to Sandage and Smith, its brightness varies by about 50 percent'.

More than 50 years later, we know that these sources are powered by Super Massive Black Holes (SMBHs) found in the centers of galaxies. When actively accreting matter, gravitational energy is released as radiation and the SMBHs are classified as Active Galactic Nuclei (AGN). As with the case of 3C273B, AGN emission is highly variable at all wavelengths, from the radio to the gamma-rays. As the radiation shines onto the surroundings of the SMBH, this material responds to the variable signal. The study of the primary variable radiation source and the response from nearby structures has been one of the most important tools used by astronomers to study the structure and physical nature of AGN.

In this review I will focus on recent advancements in our understanding of AGN variability. This includes clues on the nature of the primary variable source itself, as well as of those structures that surround SMBHs, such as the accretion disk, the Broad Line Region (BLR) and the dusty torus. I will focus on optical and infrared studies and touch upon X-ray findings, while I redirect the reader to other works that cover high-energy and radio emission, wavelengths which trace the X-ray corona and jet, such as those by Uttley (2015) and the review on radio properties of AGN by Tadhunter (2016) and the results by Mundell *et al.* (2009).

## 2. Continuum emission variability below $1\mu$ m: the primary source and the accretion disk

Most AGN show UV, optical and near-infrared variability on *short time scales*, this is, time scales of the order of days to weeks for SMBHs of masses in the  $10^6-10^8 M_{\odot}$ 

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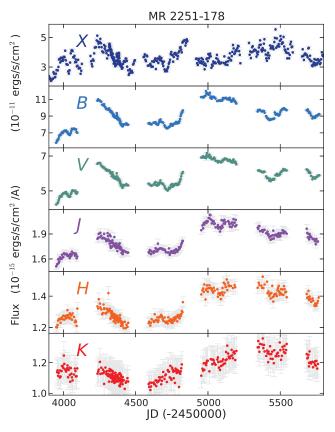


Figure 1. X-ray, optical and near-infrared light curves for the nearby quasar MR2251-178.

mass range or weeks to months for a  $10^8 - 10^{10} M_{\odot}$  mass range. Exception to this rule seem to be a not well quantified fraction of Narrow Line Seyfert 1 nuclei (NLS1s), which tend to show flat light curves despite showing variability in the X-ray and UV domain (e.g., Shemmer & Netzer 2000; Klimek *et al.* 2004; Yip *et al.* 2009; Bachev *et al.* 2009). A particular Spectral Energy Distribution (SED) might be responsible for this behaviour, as I will discuss later. Notice, however, that samples of optically varying NLS1s have also been identified and closely studied (Du *et al.* 2014).

Continuum variability below  $1\mu$ m seems to be highly correlated in all sources monitored with well sampled multiwavelength light curves (e.g., Cackett *et al.* 2007; Shappee *et al.* 2014; Edelson *et al.* 2015; Fausnaugh *et al.* 2016). An example is shown in Fig. 1, which presents X-ray, optical and near-infrared light curves for the nearby quasar MR2251-178 during ~5.5 years of monitoring. Partial results from this campaign were already presented in Arévalo *et al.* (2008) and Lira *et al.* (2011). Besides the clearly observed correlation, other results are the following : 1) there is a small but significant delay between light curves, with longer wavelengths following shorter wavelengths; 2) short time scale variability decreases with wavelength; 3) the 2–10 keV X-ray emission, while well correlated during the first 3 years of the monitoring, is not longer correlated during the remaining of the campaign.

The results on MR2251-178 are replicated in many sources. Time lag determinations between UV and optical continuum light curves give results consistent with only a few day-lights for typical Seyfert and low-mass quasars. Fitting a power-law to the observed lag versus wavelength correlation results in values close to those predicted by the classical

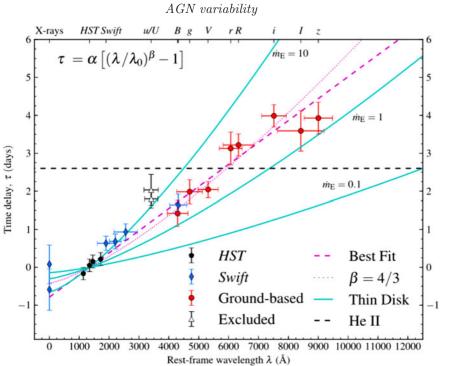


Figure 2. Time delays between X-ray, UV and continuum optical light curves determined for NGC5548. Lines represent model responses to a central illuminating source for accretion disks following the prescription  $\tau \propto \lambda^{\beta}$  (top left corner). Cyan curves represent models with a fixed  $\beta = 4/3$ , as prescribed by standard accretion disk models, and accretion rates  $\dot{m}_{Edd} = 0.1, 1$  and 10. The dotted line gives the best fit for  $\beta = 4/3$ , but its zero point requires an accretion rate much too high for what is inferred for this AGN. The best fit model, shown with a dashed line, requires  $\beta \sim 1$ . Taken from Fausnaugh *et al.* (2016).

accretion disk proposed by Shakura & Sunyaev (1973), i.e.,  $f_{\nu} \sim \nu^{1/3}$  although with a larger zero point by about a factor of 3 (see Fig. 2). In other words, the varying signal matches well the time it would take for light to travel from the center of the system to the peripheries of the accretion disk. What we are witnessing is not intrinsic disk variability, but the response from the disk to a centrally illuminating variable source. The compilation of the RMS spectrum for 849 quasars observed by SDSS-IV is astonishingly close to a  $\nu^{1/3}$  power-law, supporting again the prescription of Shakura & Sunyaev (Horne *et al.* in prep). The mismatch in the zero point, which is a function of SMBH mass and accretion rate, is a current matter of debate and could point to the failings of the simplicity of classical accretion disk models, as suggested by numerical simulations (Mishra *et al.* 2016, 2019; Sadowski 2016; Jiang *et al.* 2013, 2016, 2019; Gronkiewicz & Różańska 2020 – see further discussion in Section 4).

One very important question has not been answered yet: what is the central variable source? SED modelling can shed some light onto this question. The illuminating source must be found close to the SMBH and should therefore be highly energetic, at frequencies beyond the near-UV. Energetically, it needs to be powerful enough to imprint a variable signal which can be detected *against* the intrinsic emission of the accretion disk. Hence, the ratio between the emission seen (or estimated) in the optical/near-UV (from the disk) over that seen (or estimated) in the far-UV/X-rays (the central source?) can tell us whether short-term variability will be observed as result of the central source illumination the accretion disk. It has been argued that in the case of NLS1s, the dominant disk does not show such variability as it swamps the central varying source (Jin *et al.* 2017a,b).

MR2251-178 demonstrates that the hard 2–10 keV X-ray emission, although a good candidate during the first half of the campaign and with a delay with the B and V bands consistent with 0 days (Arévalo et al. 2008), cannot be the illuminating source, as it goes completely uncorrelated during the second half of the campaign. Lack of correlation between the X-rays and UV/optical emission is also seen in many other Seyfert galaxies, such as NGC 7469 (Behar et al. 2020), Mrk 817 (Morales et al. 2019), Mrk 335 (Gallo et al. 2018), Ark 120 (Gliozzi et al. 2017), PKS 0558-504 (Gliozzi et al. 2013). But some level of correlation is seen sometimes, as is the case of MR2251-178 (Arévalo et al. 2008, Fig. 1), NGC3783 (Arévalo et al. 2009), NGC4395 (McHardy et al. 2006; Cameron et al. 2012), NGC4593 (Pal & Naik 2018), NGC4051 (Breedt et al. 2010), NGC2617 (Shappee et al. 2014; Fausnaugh et al. 2018), NGC6814 (Troyer et al. 2016), Mrk79 (Breedt et al. 2009). In fact, some sources have shown a good correlation during some campaigns to become uncorrelated at some other time, e.g., NGC5548 (McHardy et al. 2014; Starkey et al. 2017) and F9 (Pal et al. 2017; Lohfink et al. 2014). This complex scenario can be summarized as follows: the highly variable X-ray emission arises from a different structure than the much smoother and well correlated UV/optical emission. This is in good agreement with our general hypothesis that the X-rays originate from a 'corona' of fast electrons that upscatter seed photons provenient from the accretion disk. The accretion disk, at the same time, reprocesses high-energy photons in a well behaved manner, showing variability time scales in agreement with the travel time between regions characterized by different temperatures in the stratified accretion disk. If there is correlation between the X-rays and the longer wavelength light curves, the lags are consistent with  $\sim 1$  day, although some longer lags have also been measured (Shappee et al. 2014; Edelson et al. 2017), making things harder for the reprocessing model.

Gardner & Done (2017) analyze the lack of correlation between the X-ray emission and that observed in the UV and optical bands for NGC5548. They propose that a new region has to be added to our paradigm in order to understand these observations. The new region would be found between the central high-energy source and the disk and would reprocess the X-ray central emission into far-UV photons that finally heat the accretion disk (Mehdipour *et al.* 2011; Middei *et al.* 2018; Ursini *et al.* 2019).

## 3. Continuum emission variability beyond $1\mu$ m: the dusty torus

Dust grains will survive to the high temperatures maintained by the central emission from the so-called sublimation radius and beyond. Dust exposed to the AGN radiation will absorb short-wavelength emission and re-emit it with typical temperatures of many hundreds to a thousand Kelvin (i.e., much higher than those experienced in starburst galaxies).

The assumption that this hot dust is arranged into a toroidal shape is based on observational evidence (see Netzer 2015 for a comprehensive review): the axisymmetric nature of the accretion flow suggests that, if the dust is associated to the parsec-scale gas dynamics, then a similar axisymmetric dust structure could be expected; in some objects collimation of the ionizing continuum that reaches to large distances from the central SMBH is thought to be caused by the dusty torus; the presence of polarized broad emission lines in systems that do not show these components in direct light has been considered the strongest evidence for the so-called Unification Model of AGN, where the line of sight of the observer to the central source, either intercepting or not the toroidal structure, determines whether the AGN appears as a 'type-I' or 'type-II' nucleus.

Evidence supporting the Unified Model is vast and shows that even though the situation is probably much more complex than original thought, and that not all AGN follow the model strictly, the presence of something similar to an axisymmetric dusty structure in many AGN is true. Recently, ALMA has been able to detect the presence of dust emission from the center of a few very nearby Seyfert galaxies that might indeed correspond to the torus (Combes *et al.* 2019).

The cross-over between the emission from the accretion disk and the torus is found at about  $1\mu$ m, i.e., at the limit between the optical and near-infrared domain. Hence, the K-band corresponds to the near-infrared wavelength where the torus emission dominates, while the J and H bands might still show a strong disk contribution. Cross-correlation between K-band light curves and those obtained in the optical show that the torus follows the variations seen in the accretion disk but with a lag corresponding to the light travel time to the sublimation radius (Clavel *et al.* 1989; Suganuma *et al.* 2006; Koshida *et al.* 2014; Lira *et al.* 2011, 2015; Pozo *et al.* 2014, 2015; Mandal *et al.* 2018). The lags observed in local Seyfert galaxies are in the 40–400 day range, corresponding to distances of fractions of a pc to a few pc. These distances are larger than those inferred for the BLR (see next Section), therefore supporting the most basic requirement of the Unified Model, i.e., that the presence of the dusty torus is able to hide the central region of the AGN from the viewer.

## 4. Line emission variability: the physics of the BLR

The idea that studying the response of the BLR to the variable central continuum could reveal important clues about the nature of the BLR was first suggested by Bahcall *et al.* (1973) and Cherepashchuk & Lyutyi (1973). It was put into mathematical formalism by Blandford & McKee (1982), who also coined the term Reverberation Mapping (RM). The first observational campaigns were carried out in the 1980s. The principle is simple: through spectroscopic monitoring of a galactic nuclei it is possible to isolate the continuum and line fluxes, allowing for the construction of separate light curves. The cross-correlation of continuum and line-emission light curves would then allow to determine the time lag, or light travel distance between the central engine and the BLR. Such campaigns, are however, very costly in terms of observing time, and require a very careful analysis to secure the proper flux calibration of the light curves and meaningful error estimations.

The determination of lags using cross-correlation analysis can be done in different ways. The z-transformed discrete correlation function (ZDCF – Alexander 1997) works solely with the observed values of the light curves and it is based on the discrete correlation function (DCF) of Edelson & Krolik (1988). The widely used interpolated cross correlation function (ICCF, Peterson *et al.* 1998, 2004) interpolates fluxes to a desired cadence assuming that the line and continuum fluxes in gaps between two observed points are properly approximated by a linear interpolation in time between the two. Finally, more recent methods such as JAVELIN (Zu *et al.* 2011) and CREAM (Starkey *et al.* 2016) model the light curves as a damped random walk process (see below) to determine a lag and its significance. Their basic assumption is that the emission line light curves are the result of the response to an ionizing continuum which is changing exactly in the same way as the observed continuum used in the lag determination, which however, is not always observed to be the case (Goad *et al.* 2016; Lira *et al.* 2018). For well sampled light curves all methods give results which are consistent with each other, as expected.

Early results from RM campaigns immediately yielded significant findings. The BLR was an extended and stratified structure, with different emission lines been produced by gas located at different distances from the central source. This allowed radiative transfer models to reproduce many of the BLR traits, now assuming that different 'cloud' conditions yielded the diverse family of emission lines seen in AGN spectra.

Later findings gave evidence for a 'Virialized' BLR, this is, the BLR is a gravitationally bound structure (Peterson & Wandel 1999; Peterson *et al.* 2004). This is in fact a corner stone for SMBH mass determinations based on RM of the BLR. The relation  $M_{\rm BH} = fGv^2/R$  shows that we can determine the mass of the SMBH  $(M_{\rm BH})$  if we know the location (R) and speed (v) at which an object that is under the gravitational influence of the black hole travels. While the width of the Doppler-shifted broad emission lines gives v, RM analysis yields the much sought after value of R. f, remains a fairly unconstrained factor that encompasses the details of the geometry and kinematics of the BLR and it is expected to vary from source to source (Collin *et al.* 2006), although recent progress seems to agree about a BLR in the form of a rotating flattened system (Pancoast *et al.* 2014; Grier *et al.* 2017; Williams *et al.* 2018; Mejía-Restrepo *et al.* 2018; Li *et al.* 2018; Martínez-Aldama *et al.* 2019).

One of the most important results that came from RM studies of the BLR is the socalled 'Radius-Luminosity' (RL) relation. This is an observational result that correlates the luminosity of the continuum emission with the distance at which a BLR particular line is produced. The RL relation allows to obtain SMBH masses with a single spectrum! (the so-called 'single-epoch' method), since the continuum luminosity and line width can be determined from one single, flux-calibrated spectroscopic observation. Until now, well determined RL relations based on RM results have only been established for H $\beta$  (Kaspi *et al.*, 2000, 2005; Peterson *et al.* 2004; Bentz *et al.* 2006, 2009, 2013; Grier *et al.* 2017), although cross-calibration with MgII  $\lambda$ 2798 shows that no significant biases are present when using this line (McLure & Dunlop 2004; Shen & Liu 2012; Zuo *et al.* 2015; Mejía-Restrepo *et al.* 2016), allowing the determination of  $M_{\rm BH}$  up to  $z \sim 2$  when using optical observations (e.g., Trakhtenbrot & Netzer 2012) and up to  $z \sim 5$  when observing MgII  $\lambda$ 2798 in the observed-frame near-infrared (e.g., Jiang *et al.* 2007; Kurk *et al.* 2007; Willott *et al.* 2010; Trakhtenbrot *et al.* 2011; De Rosa *et al.* 2014; Mazzucchelli *et al.* 2017; Shen *et al.* 2019; Onoue *et al.* 2019).

RM studies are now witnessing a new era, where the boundary is been pushed in terms of redshift, observed lines, and number of studied sources, with recent results (Shen *et al.* 2016; Lira *et al.* 2018; Grier *et al.* 2019; Hoormann *et al.* 2019) and future projects, such as the SDSSV Black Hole Mapper (Kollmeier *et al.* 2017) and the 4MOST Time-Domain Extragalactic Survey (TiDES) program (Swann *et al.* 2019) expanding our knowledge of the BLR. These huge datasets will also bring important challenges, as has already been shown by recent RM results presenting systematic offsets between these new determinations and the 'classical' results of Bentz et al. (2014). Czernic et al. (2019) explores these differences and argues that they can be understood if a range of spin values is allowed when interpreting RL relations.

The very best RM data can afford a more thorough investigation of the BLR using 'tomography', this is, the study of the line response as a function of velocity. As different regions of the broad lines correspond to regions traveling at different velocities, their changes as a function of time allow us to have a close view of the BLR kinematics. In fact, exquisite data might one day allow us to study the presence of spiral arms in the accretion disk of AGN, but so far tomography has been used to put constrains on the presence of bulk BLR motions, such as outflows and inflows (Horne *et al.* 2004).

The most intense RM campaign is that of the nearby Seyfter galaxy NGC5548 which took place in 2014 using ground and space-based facilities. Continuum lags once again showed that the best-fit model to the lag vs. wavelength relation corresponds to a powerlaw with index ~1/3 (Edelson *et al.* 2015), in good agreement with the prescriptions of classical accretion disks, but with a zero point, which is a function of the  $M_{BH}M_{\odot}$ product (where  $\dot{M}_{\odot}$  is the accretion rate – Fausnaugh *et al.* 2016), too large by about a factor 3, a result found in several other monitoring campaigns (McHardy *et al.* 2014; Shappee *et al.* 2014; Lira *et al.* 2015). However, arguably the most interesting result of the campaign was that of the 'BLR-holiday', corresponding to nearly half of the campaign length, where the line flux light curve did not seem to be responding to the observed continuum, but instead showed an uncorrelated behaviour (Goad *et al.* 2016 – see Fig. 3).

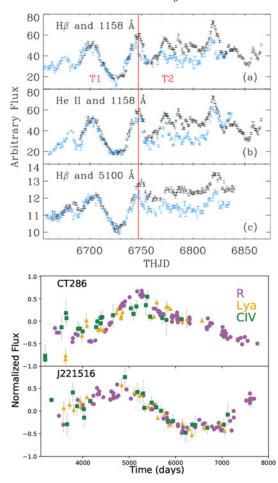


Figure 3. Continuum and emission line light curves for three different AGN. Top panel shows the observed 'BLR-holiday' (from the vertical red line onwards) of the emission lines in NGC5548, which after ~100 days of monitoring present an uncorrelated behaviour with respect to the UV and optical continuum. Taken from Pei *et al.* (2017). Bottom panel shows the response of Ly $\alpha$  and CIV emission lines to the observed R-band (rest-frame ~1500 Å) continuum. While emission lines and continuum follow each other almost perfectly in J221516, the response in CT286 is clearly more complex. Adapted from Lira *et al.* (2018).

The anomalous behaviour shown by NGC5548 highlights one of the most important shortcomings during RM analysis: that the observed continuum is not the ionizing continuum responsible for the formation of emission lines. In particular, recombination Hydrogen lines require photons with energies in excess of 13.6 eV (or wavelengths shorter than 912Å), a range which is inaccessible to us observers. Nature does not allow us to observe any photons of astrophysical origin at the Lyman edge, while Galactic emission in the very soft X-ray domain (0.001–0.01 KeV) severely limits our chances to study extragalactic sources. Hence, the true far-UV emission from AGN can only be studied by indirect methods, which represents a huge challenge and introduces significant problems in our understanding of AGN physics as it corresponds to the peak of the emission coming from the innermost region of the accretion disk. The physics of SMBH spin is largely hidden in this unobserved wavelength range.

#### 5. Variability on longer time-scales: Changing Look AGN

It is expected that while short-time scale variations are dominated by the accretion disk response to the illumination from the central variable source, changes in the accretion flow itself may introduce changes on *longer-time* scales. Depending on the physical mechanism, the accretion flow can show fluctuations with time scales ranging from many months to hundreds of years for a SMBH with  $M_{\rm BH} = 10^8 \, M_{\odot}$ , quantities that scale linearly with SMBH mass (e.g., Graham *et al.* 2020).

MR2251-178, with an estimated mass of  $\sim 2 \times 10^8 \text{ M}_{\odot}$  fits well this scenario. As proposed by Arévalo *et al.* (2008), the short-term (days to weeks) variability observed in the optical bands is well explained as illumination of the highly variable X-ray emission (or some other similar central source), while the long-term (months to years) variability shows too large amplitude variations to be explained as reprocessing of the X-ray fluctuations. Hence, changes in the intrinsic accretion flow are invoked.

Changes in the accretion flow have been claimed to be responsible for a much more dramatic behaviour observed in active nuclei: that of Changing Look (CL) AGN. The CL terminology was borrowed from X-ray astronomy (where it refers to changes in the level of obscuration at these wavelengths), to first represent the appearing or disappearing of broad components to the Balmer lines on time scales of months to years, and later to represent also significant continuum variations (e.g., Fig. 4). This redefinition allows to explore large photometric surveys to look for significant flux changes. As an example, in a study of more than 8000 quasars, Rumbaugh *et al.* (2018) found that ~10% of them exhibited more than one magnitude change in flux ( $|\Delta g| > 1$ ) within a time scale of 15 years and claimed that the true fraction could reach 30–50%. Continuum and spectral changes, however, do not necessarily go hand-in-hand, as discussed by Graham *et al.* (2020).

CL AGN has been an exploding area of research in the last decade due to the large number of time domain spectroscopic and photometric measurements now available (e.g., LaMassa *et al.* 2015; McElroy *et al.* 2016; MacLeod *et al.* 2016; Ruan *et al.* 2016; Gezari *et al.* 2017; Ross *et al.* 2018; Rumbaugh *et al.* 2018; Stern *et al.* 2018; Shapovalova *et al.* 2019; Graham *et al.* 2020), although many serendipity detections of CL activity were claimed since the mid 70s.

Occultation of the BLR and accretion disks by intervening material along the line of sight seemed since early on as a possible mechanism responsible for CL systems. Indeed, this process has been clearly identified as the origin for the rapid changes in the X-ray obscuration towards the active nucleus in the type-II Seyfert galaxy NGC1365 (Risaliti *et al.* 2005). In this case, BLR 'clouds' were responsible for producing the transitions in the amount of absorption towards the X-ray emitting region in this nucleus. A similar mechanism for CL AGN, however, usually fails. LaMassa *et al.* (2015) showed that the crossing time across the line of sight of any occulting material towards the BLR was larger than the observed CL time scales.

Using the single-epoch technique, Ruan *et al.* (2016) showed that the inferred SMBH mass estimates for the active nuclei in SDSS J015957.64 + 003310.5 remain consistent with each other when using spectroscopic observations obtained at different epochs in this CL nucleus. The observed *broader when dimmer* effect can be interpreted as the switching-off of distant BLR clouds that produce the broad lines when the continuum becomes dimmer, while new clouds, closer in to the SMBH experiencing a more intense gravitational pull and hence emitting broader lines, switches-on. This results also imply that the BLR remained bound throughout and that the presence of the broad components is not due to other external mechanisms, such as supernova explosions or Tidal Disruption Events (TDEs).

Things can never be that simple, however, as complex behaviour is appearing in some of the best studied CL AGN, as shown in Trakhtenbrot *et al.* (2019) and Ricci *et al.* (2020). They present results from the CL nucleus in 1ES 1927 + 654 which changed from

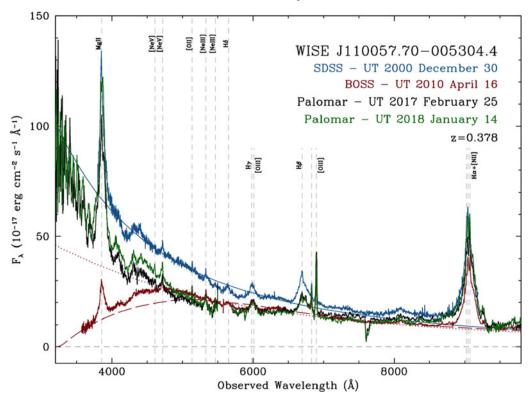


Figure 4. SDSS spectra of a quasar at  $z \sim 0.4$  presenting dramatic changes at the blue end within the time scale of 20 years, indicating changes in the inner part of the accretion disk. BLR and near-infrared torus-related emission should follow suit with the expected delays of a few years. Taken from Ross *et al.* (2018).

a "true" Type-II to a Type-I system in the time scale of months. However, its light curve looks suspiciously similar to that predicted for a TDE, while its X-ray emission showed complex and extremely variable behaviour. This could be freak a case, where the change in the look of this AGN is not due to internal changes, but instead triggered by the infall of a star or a massive cloud onto the central black hole.

The physical mechanism behind CL AGN needs to be explained withing the context of accretion physics. People have looked at what is already known in black hole hosting X-ray Binaries (BHBs), systems that vary in much shorter time scales because of their much smaller sizes. Accreting BHBs have already been united with their radioloud SMBH counterparts through the "fundamental plane for accreting black holes", a relation between  $L_{\rm xrays}$ ,  $L_{\rm radio}$ , and  $M_{\rm BH}$  which covers many orders of magnitudes in all three quantities involved (Merloni *et al.* 2003; Falcke *et al.* 2004).

BHBs are known to show two characteristic emission 'states', the Low/Hard and the High/Soft states. This denominations refer to the observed changes in their X-ray spectra, window where disk emission appears in this low-mass objects. These changes have long been understood as changes in the nature of the accretion flow, with the High/Soft state corresponding to high-accretion periods, with a luminous classical accretion disk, while the Low/Hard state is characterized by the inner truncation of the disk, been replaced by a flow that cools inefficiently during a low accretion-rate period (for reviews see Done *et al.* 2007 and Belloni 2010).

The question that then arises is whether CL AGN correspond to changes of the accretion flow onto SMBHs as seen in BHBs. Some evidence supports this scenario, as both changes are characterized by similar tracks in a Eddington ratio vs spectral-slope plane (Ruan *et al.* 2019). Some evidence seems to support that the AGN CL episodes would also happen when the accretion rate lowers below 1%, in agreement with what is seen in BHBs. However, a crucial problem remains: the time scales are seriously off when scaled based on black hole mass. State transitions in BHBs occur at time scales of ~ days, which would correspond to ~ $10^5$  years for SMBHs in AGN (Ruan et al. 2020).

In a very recent work, Graham *et al.* (2020) analysed the variability properties of a sample of CL AGN candidates selected from the CATALINA survey by requiring a change in the H $\beta$ /OIII ratio larger than 30%, besides other criteria like optical and near-infrared significant variability. They found that 111/717 of nuclei presented the required spectroscopic variability, i.e., ~15%. The variability time scales of the optical continuum in their sample are in reasonable agreement with that predicted for thermal fronts. These were proposed in the early 80s to explain the observed behaviour during outbursts in X-ray binaries and dwarf novae, as partially ionized gas can undergo abrupt changes between hot and cold conditions. The disk structure then cycles between these configurations which also carry sudden changes in mass accretion rate and therefore in the release of gravitational energy. Transition waves or thermal fronts are responsible for this structural changes, either heating or cooling the disk (Meyer 1985; Menou *et al.* 1999). In AGN, thermal fronts have been proposed to be driven also by changes in the magnetic torques exerted in the innermost part of the disk (Ross *et al.* 2018), or by instabilities in radiation pressure (Śniegowska & Czerny 2019).

Clearly, the jury is still out concerning CL AGN and it is very likely that this phenomena will in fact group together more than one type of physical mechanisms. Needless to say things will greatly change once the Vera Rubin Telescope (LSST) is operating since biases due to sparse sampling and flux limits will be greatly reduced. Spectroscopic follow-up for the study of the emission lines will require a great effort from the astronomical community but it will no doubt be needed to secure a thorough understanding of these systems.

# 6. Selecting AGN through variability

Traditionally, AGN have been found by their blue optical colors. In color-color plots (e.g., Richards *et al.* 2009), AGN have been found to occupy a well defined region that separates them from other sources such as stars and galaxies, which usually look redder. This selection works if the AGN are 1) high luminosity (so that the redder colors of their hosts do not contaminate the AGN emission), 2) unobscured (so that the AGN is not reddened by dust absorption), 3) below z < 3 (so that AGN remain well separated from stars). To find AGN that do not fulfill these requirements is a challenge.

AGN are, however, ubiquitously variable. Hence, variable sources located in the centers of galaxies are excellent candidates for accreting SMBH, even if they do not meet one or more of the requirements listed above. With this in mind, many works have focused on the idea of finding AGN using variability as a search criterion (e.g., Butler & Bloom 2011; Myers *et al.* 2015; Peters *et al.* 2015; Palanque-Delabrouille *et al.* 2016; Tie *et al.* 2017).

More recently Sánchez-Sáez *et al.* (2019) have shown that using variability and colors not only is more effective than using colors only, but that a new population of redder AGN can be identified. This population appears to be dominated by low-luminosity AGN, with colors some times completely dominated by the host emission. Obscured AGN are however, mostly missed by this technique, as the presence of significant amounts of dust can completely quench the AGN variable continuum.

Variability searches also open a new window for the detection of Intermediate Mass Black Holes (IMBHs), those with  $M_{\rm BH} < 10^6 {\rm M}_{\odot}$  and above the stellar mass range. This

population is particularly interesting, as it might help to disentangle between possible scenarios for the formation of SMBHs at high redshift, the so-called 'BH seeds' (Greene, Strader & Ho 2019). The demographics of black holes that have not grown much since their formation will help to constrain which of these scenarios are viable by confronting occupation numbers and mass distributions with theoretical predictions (Bellovary *et al.* 2019). Also, BH masses in the  $10^5-10^6$  M<sub> $\odot$ </sub> represent the primary targets for the Laser Interferometer Space Antennae (LISA), which is expected to be launched in the next decade. LISA will detect Gravitational Waves from merging black holes from  $z \sim 0$  to  $z \sim 20$ , overlapping with the cosmological times when the *seeding* of black holes occurred and mapping their presence all the way to the present universe.

Finding IMBHs has not been an easy task and the examples known today probably constitute the tip of the iceberg of the underlying population. The most successful method to probe *bona-fide* IMBHs is to look for broad but weak emission Balmer lines, which together with the radius–luminosity scaling relations, has yielded masses for a few hundred sources after combing the whole SDSS (Liu *et al.* 2018; Chilingarian *et al.* 2018). The search for faint and compact radio sources ('radio cores') associated to the nuclei of galaxies in the COSMOS field has successfully determine the presence of AGN in 35 dwarf galaxies (Mezcua *et al.* 2019), although their bolometric luminosities are found beyond that predicted by numerical simulations

More recently, variability has shown to be able to identify many candidate IMBHs. Martínez-Palomera *et al.* (2020) have selected galaxies which show rapid, low-amplitude variability in their nuclei and shown that this method increases the number density of IMBHs by a factor of 40-50, when compared with spectroscopic searches. About 20% of this candidates have already been confirmed as AGN by the presence of broad Balmer lines, radio cores or by their emission line ratios, typical of AGN.

#### 7. Statistical analysis and description of AGN variability

As has already been noted in this article, many things we have learnt about AGN are the result of what was already known for black holes in stellar systems. The Power Spectrum (or Power-Spectral Density, PSD), this is, the amount of variance as a function of frequency, had already been used to describe the wide range of variability time scales observed in the X-ray light curves of accreting compact stellar systems (e.g., Nolan *et al.* 1981) before been applied to observations of X-rays from AGN (e.g., Lawrence *et al.* 1987).

The PSD of AGN is in fact a close relative to those seen in BHBs during their Soft/High state (for a review see Uttley 2006). They are characterized by  $P(\nu) \propto \nu^{-2}$ , the so called 'red noise', but break to a  $P(\nu) \propto \nu^{-1}$ , a 'flicker noise' regime, at low frequencies. In fact, this characteristic break frequency has been shown to strongly correlate with  $M_{\rm BH}$  (and possibly inversely with accretion rate), a correlation that spans many order of magnitude (McHardy *et al.* 2016; González-Martín & Vaughan 2012) from BHBs to massive AGN.

In the UV, optical and near-infrared, most of the statistical analysis of light curves has been done in the time domain, instead of frequency. This is because light curves are usually not dense enough and do not have enough data points to apply Fourier Transform techniques. Instead, auto-correlation, cross-correlation and structure function analysis are usually adopted. In particular, the structure function (SF) has been widely used to describe AGN light curves as it is closely related to the PSD (see Fig. 5). For example, a single power law description of the PSD can be recovered by the slope observed in the SF. The problem with techniques that work in the time domain, however, is that the measurements are strongly correlated by design, and therefore can fail to provide a robust description of more complex structure in the variability signal (Emmanoulopoulos *et al.* 2010). Dense sampling of AGN in the optical have been achieved only recently

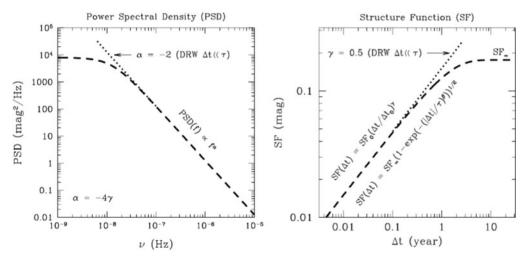


Figure 5. Power Density Spectrum and Structure Function of a DRW signal. Taken from Kozłowski 2016.

using space missions for the search of transiting planets, such Kepler and TESS, allowing for the determination of PSDs for individual sources Smith *et al.* (2018a) show that 6/21 sources present breaks in their PSDs at low frequency, the remaining sources been consistent with a single power-law. High-frequency exponents are found to be  $\sim -2.5$ .

A PSD characterized by  $P(\nu) \propto \nu^{\alpha}$ , with  $\alpha \leq -2$  corresponds to a highly correlated behaviour, such as those displayed by 'random-walk' or 'Brownian motion', or as already mentioned, 'red noise'. Here we understand correlation as the level of independence between *events* (or flux values in a light curve). Random-walk, for example, ties subsequent events, this is, the event that is occurring now has memory of the previous event, and so on. No correlation (characterized by a PSD with  $\alpha = 0$ ), on the other hand, is referred to as 'white-noise'.

The level of correlation or 'memory' in a time sequence can be expressed mathematically in a simple manner when the events are equally spaced in time (fixed  $\Delta t$ ). So for example, if the process driving a light curve is that of a random-walk, then every event  $x_i$  at time  $t_i$  can be expressed as  $x_i = \mu + \phi x_{i-1} + \epsilon_i$ , where  $\mu$  is the mean,  $x_{i-1}$  corresponds to the event that occurred at  $t_{i-1}$ ,  $\phi = 1$  and  $\epsilon_i$  is a random term usually drawn from a Gaussian distribution (Moreno *et al.* 2019). If  $\alpha = 2$  the process is called 'damped random-walk' (DRW) and  $\phi^{-1}$  corresponds to the characteristic time scale of the break in the PSD. DRW has been found to be a very good description of AGN light curves (Kelly *et al.* 2009), although deviations are also found (Kasliwal *et al.* 2015; Simm *et al.* 2016; Sánchez *et al.* 2017). As real data show gaps and unequal time spacing, instead of a using a discrete characterization of the underlying process, a continuous mathematical description is necessary. In its more generic form, this is termed a CARMA process. A CARMA(1,0) process corresponds to a DRW.

Further correlation is found when  $x_i$  depends on the last two previous events and the previous perturbation terms (Moreno *et al.* 2019). In its continuous form this is a CARMA(2,1) process, also known as a 'damped harmonic oscillator' (DHO), characterized by two time scales related to the terms  $\phi_1$  and  $\phi_2$ . A particular case of a DHO corresponds to a 'quasi-periodic oscillator' (QPO). High and low-frequency QPOs have been observed in a few BHBs, IMBH and AGN (Smith *et al.* 2018a and references therein) and growing evidence suggests that these are good black hole mass estimators. With the future of time-domain astronomy and data science looking extremely bright, in the next decade we will be able to learn about the physics of black holes and accretion at an increasing pace, and perhaps unravel some of the best kept secrets of these fantastic systems.

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