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CORRIGENDUM

Hydrodynamics of a quantum vortex in the presence of twist – CORRIGENDUM

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In Foresti & Ricca (2020) (hereafter referred to as FR20) we derived a modified form of the Gross–Pitaevskii equation for a defect subject to twist. A mistake was introduced by the wrong use of the operator $\widetilde{\nabla} = \nabla - \mathrm{i}\nabla\theta_{tw}$. By repeating the same calculations we can see that the mGPE (2.6) must be replaced by the following equation:

$$\partial_t\psi_1 = \frac{\mathrm{i}}{2}\nabla^2\psi_1 + \frac{\mathrm{i}}{2}\left(1 - |\psi_{tw}|^2 - |\nabla\theta_{tw}|^2\right)\psi_1 + \mathrm{i}(\partial_t\theta_{tw})\psi_1 + \frac{1}{2}\nabla^2\theta_{tw}\psi_1 + \nabla\theta_{tw}\cdot\nabla\psi_1. \tag{0.1}$$

Note the extra terms that come from the broken symmetry of the theory under superposition of a local phase.

The Hamiltonian (3.1) then becomes

$$H_{tw} = \tfrac{1}{2}\boldsymbol{p}^2 - \tfrac{1}{2}(1 - |\psi_{tw}|^2) + V_{tw}, \tag{0.2}$$

where $\boldsymbol{p} = -\mathrm{i}\nabla$ is the momentum operator, and

$$V_{tw} = \frac{\mathrm{i}}{2}\nabla^2\theta_{tw} + \frac{1}{2}|\nabla\theta_1|^2 - \partial_t\theta_{tw} - \nabla\theta_{tw}\cdot\boldsymbol{p} \tag{0.3}$$

is the twist potential. It can be directly verified that the above Hamiltonian is also non-Hermitian.

The energy expectation value E_{tw} is given by the contribution of the unperturbed state ψ_0 and twist. Since the twist contribution is linear in ψ_1, it can be obtained from the expectation value of V_{tw} and the kinetic part that depends on θ_{tw}; thus, (3.5) must be

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replaced by

$$E_{tw} = \int \left[\left(\frac{1}{2} |\nabla \theta_{tw}|^2 - \partial_t \theta_{tw} + \frac{\mathrm{i}}{2} \nabla^2 \theta_{tw} \right) |\psi_1|^2 + \mathrm{i} \nabla \theta_{tw} \cdot \nabla \psi_1 + \frac{1}{2} |\nabla \psi_1|^2 - \frac{1}{2} |\psi_1|^2 + \frac{1}{4} |\psi_1|^4 \right] \mathrm{d}V. \tag{0.4}$$

Upon application of the Madelung transform $\psi_1 = \sqrt{\rho} \exp(\mathrm{i}\chi_1)$, taking $\nabla \theta_{tw} \cdot \nabla \rho = 0$ in the neighborhood of the defect, we have

$$E_{tw} = \int \left[\left(\frac{1}{2} |\nabla \theta_{tw}|^2 - \partial_t \theta_{tw} - \nabla \theta_{tw} \cdot \nabla \chi_1 + \frac{1}{2} |\nabla \psi_1|^2 - \frac{1}{2} + \frac{1}{4} |\psi_1|^2 \right) + \frac{\mathrm{i}}{2} \nabla^2 \theta_{tw} \right] |\psi_1|^2 \, \mathrm{d}V. \tag{0.5}$$

As in FR20, the imaginary term above makes the Hamiltonian non-Hermitian, and the twisted state remains unstable. Following what is done in FR20 (§ 3), by the same procedure we obtain the correct dispersion relation

$$\nu = \frac{1}{2} \left[\left(|\boldsymbol{k}|^2 - 2 \nabla \theta_{tw} \cdot \boldsymbol{k} + |\nabla \theta_{tw}|^2 - 1 - 2 \partial_t \theta_{tw} \right) + \frac{\mathrm{i}}{2} \nabla^2 \theta_{tw} \right]. \tag{0.6}$$

The instability criterion of § 3 remains unaltered.

Since injection of negative twist is given by a rotation of the twist phase opposite to the vortex orientation, if we replace $\theta_{tw} \rightarrow -\theta_{tw}$ we evidently have instability when $\nabla^2 \theta_{tw} < 0$ as $t \rightarrow \infty$.

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