A NEW METHOD OF RELATIVE COORDINATE-AND-TIME DETERMINATION

V.S.GUBANOV, Z.M.MALKIN, N.I.SOLINA
Institute of Applied Astronomy
8, Zhdanovskaya st., St.-Petersburg
197042, Russia

ABSTRACT. The paper deals with the relative coordinate-and-time determination in a ground based reference frame by using the NAVSTAR/GLONASS radio interferometry and duplex clock comparison techniques.

1. REGIONAL SPACE-TIME REFERENCE FRAME SPECIFICATION

Let us introduce a regional Space-Time Reference Frame (STRF) as the aggregate (C,t) of $n{\times}3$ matrix $C=(x_i,y_i,z_i)$ of the adopted coordinates of the feducial stations $A_i(i=1,2,...,n)$ and of the vector of some reference time scale t. For example, we can take $t=t_j(j\in i)$. The STRF is given if both the matrix C and the vector of current differences $\Delta t_i=t_i-t$ between local and reference atomic time scales are known at any time.

The best combination of techniques of the STRF feducial stations is the collocation of some VLBI-network and the LAGEOS laser ranging. It is proposed that all laser ranging stations to be equipped by clock comparison technique, for example, the duplex method via space communication (Imae et al.(1983) and Gubanov, Kajdanovskij and Umarbaeva(1989)). Moreover, no less than four of the feducial stations must be equipped with GPS/GLONASS radio interferometric techniques of SYRIUS type (Umarbaeva et al.(1991)). This approach is illustrated on the Figure 1.

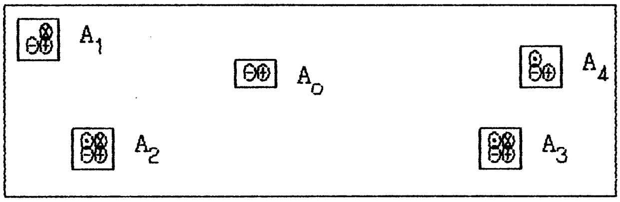

Fig.1. The example of the techniques collocation on the feducial and mobile stations. Designations: ⊗ VLBI, ⊖ SYRIUS, ⊙ Laser ranging, ⊕ DUPLEX

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2. MOBILE STATION

The Mobile Station (MOST) is an aggregate of two transportable measuring systems that are the SYRIUS and DUPLEX techniques using the 1.5÷2 metre diameter antennas. Potential accuracies of both techniques in time-delay measurements are about 0.1 ns (Gubanov, Kajdanovskij and Umarbaeva (1989) and Gubanov, Brumberg and Solina (1989)). At present these accuracies achived are about 1 ns (Imae et al. (1983) and Umarbaeva et al. (1991)).

The MOST A_o has to operate together with not less than four feducial stastations A_i as several very-long-baseline interferometers and has to observe synchronously the GPS/GLONASS navigation satellites. The atomic clock of MOST is compared periodically with one of feducial station A_j by the DUPLEX technique. As long as the differences $\Delta t_i = t_i - t$ and $\Delta t_{ij} = t_i - t_j$ $(i,j=1,2,\ldots,n)$ are known at any time for all the feducial stations, it is sufficient to compare the MOST clock with only one feducial station A_j because

$$\Delta t_{oi} = t_o - t_i = (t_o - t_j) - (t_i - t_j) = \Delta t_{oj} - \Delta t_{ij}.$$

3. RELATIVE COORDINATE-AND-TIME DETERMINATION

The problem is to determine the MOST local time and coordinates in the STRF using the above mentioned techniques.

Let us consider the VLBI observations of the GPS/GLONASS satellite S_k by the SYRIUS tecnique at a few interferometers A_oA_i simultaneously. The equation system for time-delay measurements is

$$c\tau_{ik} = R_{ok} - R_{ik} + c\Delta t_{oi}, \qquad (i=1,2,\ldots,n), \tag{1}$$

where c is the light velocity, R_{ok} and R_{ik} are the instantaneous distances A_oS_k and A_iS_k, respectively.

Introducing into eqs.(1) the increments of rectangular coordinates of the stations $A_o(x_o,y_o,z_o)$, $A_i(x_i,y_i,z_i)$ and satellite $S_k(x_k,y_k,z_k)$ and taking into account that the differences Δt_{oi} and possible corrections $(\Delta x_i,\Delta y_i,\Delta z_i)$ to coordinates of feducial stations are known, we can rewrite the system of equations (1) in the matrix form as follows

$$\mathbf{l}_k = \mathbf{B}_k \mathbf{u}_o + \mathbf{C}_k \mathbf{v}_k, \tag{2}$$

where $\mathbf{l}_k = (l_i)_k$ is the data vector

$$(l_i)_k = c\Delta\tau_{ik} - c\Delta t_{oi} - (a_{ik}\Delta x_i + b_{ik}\Delta y_i + c_{ik}\Delta z_i), \tag{3}$$

$\mathbf{u}_o = (\Delta x_o,\Delta y_o,\Delta z_o)^T$, $\mathbf{v}_k = (\Delta x_k,\Delta y_k,\Delta z_k)^T$ are the vectors of unknown corrections to coordinates of the MOST and observable satellite, respectively;

$\mathbf{B}_k=(a_k,b_k,c_k)$, $\mathbf{C}_k=(a_{1k},b_{1k},c_{1k})$ are $n\times 3$ matrices of the coefficients

$$a_k=(x_o-x_k)/R_{ok},\ b_k=(y_o-y_k)/R_{ok},\ c_k=(z_o-z_k)/R_{ok},$$

$$a_{1k}=(x_1-x_k)/R_{1k}-a_k,\ b_{1k}=(y_1-y_k)/R_{1k}-b_k,\ c_{1k}=(z_1-z_k)/R_{1k}-c_k.$$

The system of normal equations in two-group form is

$$\left.\begin{aligned} \mathbf{F}_k\mathbf{u}_o + \mathbf{H}_k\mathbf{v}_k &= \mathbf{f}_k, \\ \mathbf{H}_k^T\mathbf{u}_o + \mathbf{G}_k\mathbf{v}_k &= \mathbf{g}_k, \end{aligned}\right\} \tag{4}$$

where

$$\mathbf{F}_k=\mathbf{B}_k^T\mathbf{P}_k\mathbf{B}_k,\ \mathbf{H}_k=\mathbf{B}_k^T\mathbf{P}_k\mathbf{C}_k,\ \mathbf{G}_k=\mathbf{C}_k^T\mathbf{P}_k\mathbf{C}_k,\ \mathbf{f}_k=\mathbf{B}_k^T\mathbf{P}_k\mathbf{l}_k,\ \mathbf{g}_k=\mathbf{C}_k^T\mathbf{P}_k\mathbf{l}_k$$

and $\mathbf{P}_k$ is the data weighting matrix

$$\mathbf{P}_k= \sigma_o^2\mathbf{Q}_k^{-1} = \sigma_o^2(c^2\mathbf{Q}_\tau + c^2\mathbf{Q}_t + \mathbf{C}_k\mathbf{Q}_r\mathbf{C}_k^T)^{-1}, \tag{5}$$

which according to (3) is expressed through the data variance-covariance matrices $\mathbf{Q}_\tau$, $\mathbf{Q}_t$ and $\mathbf{Q}_r$ of time-delay measurements, clock synchronization and adopted coordinates of the feducial stations, respectively. Excluding the vector $\mathbf{v}_k$ from eqs.(4), we obtain

$$\mathbf{D}_k\mathbf{u}_o = \mathbf{h}_k, \tag{6}$$

where

$$\mathbf{D}_k =(\mathbf{F}_k- \mathbf{H}_k\mathbf{G}_k^{-1}\mathbf{H}_k^T), \quad \mathbf{h}_k=(\mathbf{f}_k- \mathbf{H}_k\mathbf{G}_k^{-1}\mathbf{g}_k).$$

If $n\geqslant 4$ we have $\det\mathbf{G}_k\neq 0$ and $\mathbf{D}_k\neq[\mathbf{0}]$, though $\det\mathbf{D}_k\equiv 0$. However, if the same and other satellites are observed in this manner several times we can sum up eqs.(6) and obtain

$$\mathbf{D}\mathbf{u}_o= \mathbf{h},$$

where

$$\mathbf{D} =\sum_{k=1}^{m}\mathbf{D}_k, \quad \mathbf{h} =\sum_{k=1}^{m}\mathbf{h}_k .$$

It is easy to see that $\det\mathbf{D}\neq 0$, and thus, according to the least-squares technique we have the following estimates

$$\mathbf{u}_o=\mathbf{D}^{-1}\mathbf{h}, \quad \mathbf{Q}_u=\sigma_o^2\mathbf{D}^{-1},$$

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$$\sigma_o^2=S/(mn-3m-3), \quad S=\sum_{k=1}^{m}(\mathbf{l}_k^T\mathbf{P}_k\mathbf{l}_k-\mathbf{f}_k^T\mathbf{F}_k^{-1}\mathbf{f}_k)-\mathbf{u}_o^T\mathbf{h}.$$

4. NUMERICAL SIMULATION RESULTS

Due to the fact that the spatial distribution of the GPS satellites relative to ground stations is repeated from day to day, it is enough to consider one period of twenty-four hours duration. The program of the GPS observations during this period is simulated in the same way as that described by Gubanov, Brumberg and Solina (1989). However, here only the following Asian feducial stations $A_i(\varphi_i,\lambda_i)$ will be used: Ashkhabad (38°, 58°), Irkutsk (52°, 103°), Bangalour (13°, 78°) and Kungming (25°, 102°).

The results of numerical simulation are the correlations r/φ, r/λ and φ/λ between the MOST spherical coordinates corrections (Δr_o, $r\Delta\varphi_o$, $r\Delta\lambda_o\cos\varphi_o$) and their RMS, recpectively. They are shown in Tables 1 and 2.

TABLE 1. Correlations between the MOST coordinates

φ \ λ	50°	60°	70°	80°	90°	100°	110°
60°	.0	.0	-.1	-.1	.0	.1	.2
	.0	.0	.0	-.1	-.1	-.1	-.1
	-.1	-.1	-.1	.0	.0	.0	.0
50°	-.1	-.1	.1	.2	.2	.1	.1
	.0	.0	.0	-.1	-.1	-.1	-.1
	-.1	-.1	-.1	-.1	-.1	.0	.0
40°	.0	-.1	.0	.0	.0	.0	-.1
	.0	.0	-.1	-.1	-.1	-.1	-.1
	-.1	.0	-.1	-.1	.0	.0	.0
30°	.0	-.3	-.2	-.1	-.1	-.2	-.2
	.0	.0	-.1	-.1	-.1	-.1	-.1
	-.1	.0	-.1	-.1	-.1	.0	.0
20°	.0	-.1	-.2	-.2	-.2	-.2	-.2
	.0	.1	-.1	-.1	-.1	-.1	.0
	.0	-.1	.0	-.1	-.1	.0	-.1
10°	-.2	-.1	-.1	-.2	-.2	-.2	-.1
	.2	.0	.0	-.1	-.1	.1	.1
	-.1	-.1	-.1	.0	.0	-.1	-.1

TABLE 2. RMS of the MOST coordinates in cm

φ \ λ	50°	60°	70°	80°	90°	100°	110°
60°	7	5	3	3	3	4	6
	7	5	3	3	3	4	6
	6	4	3	3	3	4	5
50°	4	2	2	3	3	2	3
	4	2	2	3	3	2	3
	4	2	2	2	2	2	2
40°	4	2	4	4	4	3	2
	3	2	4	4	4	3	2
	3	2	4	4	4	3	2
30°	4	2	3	4	4	3	2
	4	2	3	4	4	3	2
	4	2	3	4	4	3	2
20°	7	3	2	2	2	2	4
	6	2	1	2	2	2	4
	7	3	2	3	2	2	3
10°	20	8	5	3	3	6	12
	12	6	3	2	2	5	10
	13	8	5	3	3	5	10

The estimates given in Tables 1 and 2 are obtained by suitable transformation of the unknown parameters vector $\mathbf{u}_o$ with the variance-covariance matrix $\mathbf{Q}_u$ under the condition that the data variance-covari-

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ance matrices in (5) are

$$\mathbf{Q}_\tau = \mathrm{diag}(\sigma_\tau^2), \quad \mathbf{Q}_t = \mathrm{diag}(\sigma_t^2), \quad \mathbf{Q}_f = \mathrm{diag}(\sigma_f^2)$$

and

$$c\sigma_\tau = c\sigma_t = \sigma_f = \sigma_o = \pm 3\mathrm{cm}.$$

REFERENCES:

Gubanov,V.S.,Kajdanovskij,M.N.,Umarbaeva,N.D.(1989), 'Application of interferometric technique in clock comparison via geostetionary retranslator', Kinematic and Phys. Cel. Bodies, Allerton Press, New York, 5, No.6, 84-88.

Gubanov,V.S.,Brumberg,E.V.,Solina,N.I.(1989),'Geometric method to determine geocentric coordinates of the ground stations on the reference frame of a VLBI complex', Proc. of 6th Internat. Symp. "Geodesy and Physics of the Earth", Potsdam, 1988, Pt.2, 309-318.

Imae,M.,Okazawa,H.,Sato,T. et al. (1983), 'Time comparison experiments with small K-band antennas and SSRA equipment via a domestic geostationary satellite', IEEE Trans. Instrum. Meas., IM-32, No2, 199-203.

MacDoran,P.F.(1980),'Satellite emission radio interferometric Earth surveying (SERIES)', CSTG Bulletin, No2, 118-119.

Umarbaeva,N.D. et. al.(1991), 'SYRIUS-A observs NAVSTAR in the compaign GIG-91', Chapman Conference "Geodetic VLBI: Monitoring Global Change", Washington, D.C., April 22-26 (in press).

https://doi.org/10.1017/S0074180900173656 Published online by Cambridge University Press