RR LYRAE AND BL HERCULIS VARIABLES

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The RR Lyrae variables are currently believed to have masses between about 0.5 and 0.8 $M_\odot$, effective surface temperatures between 6350 and 7500K, radii from about 4.0 to 6.0 $R_\odot$ and luminosities between $\log L/L_\odot$ of 1.5 and 2.0. Since they are found in population II locations, they generally have $Y = 0.3$ and $Z = 10^{-3}$, but there are exceptions for both higher Z like the sun and lower Z like 0.0002. In globular clusters the periods range from 0.25 - 0.45 day for the first overtone pulsators and 0.40 - 0.80 day for those in the fundamental mode, depending on their luminosity. At transition lines, to be discussed in detail below, the switch from fundamental to first overtone, or maybe vice versa, involves a period change factor of about 0.74 - 0.75.

Evolution tracks by Sweigart and Gross (1976) show blueward evolution through the radial pulsation instability strip at RR Lyrae luminosities for $Y \geq 0.3$ if $M < 0.7\ M_\odot$. At lower helium mass fraction ($Y < 0.3$) evolution goes in both directions. Since later we will be wanting RR Lyrae variables to evolve only blueward to reconcile observations and pulsation theory, the low Y and simultaneous high mass seem to be excluded. Just the occurrence of first overtone pulsators implies by pulsation theory that Y must exceed about 0.22. The evolution tracks do return redward, but at our suggested Y and M only when $\log L/L_\odot > 1.9$, the realm of the H and He shell burning BL Herculis variables at periods about 1 day.

In the last 10 years, RR Lyrae variables have frequently been discussed in terms of the two clearly separated Oosterhoff groups I ($<\Pi_0> = 0\overset{d}{.}54$, $<\Pi_1 > 0\overset{d}{.}32$) and II ($<\Pi_0> = 0\overset{d}{.}64$ $<\Pi_1> = 0\overset{d}{.}37$). Stobie (1971) considered that these groups implied a RR Lyrae variable mass difference with the Oosterhoff group II clusters having a larger value. Butler, Dickens and Epps (1978) conclude that indeed the group II clusters have more massive and luminous RR Lyrae variables by 0.1 $M_\odot$ and are very metal poor based on their discovery that apparently both groups actually exist together in ω Cen. They discuss a composition dependent mass loss which would mean that the basic parameter

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is composition, with therefore a smaller mass loss for the extremely metal poor Oosterhoff group II variables. ω Cen evidently consists of two different composition condensations to give the two Oosterhoff groups. Normally a globular cluster has only one composition condensation.

Van Albada and Baker (1973) proposed that the difference between the two Oosterhoff groups could be the evolution direction: group I to blue and group II to red. For group I the transition from the fundamental mode to the first overtone occurs just a bit redward of the fundamental blue edge as shown as the lower horizontal line in the H-R diagram of Figure 1. In the group II case given as the upper horizontal line, evolution to the red gives a transition at a lower T_e and higher period due both to the higher luminosity and lower T_e. A younger group I cluster like M3 would have shorter average periods and a sharp transition period very near the fundamental blue edge. Older or lower metal group II clusters would have some redward evolution giving longer average periods and even an overlap of both modes occurring at a given color or T_e. This overlap for ω Cen is shown in Figure 2 where P' is the observed period corrected for the small luminosity differences between stars. Persistence of the overtone (the hysteresis effect) in redward evolution is alleged to occur in the Oosterhoff group II clusters.

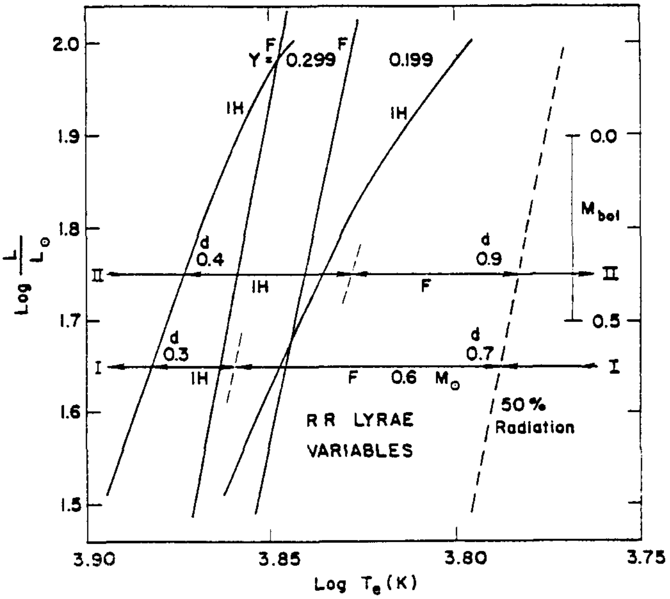


Figure 1: Hertzsprung-Russell diagram for Oosterhoff groups

The theoretical justification for the hysteresis comes from calculations by Von Sengbusch (1975) and Stellingwerf (1975), the later results given in the H-R diagram of Figure 3. It is the dashed

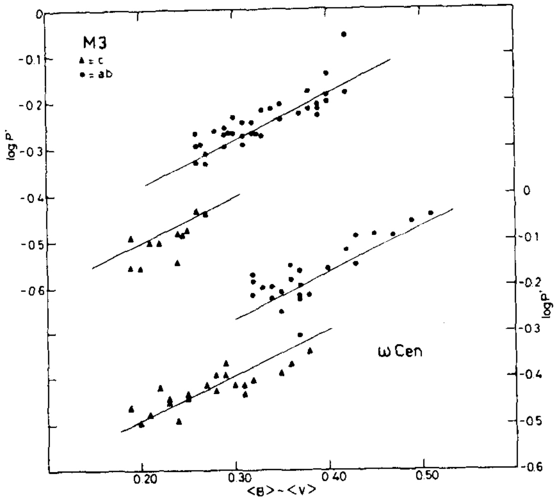
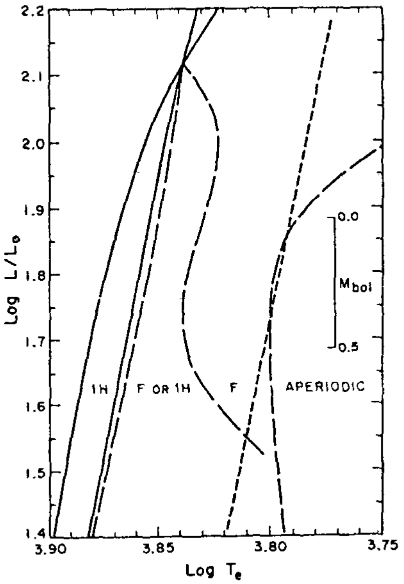


Figure 2: Period versus color for RR Lyrae variables in M3 and ω Cen



—H-R diagram showing the overall stability results for the main survey, $M = 0.578\ M_{\odot}$, $X = 0.7$, $Y = 0.299$, $Z = 0.001$. *Solid lines*, linear blue edges; *dashed lines*, nonlinear transition lines (type of behavior indicated); *dotted line*, estimated red edge.

Figure 3: Stellingwerf instability strip

transition line between the F or 1H and F regions in the center of the figure that is now in question along with the interpretation of Van Albada and Baker. Cox and Simon have used the strictly periodic method of Stellingwerf (1974) to map the stability of static and nonlinear full amplitude solutions as a function of T_e. Figure 4 gives growth rates of linear theory perturbations in static models (labelled 1H and F) and in full amplitude models F in 1H and 1H in F. The undisputed transition line is at about T_e = 7100 K where blueward evolution converts fundamental pulsations to the first overtone. The instability of overtone (1H) pulsations shown at 6600 K disappears if the artificial viscosity is reduced giving a larger and more stable 1H amplitude. These new results give no transition line and F or 1H behavior in almost all the instability strip, just as Spangenberg (1975) displayed earlier (Figure 5).

The Stellingwerf (1974) program has been used to investigate the difference between his shallow models and those of Spangenberg and of Cox and Simon. Figure 6 gives the conclusions in the form of a growth rate versus Lagrangian zone number diagram. The temperature in units of 10^6 K is given uniquely for the zones which run from 1 to 29 for the + and o cases and 1 to 48 for the deeper model x case. The + signs give the work for each zone to promote growth of the fundamental mode in a static model and is obtained from standard

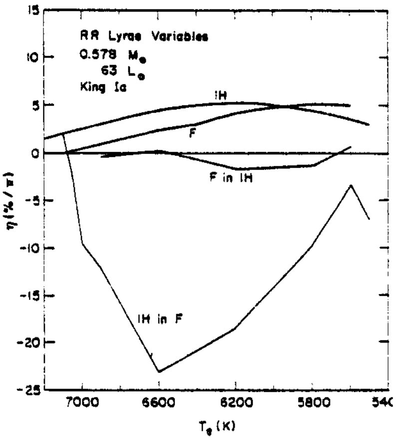


Figure 4: Growth Rate versus effective temperature

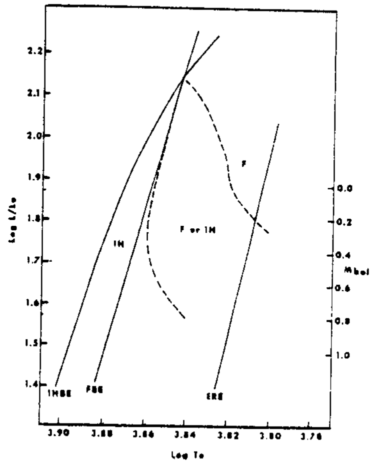


Figure 5: Spangenberg instability strip

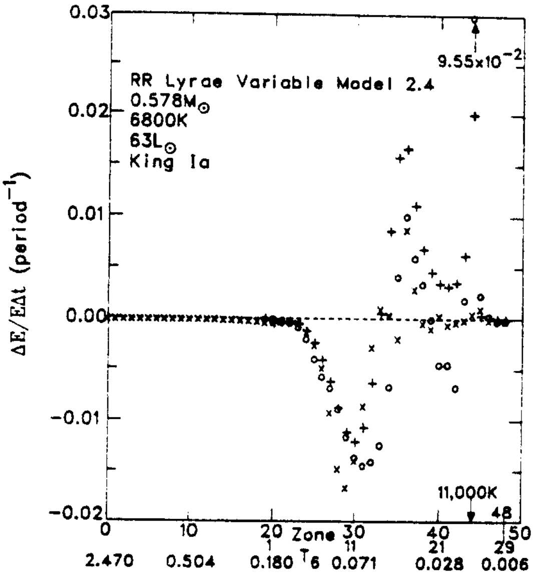


Figure 6: Work versus zone number

linear theory. Driving peaks from the H + He I and the He II ionization regions is seen in, respectively, zones 25 (11,000 K) and 17. Interpretation of the Floquet matrix generated during integration of a first overtone full amplitude periodic solution in terms of its eigenvalues and eigenvectors gives the work to promote mode switching from 1H to F. For the same shallow model the o symbols give this driving per mass zone. The single point at 11,000 K gives enough driving to cause positive net work and a mode switch. Deeper zoning (x symbols) gives too little driving and therefore no mode switching as found in other hydrodynamic programs by Spangenberg and Cox and Simon.

The Stellingwerf program has an adiabatic spring in zone 1 which is not supposed to influence the driving as it simulates the pulsations of the interior. How this spring causes so much driving in zone 25 is under investigation. It appears that no transition line is theoretically predicted between the F or 1H and the F regions, but the Stellingwerf shallow model still needs explanation.

If the Van Albada and Baker interpretation is not correct, then how are Oosterhoff groups explained? Table 1 gives periods at various places in the H-R diagram for Y = 0.299, Z = 0.001 and for 0.55 $M_\odot$ and 0.75 $M_\odot$. Periods at the assumed red edge of Figure 1 are in Column 3 for six luminosities. For 0.55 $M_\odot$ at log $L/L_\odot$ = 1.70 the transition edge fundamental period is $0^{d}.50$ (circled) and overtone period is $0^{d}.37$. Also, the mean overtone period is $0^{d}.34$ (circled). Blueward evolution at about log $L/L_\odot$ = 1.66 (Oosterhoff group I?) would give a mean Π_0 = 0.63 to be compared to $\langle\Pi_{ab}\rangle$ = 0.54, but this is uncertain and probably too large because the red edge of figure 1 is redder (longer period) than Deupree (1977) gives. The Π_0 at the transition edge (TE) is 0.45 and the mean Π_1 is 0.31 just about as observed for group I clusters. Again blueward evolution at about log $L/L_\odot$ = 1.74 (group II?), gives a too large mean Π_0 = 0.75, but Π_0 at the TE is just right at $0^{d}.55$ and the mean Π_1 = $0^{d}.38$ is just slightly too long. For 0.75 $M_\odot$ the two Oosterhoff group luminosities are log $L/L_\odot$ = 1.77 and 1.85.

Table 1

Transition Periods (days)

Y = 0.299 Z = 0.001

		0.55 $M_\odot$						0.75 $M_\odot$					
		RE		TE			BE	RE		TE			BE
Log $L/L_\odot$	$L(\times 10^{35}$ erg/s)	Π_0	$\langle\Pi_{ab}\rangle$	Π_0	Π_1	$\langle\Pi_c\rangle$	Π_1	Π_0	$\langle\Pi_{ab}\rangle$	Π_0	Π_1	$\langle\Pi_c\rangle$	Π_1
1.50	1.23	0.53	0.42	0.32	0.24	0.22	0.20	0.53	0.38	0.24	0.19	0.18	0.17
1.60	1.55	0.71	0.55	0.39	0.28	0.26	0.25	0.71	0.51	0.31	0.24	0.22	0.20
1.70	1.95	0.88	0.69	0.50	0.37	0.34	0.32	0.88	0.64	0.39	0.30	0.28	0.25
1.80	2.46	1.05	0.84	0.63	0.48	0.45	0.42	1.05	0.76	0.48	0.37	0.34	0.31
1.90	3.10	-	-	0.79	0.58	0.56	0.53	-	-	0.62	0.46	0.43	0.40
2.00	3.90	1.7	-	1.03	0.75	0.74	0.72	1.7	1.2	0.79	0.59	0.56	0.52

		$\langle\Pi_{ab}\rangle$	$\langle\Pi_{tr}\rangle$	$\langle\Pi_c\rangle$	$r = n_c/n_c + n_{ab}$
Oosterhoff	I	0.54	0.45	0.32	0.18
	II	0.64	0.55	0.37	0.49

Table 2

Transition Periods (days)

$Y = 0.219$ $Z = 0.001$

	0.55 $M_\odot$				0.75 $M_\odot$			
	TE			BE	TE			BE
Log $L/L_\odot$	Π_0	Π_1	$\langle \Pi_c \rangle$	Π_1	Π_0	Π_1	$\langle \Pi_c \rangle$	Π_1
1.5	0.33	0.25	0.24	0.22	0.26	0.20	0.18	0.17
1.6	0.41	0.30	0.29	0.28	0.33	0.25	0.24	0.23
	X		X					
1.7	0.52	0.39	0.38	0.37	0.41	0.31	0.30	0.28
					X		X	
1.8	0.66	0.48	0.48	0.49	0.51	0.39	0.38	0.36
1.9	-	-	-	-	0.65	0.48	0.47	0.46

		$\langle \Pi_{tr} \rangle$	$\langle \Pi_c \rangle$
Oosterhoff	I	0.45	0.32
	II	0.55	0.37

Table 2 gives the same kind of data for $Y = 0.219$ $Z = 0.001$. Any lower Y than this would give no overtone (Baily type c) pulsators at all at the periods observed. Figure 1 shows how the overtones disappear at lower Y. Red edge data have not been included for this lower Y case because our red edge is not well determined. At $0.55M_\odot$ the Oosterhoff group I luminosity could be $\log L/L_\odot = 1.64$ and at $0.75M_\odot$ $\log L/L_\odot = 1.73$ (given in the table by x marks). The Oosterhoff group II luminosity would be at these two masses $\log L/L_\odot = 1.71$ and 1.82.

Since similar completely blueward evolution can explain both Oosterhoff groups at 0.55 and 0.75 $M_\odot$ also, it appears that the hysteresis postulate of Van Albada and Baker is not necessary and not even viable if no transition line between F or 1H and F exists.

Due to the shortage of time and space not all the current research in RR Lyrae variables has been reviewed. I only note that the synthetic globular cluster work of Caputo, Castellani and Tornambe (1978, 1980) finds that the here disputed Stellingwerf transition line is too red and that maybe evolution to the red, if it does occur at the RR Lyrae luminosities is much faster than given by Sweigart and Gross. Indeed, if the transition line does not exist, as here shown, then rapid evolution at $\log L/L_\odot < 1.9$ seems necessary indeed.

Important current nonlinear BL Herculis variable research to show light and velocity curve bumps is underway. Other results not reported at this meeting are given by Carson, Stothers, and Vemury, and they will appear in the 1981 Astrophysical Journal. The Petersen linear theory results on these stars are reported by him below. Linear and nonlinear work of Cox. King and Hodson using both the Los Alamos and Carson opacities is given by Hodson.

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DISCUSSION

J. COX: What is your opinion as to the difference between Stellingwerf's results and yours in regard to the mode switching tendency in the hydrogen ionization zone?

A. COX: I don't know. We are discussing this difference. The interesting thing about the work done each cycle to promote mode switching is that it has more or less the same shape as the work done to promote growth of the mode from the static model in linear theory. The reason why we get such large work in a shallow model and not with a deep model, both cases with Stellingwerf's code, is something we are working on.

STELLINGWERF: The nonlinear stability, that is, the linear perturbation from the nonlinear limit cycle analysis has very coarse zoning near the hydrogen ionization zone. One thing that has to be checked is whether shifting zones back and forth just a little bit will affect the driving in that zone. It is not clear to us why changing the lower boundary position changes things in the hydrogen zone. It is obviously a very subtle point.

A. COX: Hodson suggests that a change in the phasing of the P and V eigenvectors can change the looping direction in the P-V diagram and a difference in the work.

SIMON: It's not encouraging for future hydrodynamic calculations if all you do is move the inner boundary in or out a little bit to change modal stability.

A. COX: We had a nice transition line a long time ago and now we've lost it. I think we maybe can save the day if both Oosterhoff groups have evolution to the blue with redward evolution much more rapid or at a higher luminosity than we now think.