THEORETICAL RESULTS ON WR INSTABILITIES

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ABSTRACT. The instabilities of WR stars are discussed in the framework of both the quasi–adiabatic and non–adiabatic methods. We show that a very careful treatment of stellar opacities is necessary. Both methods show that WR stars, with little or no H left, are vibrationally unstable in the fundamental radial mode due to the ϵ–mechanism, confirming earlier results by Maeder (1985). Thus, from the point of view of stability, WR stars are fundametally different from O stars. Our new analysis also supports our previous view that WR stars evolve, keeping at the edge of vibrational instability.

1 INTRODUCTION

What is the main physical reason for the differences between an O star and a WR star? Clearly, surface abundances are a key point, but we also show here that with respect to stellar instabilities, WR star models are in a completely different situation than O stars: WR stars have a deep seated instability. Whether this instability is the main source of the WR phenomenon is not known at present. However, this instability is a major physical property of WR models and it is likely to determine the evolution of WR stars keeping at the edge of vibrational instability. We also show that the nuclear evolution of stars at the edge of vibrational instability implies very high mass loss rates $\dot{M}$ as currently observed for WR stars, and also implies a $\dot{M}$ vs. M relation.

2 NUCLEAR ENERGIZING AND RADIATIVE DAMPING IN RADIAL PULSATIONS: QUASI–ADIABATIC APPROACH

Numerous analyses of vibrational stability in helium stars have been performed (e.g. Boury and Ledoux, 1965; Noels-Grötsch, 1967; Simon and Stothers, 1969, 1970; Stothers and Simon, 1970; Noels and Masereel, 1982; Noels and Magain, 1984; Maeder, 1985). These works clearly show that the results about stability or instability of the radial mode in He–stars depend critically on the relative importance of two terms:

1. The nuclear driving
2. The radiative damping

The nuclear driving occurs in the central region: compression implies heating, which in

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turn implies an increase of ϵ, the rate of nuclear energy generation, which means mechanical expansion until temperature and ϵ go down and the restoring force of gravity again produces a compression. The radiative damping is the loss of pulsational energy radiated by the temperature excess of the waves. Radiative damping usually dominates in the external stellar layers. If the nuclear driving overcomes the radiative damping, the star is unstable. The fundamental mode of radial pulsation is the most unstable mode in this case.

This instability mechanism is known as the Eddington ϵ–mechanism. For an He–star, Noels and Masereel (1982) found a critical mass of 16 $M_\odot$: above this mass limit, the ϵ–mechanism produces radial pulsation instability. For more realistic WR models, Maeder (1985) found an instability limit of about 12 $M_\odot$. On the zero–age sequence of Pop. I stars, the critical mass is around 100 $M_\odot$.

The relative importance of the nuclear driving and radiative damping depends essentially on the relative size of the pulsation amplitudes ξ_c in the center to the amplitudes ξ_s at the stellar surface. A relatively large ξ_c favours the pumping of nuclear energy in central regions, while a small ξ_c produces little driving. In turn, the amplitude ratio ξ_c/ξ_s essentially depends on the density contrast $\rho_c/\overline{\rho}$ in the star. Here ρ_c is the central density and $\overline{\rho}$ the average density. One has the following connexion:

$$\text{if } \frac{\rho_c}{\rho} \searrow \implies \frac{\xi_c}{\xi_s} \nearrow$$

Thus a low density means a relatively high central amplitude, which favours pulsational instability. Usually the ξ–values have been taken from linear adiabatic calculations and this is why this approach is called the quasi–adiabatic approximation; let us also note that in the linear approximation, ξ_s is normalised to 1.

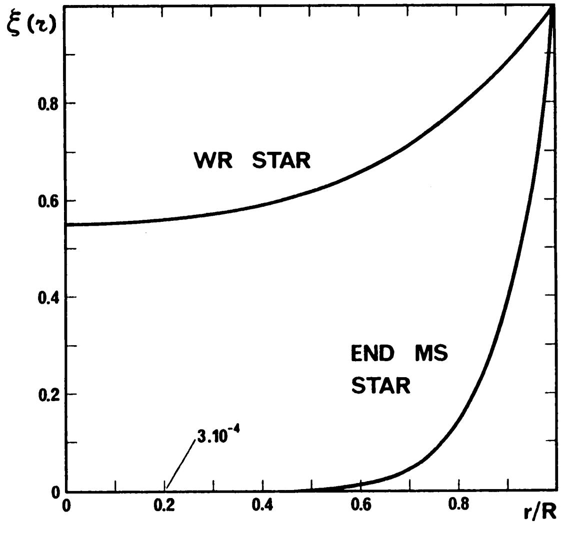

Figure 1. Comparison of the pulsation amplitudes from the center to the surface in a WR star model and a star at the end of the main sequence.

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Figure 1 compares the pulsation amplitudes (cf. Maeder, 1985) from the center to the surface in a WR star model (WC star with 33.7 $M_\odot$, resulting from an initial 120 $M_\odot$ model) with the pulsation amplitudes for an initial 120 $M_\odot$ model at the end of the Main Sequence (actual mass 98.7 $M_\odot$). In the WR model, the relatively large amplitude in the center strongly favours the nuclear driving and makes the WR model unstable. On the contrary, the MS star (even for an initial 120 $M_\odot$ model) has very small central amplitudes; thus nuclear driving is negligible there compared to radiative damping in the external layers.

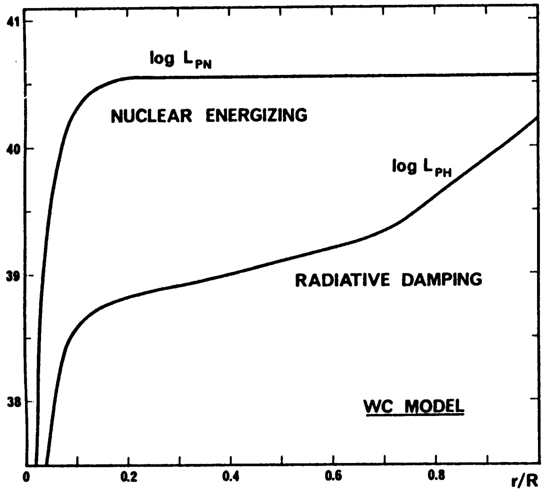

Figure 2. Comparison of the integrated values (from the center to the surface) of the nuclear energizing and radiative damping in a WC star model.

Figure 2 shows the integral of L_{PN}, the rate of gain of pulsation energy from nuclear sources, integrated from the center to the point considered. The integral L_{PH} is also shown, where L_{PH} is the rate of loss of pulsation energy by heat leakage. From Fig. 2 we conclude that in a WR star the contribution of the nuclear energizing is very large in central regions and is not overcome by radiation damping in the outer layers. WR stars are likely to be the only case where this situation occurs.

The analyses made in the quasi–adiabatic approximation (cf. Maeder, 1985) have shown the following results:

- The variability of the Hubble–Sandage variables is not due to vibrational instability, since the density contrast of supergiants is too high.
- In the WR star models considered, the nuclear energizing of pulsations largely overcomes the radiative damping due to heat leakage and the stars are radially vibrationally unstable in the fundamental mode.
- For models with observed mass loss rates and standard treatment of convection, entry into the unstable regime occurs when (H/He)=0.3 (in numbers) at the stellar surface.

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- The pulsation periods of the inner hydrostatic star (without the wind) are in the range of 15–60 minutes.
- Any mixing process such as overshooting or turbulent diffusion would enhance the vibrational instability of WR stars.
- Vibrational instability characterizes stellar models of WR stars that occupy a well–defined mass–luminosity relation, showing a large overluminosity with respect to the main sequence.

3 THE CASE FOR NONRADIAL PULSATIONS

Let us examine here the theoretical status about the nonradial pulsations of WR stars. The driving of nonradial oscillations by central nuclear energizing is in a very unfavourable situation, because the amplitudes $\delta T/T, \delta\rho/\rho$ tend to be zero at the stellar center and no efficient pumping of energy can occur there (cf. Simon 1957). This explains why Kirbiyik et al. (1984), in an investigation of nonradial oscillations for WR models, found these stars to be stable for the low harmonics ℓ; they noticed, however, the appearance of instabilities for high degrees of harmonics ($\ell = 15$). Noels and Scuflaire (1986) interestingly showed that, while there is no driving of nonradial oscillations in WR stars to be expected from He–cores, the H–burning shells may produce some efficient driving of nonradial pulsations. The periods found are of the order of a few hours. There is, however, a limitation: the H–shell is, if any, only present for a very short time in WR stars. In the case studied by Noels and Scuflaire it lasts only 6000 years. Indeed, most WR models (cf. Maeder, 1981) even do not exhibit an H–burning shell. This is obvious if we remember that WNE as well as WC stars no longer have hydrogen at their surface; thus only a fraction of the WNL stars could have an efficient driving from the H–shell. Studying various WR models, Cox and Cahn (1988) found no unstable g–type nonradial models. The nonradial modes have large amplitudes outside the convective core where there would be an H–burning shell. But even there Cox and Cahn (1988) find the WR models to be stable, with respect to nonradial oscillations, because of the large radiative damping in the outer layers. Thus, on theoretical grounds, nonradial instabilities due to the ϵ–mechanism in WR stars seem to be very unlikely.

4 THE ϵ AND κ–MECHANISMS IN WR STARS

The κ–mechanism, which is responsible for cepheid pulsations, could also play a destabilizing role in WR stars. The study of this problem has been undertaken in the framework of the linear non–adiabatic method for the case of radial oscillations. The pulsation calculations are based on evolutionary models taking into account the entire stellar structure from the very center to the surface allowing us to model an ϵ driving mechanism as well as a κ–mechanism (cf. Cox and Cahn, 1988). An effective opacity as described in Stellingwerf (1975) is taken for the finite–difference scheme of the radiative transfer equation.

The effects of stellar opacities on the radial pulsations were firstly explored. Our standard opacity tables (called Iben XVIII et XIX mixtures) led to no destabilizing contribution by the κ–mechanism and predicted, with that method, a critical mass limit of 43 $M_{\odot}$. We have also established a set of new opacity tables computed with the Los Alamos Opacity

Programme for abundances in He, C, N, O, Ne etc. well adapted to WR stars. The new tables produce a small increase of the opacity coefficient in the outer layers (κ increases from 0.24 to 0.28), mainly due to partially ionized oxygen.

Using the new opacity tables we found radial vibrational instability at lower masses. For example, we found the critical mass at 32 $M_\odot$ instead of 43 $M_\odot$ with standard tables for an initial model of 120 $M_\odot$ at the WC stage. Figure 4 shows the corresponding changes in the cumulative work integral (here the integration is performed from the surface to the center). In Fig. 4 we clearly see the positive driving due to the κ–mechanism in the outer layers (continuous curve), while with standard opacities no such driving could be observed. The central T and ρ are also slightly increased and thus the nuclear energizing is also larger in this model. It appears that a small change in the values of the opacities, in our case a few percent, plays a determinant role in the stability results. These critical mass values are given only to show the sensitiveness of the stability to a change in the $\dot{M}$–rate or opacity, but the right value is certainly lower, maybe as low as 10 $M_\odot$. This is especially true if we account for the fact that opacity coefficients are expected to be increased by future developments in the field.

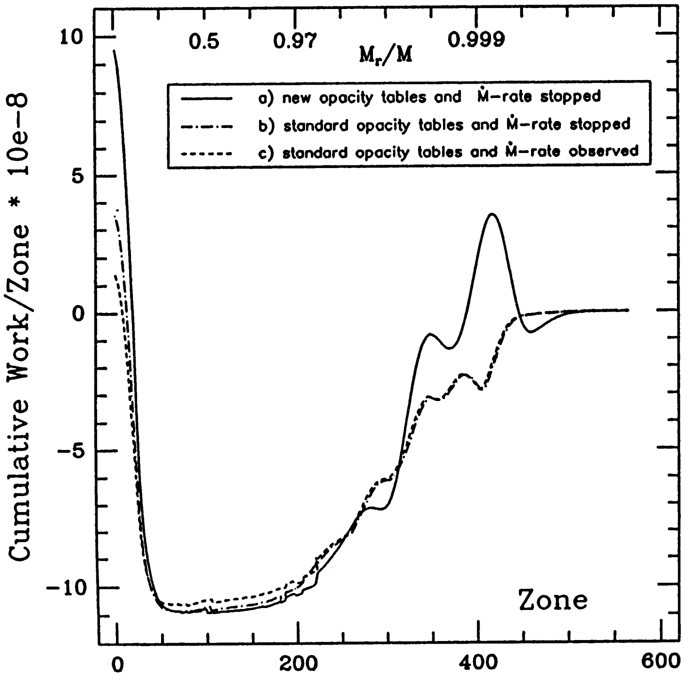

Figure 3. Cumulative work integral normalized by the kinetical energy of the pulsation vs. the zone number labeled from the center to the surface for the fundamental mode in a 56 $M_\odot$ model of an initial 120 $M_\odot$ star.

We have also explored the effects of changes in the mass loss rates $\dot{M}$ on WR stability. If the $\dot{M}$–rate is increased by a high factor the instability declines because central T and ρ are slightly reduced and therefore also the nuclear energizing. On the contrary, reduction

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of $\dot{M}$–rate leads to an enhancement of the instability. In Fig. 3 we notice the differences between the dashed and the dashed–dotted curves near the stellar center. This numerical result well supports the view (cf. Maeder 1985) that WR stars evolve, keeping to the edge of vibrational instability. It was also shown analytically that if WR stars evolve quasi–homogeneously and keep to the edge of vibrational instability with $M\overline{\mu}^2 \simeq$ const, the pulsations would be able to sustain the observed mass flux. Indeed, for a star evolving at the edge of vibrational instability, we get

$$\dot{M} = \frac{1}{2}\overline{\mu}\, M\dot{Y}$$

and by using the relation between the luminosity L and the change $\dot{Y}$ of helium content we obtain

$$\dot{M} \simeq -\frac{1}{2}\overline{\mu}\frac{L}{E_{3\alpha}q_c}$$

which corresponds numerically to $\dot{M} = -\,(2 \cdot 10^{-5} - 10^{-4})\, M_{\odot}y^{-1}$. This demonstrates the ability of a star evolving at the edge of vibrational instability to produce the observed mass loss rates. Moreover, since there is a mass–luminosity relation for WR stars (cf. Maeder, 1983), it turns out that there is also a $\dot{M}$ vs M^{α} relation for WR stars, with $\alpha \simeq 2$ for WR stars with masses $M \leq 10\ M_{\odot}$.

Finally, let us emphasize that these results on WR instabilities are based for now on WR models, which do not account for the optically thick wind. The wind is likely to affect the visibility of the inner pulsation, the boundary conditions, the stability limit as well as the pulsation periods. The study of these effects will demand complete stellar models of both the stellar interior and of the non–static and non–stationary envelope. Years of work may still be needed until we reach that stage of development in pulsation analysis.

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DISCUSSION

Owocki: What driving mechanism lifts the mass to infinity in your scenario, in which you assume $\dot{M} \approx 10^{-5} - 10^{-4} M_{\odot} yr^{-1}$, in order to stay close to your instability line?
Maeder: My statement was that if a WR star keeps at the edge of vibrational instability during its evolution, its mass has to decrease with a mass loss rate comparable to the currently observed ones. In this case of vibrational instability and purbational motions are likely to turn, as shown by Appenzeller in the early '70-s, into outward bulk motions in the very outer layers.

Sreenivasan: Most people who do pulsation calculations now believe that opacities have to be increased by a certain factor. But no explicit calculations of additional sources of opacity are generally available. Did you simply scale the opacities upwards in an *ad hoc* manner? Have you looked for stability of non-radial pulsations in view of the κ-mechanisms in your outer layers? If you made more realistic calculations you would also see non-radial modes to be overstable.
Maeder: On one hand, we have calculated opacity tables appropriate to the WC compositions. On the other hand, we are also exploring the effect of enhanced opacities. Non-radial modes driven by the ϵ-mechanism are unlikely as shown by Ledoux.

Cassinelli: I would like to follow up on a question I had after van Genderen's talk yesterday. Observations show no pulsation effects. Yet it seems that it is only by pulsation that the interior of the star can directly influence the atmosphere and cause the star to remain at the vibrational instability limit. Should we not see some effect of the pulsations required?
Maeder: This is a point I am also wondering about. Certainly, the optically thick wind may affect the visibility of pulsations, but it also reflects that the outer boundary conditions are not the usual ones in stellar pulsations. On the whole, I wonder whether the very high wind of WR stars is not just an atmospheric response to internal pulsations.

Underhill: With the eclipsing variable V444 Cyg you can obtain an independent estimate of $\dot{M}$ from $\dot{P}$, see Underhill *et al.* (1990). We find $\dot{M} < 6 \times 10^{-6} M_{\odot} yr^{-1}$. This makes it doubtful that $\dot{M}$ is of the order of $5 \times 10^{-5} M_{\odot} yr^{-1}$ for WR stars. [But see van der Hucht, p. 28, and Willis, p. 279-280 (eds.)]
Maeder: I think your comment is more relevant to the problem of the current mass loss determinations, on which I am relying.

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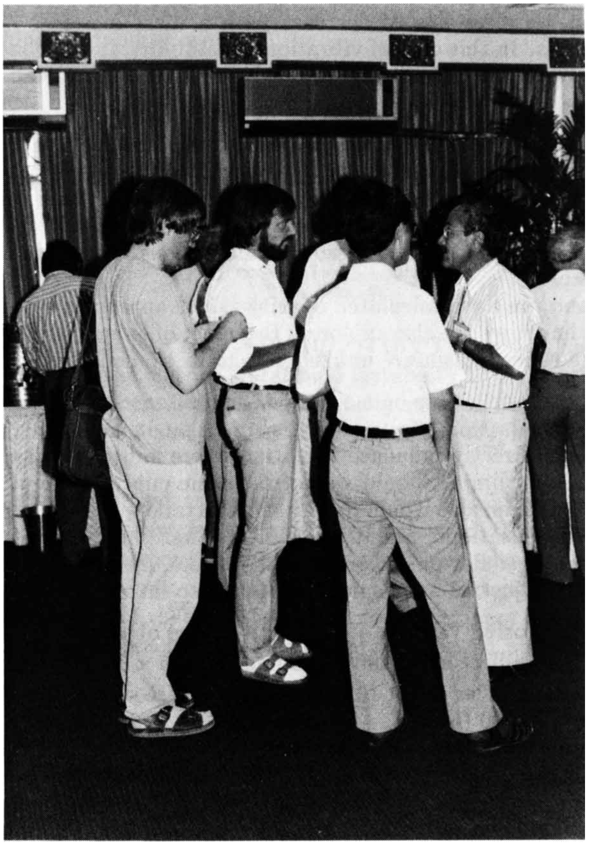

Maeder explaining it to Langer once more

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