EVOLUTION OF INHOMOGENEITIES IN THE INFLATIONARY UNIVERSE -NO HAIR THEOREM OR MULTI-PRODUCTION OF UNIVERSES?-

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ABSTRACT Recent investigations on the evolution of the inhomogeneities in the inflationary universe are reviewed. 1) Strict cosmological no hair theorem does not hold, but the class of inhomogeneous universe which evolve to homogeneous de Sitter universe is finite, i.e, "weak cosmic no hair theorem" holds. 2) High density regions in the inhomogeneous universe evolve to wormholes provided that i) the size of the regions is greater than the horizon length, but smaller than a critical length which is the function of the density contrast, and ii) the density is three times higher than that of surrounding region. 3) If wormholes are formed copiously in the period of inflation, they evolve to causally disconnected "child- universes". In this scenario, the universe we are now observing is one of the locally flat regions.

1. Introduction

About 7 years ago, exponentially expanding universe model was proposed as a consequence of grand unified theories by Guth(1981), Sato, (1981a, 1981b) and Kazanas (1980). Independently of GUTs, Starobinski (1980) pointed out the universe expand exponentially as a consequence of R^2 gravity. This model is now well known as Inflationary Universe Model (Guth, 1981) and almost became a standard model of the very early universe. The important consequence of the inflationary universe model is that the flatness and horizon problems which were essential difficulties in the "Standard Big Bang Model" are solved.

The horizon problem is to question why our universe is so homogeneous over the horizon. Paradoxically, however, inflation is usually analyzed in the context of Robertson- Walker metric which is homogeneous and isotropic metric. Many people criticized that the above consequence is nothing but the assumption. In order to answer this criticism, inflationary model should be analyzed for more general universe models and it must be shown that the universe becomes isotropic and homogeneous by inflation independently of the initial conditions. The conjecture that all the inhomogeneous and anisotropic universes with cosmological

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constants(the vacuum energy density) evolve towards the de Sitter universe is now called cosmic no hair theorem. Until now, many investigations have been done in order to make clear whether this theorem is true or not.

2. Cosmic no hair theorem

First investigation of no hair theorem was done for the anisotropic but homogeneous universe model, because this is the most simple case. Wald (1983) showed that all the Bianchi types except IX evolve exponentially towards the de Sitter solution. Extension of the Wald's work was carried out by many peoples, Moss and Sahni (1986), Turner and Widrow (1986), Jensen and Stein-Schabes (1986), Ellis and Rothman (1986), Martines- Gonzares and Jones (1986), Belinski et al (1986).

For the case of inhomogeneous universes, it is generally very hard to investigate the evolution, but Starobinski (1983), first found the solution of the inhomogeneous universe which evolve towards the de Sitter solution. Recently Barrow and Gron (1986) and Stein-Schabes (1987) also found solutions which evolve to homogeneous and isotropic universe.

Recently Jensen and Stein-Schabes (1987) showed that any inhomogeneous universe will tend towards the de Sitter solution if the following three conditions are satisfied, i.e., 1) the dominant energy condition, $\rho > p$, 2) the strong energy condition, $\rho + 3p > 0$, and 3) the scalar spatial curvature is not positive in all the spacetime. The first and second conditions are very general, but the third condition cannot be accepted generally because usually density fluctuations contain positive curvature regions. The justification given by them is, therefore, limited in very narrow class of cosmological models.

In spite of these efforts, we can show a simple counter example against the cosmological no hair theorem. That is the existence of the Schwarzschild - de Sitter solution, which describes a black holes in the de Sitter universe. The metric is given by

$$ds^2 = -(1-2M/r -(r/\ell)^2) dt^2 + (1-2M/r -(r/\ell)^2)^{-1} dr^2 + r^2 d^2\Omega \tag{1}$$

where ℓ is the horizon length of the de Sitter universe, which is given by

$$\ell = (8\pi G\rho_v/3)^{-1/2} = (\Lambda/3)^{-1/2} \tag{2}$$

where ρ_v is the vacuum energy density and Λ is the cosmological constant.

If a black hole exists from the initial of the universe, it never disappears in the classical level. Furthermore even if no black hole exists from the initial, we can set an inhomogeneity which evolves towards a black hole. It is obvious that once black holes were formed, the universe cannot evolve to homogeneous state in the classical physics level. We may, therefore, conclude that no hair theorem does not hold in general cases. (Another counter example against no hair theorem was shown by Barrows (1987).)

3. Evolution of inhomogeneous inflationary universe

If no hair theorem does not hold, how can we investigate the evolution of inhomogeneities in the inflationary universe ? Recently Piran and Williams (1983) carried out the numerical simulation of the evolution of inhomogeneous universe by using [3+1] Regge calculus formalism. In their model, the universe is composed of 5 vertex and 10 edges. They showed that anisotropies and inhomogeneities are considerably decreased by inflation, but they have frozen during the inflation. In spite that this is a pioneering work, the result is not clear and it is obvious that our universe cannot be described by only 5 vertexes. At present we may conclude that it is difficult to get reliable result by numerical simulation.

Then how the fate of the very inhomogeneous universe can be investigated ? In order to investigate this problem, daring simplification or idealization is necessary. In the present talk, we discuss assuming that high density regions are spherical symmetric and the densities in them are spatially constant.

The evolution of the spacetime structure of the bumpy universe was investigated in the original inflationary universe model (the first order phase transition model, Sato, 1981, Guth, 1981) in detail in a series of papers, Sato et al (1981, 1982), Sato (1981c), Maeda et al, (1982), Kodama et al (1982). When the amplitude of the density fluctuations is very large, the evolution of the spacetime structure in the inflationary universe is essentially same even if the origin of the fluctuations is classical or quantum. Therefore the result obtained in the phase transition model essentially describes the evolution of the spacetime structure in the new inflation model (Linde, 1982, Albrecht and Steinhardt, 1982) and chaotic inflation model (Linde, 1982) also. Here let's discuss the evolution by using phase transition model.

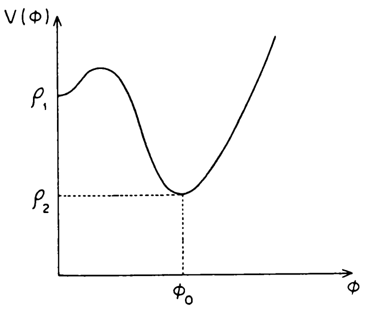

Fig.1. A model of the potential of scalar (inflaton) field. The vacuum energy density of low density state is assumed to be ρ_2.

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1) Spacetime structure in a bubble or a low density hollow

As is well known, phase transition proceeds by nucleation of bubbles and subsequent expansion of bubbles. If we take a phase transition model shown in Fig. 1, a bubble also contains the vacuum energy density, ρ_2. We can consider that these bubbles correspond to low density regions in the new inflation model and chaotic inflation model.

By virtue of generalized Birkoff theorem, the metric in a spherical symmetric bubble (a hollow) must be the Schwarzschild- de Sitter solution (Eq. 1), but with M= 0, i. e., the metric is the de Sitter metric with the cosmological constant $\Lambda_2 = 8\pi G\rho_2$, which is smaller than that of the outer high density region.

2) Spacetime structure of a high density region surrounded by bubbles

In the phase transition model, there exist infinite size networks of the false vacuum (the high energy state) in the early stage of the phase transition. However, in the cause of the phase transition, the high density vacuum regions (ρ_1) are eventually divided into pieces and surrounded by bubbles. This also occurs in the new inflationary universe model and the chaotic inflation model with inhomogeneities in the cause of rolling down of the scalar field.

In order to make clear the metric, let's consider a following simplified model: At $t = t_0$, infinite number of bubbles are created on the sphere of radius $r = r_0$. Then the universe is divided into three regions as shown in Fig. 2. region A is the inner high density (ρ_2) vacuum region. The metric in this region is given by usual de Sitter metric, . The region B is the shell like low density vacuum region (bubble region), and the inner surface W_- and outer surface W_+ of this region are expanding at light velocity. The metric is the Schwarzschild- de Sitter metric (Eq. 1) with the mass $M = 4\pi r_0^3 \rho_1/3$ and the cosmological constant $\Lambda_2 = 8\pi G\rho_2$. The region C is the outer high density vacuum region, and the metric is described by usual de Sitter metric.

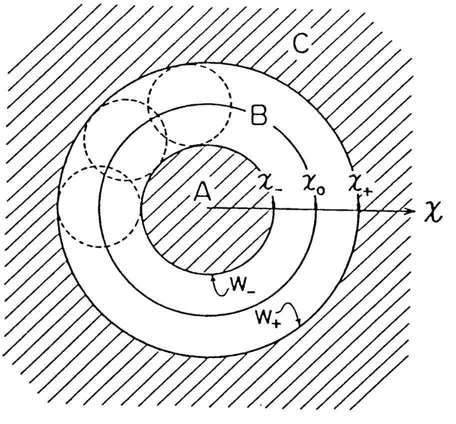

Fig. 2. A simplified model for high density vacuum region A, which is surrounded by low density vacuum region (bubble region) B.

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Table I.

	$1 < \rho_1/\rho_2 < 3$	$3 < \rho_1/\rho_2$
$r_0 < \ell$	BLACK HOLE Case I	Case I
$\ell < r_0 < r^*$	Case IIa	WORMHOLE Case IIb
$r^* < r_0$	DE SITTER LIKE SPACE Case III	Case III

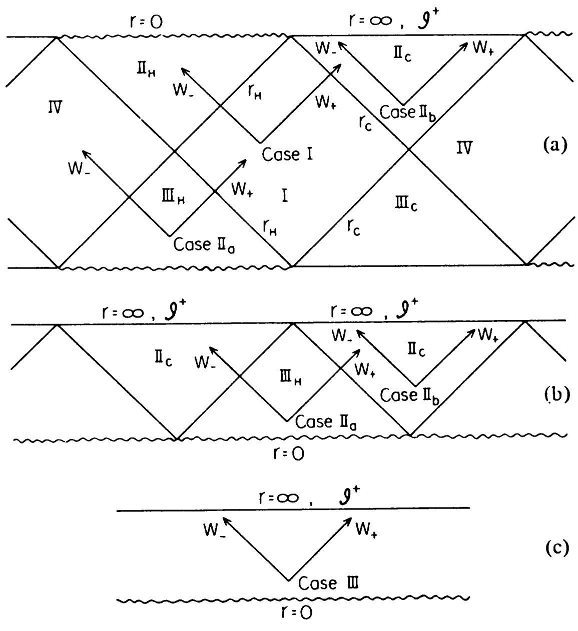

Fig. 3. The conformal diagram for the Schwarzschild-de Sitter geometry. (a): Case of $r_0 < r^*$, (b): Case of $r_0 = r^*$, (c): Case of $r_0 > r^*$, where r^* is the critical radius defined by Eq. (4). The path of the walls, W_- and W_+ are also shown (see Fig. 1 and Table I).

The final spacetime structure depends on two parameters, the initial radius r_0 and the density contrast ρ_1/ρ_2. The result is summarized in Table I.

As shown in Table I, black holes are formed when the initial radius r_0 is smaller than the horizon length of the de Sitter universe,

$$\ell = (8\pi G\rho_1/3)^{-1/2}, \qquad (3)$$

irrespective to the density contrast ρ_1/ρ_2.

The most interesting case is that wormhole structure is formed. As shown in Table I, wormholes are produced when the density contrast ρ_1/ρ_2 is greater than three and also the initial radius r_0 is greater than the horizon length ℓ but smaller than the critical radius which is defined as

$$r^* = 3^{-1/2}\ (2x^2/(x^2-1))^{1/3} \qquad (4)$$

where $x = \rho_1/\rho_2$.

In Fig. 4, a space like hypersurface with constant conformal time is shown schematically. As shown in this figure, the inner region A remains as an ever expanding de Sitter like sub- universe connected with the outer universe C by an Einstein Rosen (wormhole) bridge B. Because the region A is causally disconnected from the original universe, we can call the region A as **"a child universe"** and the original universe C as **" mother universe"** (Sato et al, 1981, 1982).

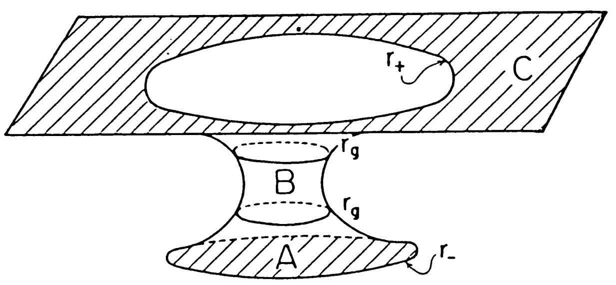

Fig. 4 A schematic picture of wormhole created from the large amplitude inhomogeneities. Because the region A is causally disconnected from the original universe, we can call the region A as "a child universe" and the original universe C as " mother universe".

5. Fate of inhomogeneous inflationary universe

In the preceding section, we showed that trapped high density regions evolve to wormholes by assuming the spatial structures are spherical symmetric. In reality, the high density regions are highly asymmetric and new bubbles are nucleated (in the phase transition scenario) or low density regions are formed (in the new or chaotic inflation scenarios) therein. However the essential feature of the above model is still applicable to the actual situation. The very reason why black holes and wormholes are created is that the high density regions are trapped by low density vacuum. In case the relative size of the a trapped domain to the surrounding low density region is small when it is formed, the domain wall continues to contract at light velocity and the energy released by the rolling over of the scalar field may well be concentrated in a very small region. Hence a black hole is generally created regardless of the initial shape of the high density domain. On the other hand in case the domain size is larger than the horizon length, ℓ, the domain wall continues to expand to infinity and consequently the universe attain to have two asymptotic (flat) regions. This anomalous structure occurs independent of the domain shape and strongly indicates the general formation of wormhole-bridge structure.

Now we can speculate the evolution of the spacetime structure of the very inhomogeneous universe:

i) If wormholes, which are produced from the inhomogeneities, evapolate as black holes do, the Einstein- Rosen bridge disappears and **child universes** will become entirely spatially disconnected. Here " spatially disconnected" means that there is a connected spacelike slice Σ of the universe such that it does not intersect with the causal boundary of the universe and its causal future, J_+ (Σ) is composed of topologically disconnected components.

ii) **The child- universes** also expand exponentially, and phase transition (in the phase transition scenario) or inhomogeneous rolling over of scalar field (in the new and chaotic inflation scenarios) proceeds also therein. As the results **grand -child universe** are formed. This sequential production of universes may continue on and on.

Now we easy arrive at an idea of multi-production of the universe; although the creator might have made a unitary universe, the universe itself is capable of bearing child- universes, which are again capable of bearing universes and so on (Sato, 1981, Sato et al. 1982)

Recently, Blau, Guendelman and Guth (1987) and Farhi and Guth (1987) discussed the creation of wormholes, but essential scenario is the same one of previous investigation (Sato et al, 1981, Kodama et al 1981, Maeda et al, 1982, Sato et al 1982).

Recently Linde (1986) proposed a interesting scenario about the spacetime structure of chaotic inflationary universe; the eternally existing self-reproducing chaotic inflationary universe. He claims that the scalar field not only roll down but also roll up due to the quantum density fluctuations. Consider one domain with the vacuum energy density $D^{(i)}$. After a Hubble expansion time, the domain is divided into many domains by quantum density fluctuations. Then the density of a high density domain would be given by $D^{(i+1)} = D^{(i)} + \delta$, which is higher than that of the original domain. By repeating this process, the scalar field not only roll down but also roll up. He claims by applying the creation mechanism of child universes which were shown by Sato et al, 1981 and 1982) that child universes are produced from the high density regions and child universes also have the same generic code as their mother universe.

6. Summary

i) In the present talk, we showed that strict no hair theorem does not hold because there is a simple counter example against this theorem, the existence of the Schwarzschild-de Sitter solution. On the other hand, however, there are some inhomogeneous solutions which evolve to homogeneous isotropic universe. This suggestes that the class of these solutions is not measure zero. In order to make clear the condition for this "weak no hair theorem" to hold, more careful investigation is necessary.

ii) The high density regions in the homogeneous universe evolve to wormholes if 1) the size of the regions is greater than the horizon length but smaller than a critical radius r^* (Eq. 4), and 2) the density of the region is at least three times higher than that of surrounding regions.

iii) If wormholes are formed copiously in the period of inflation, we can arrive ideas of multi-production of universes (Sato et al 1981,1982) or eternally existing self- reproducing universe (Linde, 1986). In these scenario, the universe we are now observing is one of the locally flat regions.

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