The value of cos2π/17 expressed in quadratic radicals.*

By Professor STEGGALL.

Call $2\pi/17$ a, and let

$$x_1 = 2\cos a,\ x_2 = 2\cos 2a, \text{ etc.}$$

Then from trigonometry

$$x_1x_2 = x_1 + x_3,$$
$$x_1x_3 = x_2 + x_4, \text{ etc };$$

and
$$x_1x_2x_4x_8 = \frac{\sin 16a}{\sin a} = \frac{\sin(17a - a)}{\sin a} = \frac{\sin(2\pi - a)}{\sin a}$$
$$= -1$$
$$= (x_1 + x_3)(x_4 + x_5)$$
$$= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8.$$

Now let
$$x_1x_4 = x_3 + x_5 = y_1,$$
$$x_2x_8 = x_6 + x_7 = y_2,$$
$$x_6x_7 = x_1 + x_4 = y_3,$$
$$x_3x_5 = x_2 + x_8 = y_4.$$

Then x_1, x_4 are the roots of

$$x^2 - y_3x + y_1 = 0 \quad \ldots \quad \ldots \quad (1)$$

and we require the four y's.

We have already seen that

$$y_1 + y_2 + y_3 + y_4 = -1;$$

and we have

$$(y_1 + y_2)(y_3 + y_4) = (x_3 + x_5 + x_6 + x_7)(x_1 + x_2 + x_4 + x_8)$$
$$= 4(x_1 + x_2 + x_3 + \quad \ldots \quad \ldots \quad + x_8)$$
$$= -4.$$

Therefore $y_1 + y_2$, $y_3 + y_4$ are the roots of

$$\xi^2 + \xi - 4 = 0 \quad \ldots \quad \ldots \quad (2)$$

$$\therefore \quad y_1 + y_2 = \tfrac{1}{2}(-1 - /17),$$

(the negative root being taken, because

$$y_1 + y_2 = x_3 + x_5 + x_6 + x_7$$
$$= x_2x_5 + x_5 + x_6,$$

where x_2 is positive and x_5, x_6 are negative)

And since
$$y_1y_2 = x_1x_2x_4x_8 = -1$$

Therefore y_1, y_2 are the roots of

$$y^2 + \tfrac{1}{2}(1 + \sqrt{17})y - 1 = 0 \quad \ldots \quad \ldots \quad (3)$$

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* This paper is merely intended to show how the solution of this interesting case of the binomial equation may be exhibited in a form suitable for a course of Elementary Trigonometry.

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Similarly y_3, y_4 are the roots of

$$y^2+\tfrac{1}{2}(1-\sqrt{17})y-1=0 \quad \ldots \quad \ldots \quad (4)$$

Then from equations (3) and (4) attending to the proper signs we have

$$y_1=\{-1-\sqrt{17}+\sqrt{(34+2\sqrt{17})}\}/4 \quad \ldots \quad (5)$$
$$y_3=\{-1+\sqrt{17}+\sqrt{(34-2\sqrt{17})}\}/4 \quad \ldots \quad (6)$$

Substituting these values in equation (1) and solving, we have finally

$$x_1=2\cos 2\pi/17$$
$$=\Big\{\sqrt{17}-1+\sqrt{(34-2\sqrt{17})}$$
$$+\sqrt{\{68+12\sqrt{17}+2(\sqrt{17}-1)\sqrt{(34-2\sqrt{17})}-16\sqrt{(34+2\sqrt{17})}\}}\Big\}/8.$$

A short notice of the additions to the Mathematical Theory of Heat since the transmission of Fourier's Memoir of 1811 to the French Academy.

By George A. Gibson, M.A.

What is here printed contains merely a list of the memoirs and treatises that may perhaps be found useful for one who wishes to trace the progress of the mathematical theory of heat beyond the stage at which Fourier left it. As discussions of the Fourier series and integrals occur in almost every treatise on the Integral Calculus, I have omitted reference to these. Similar considerations have led me to omit references to the discussion of differential equations, except where these specially dealt with the problem of the conduction of heat.

Poisson.—Mémoires sur la Distribution de la Chaleur dans les Corps Solides. (*Journal de l'École Polytechnique*, t. xii., cah. 19, 1823.)

There are two memoirs, the first of which was presented to the Institute in 1815, and the second in 1821.

Poisson.—Théorie Mathématique de la Chaleur. (Paris, 1835.)

The problems Poisson discusses are in the main those of Fourier generalised. The methods he gives of proving the possibility of the expansion of a function in a Fourier series, or in a series of spherical harmonics, are those usually given in English text-books. The treatise contains little of importance that is not to be found in the memoirs.