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Reflections on the concept of symmetry

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The concept of symmetry is omnipresent, although originally, in Greek antiquity, distinctly different from the modern logical notion. In logic a binary relation R is called symmetric if $x\mathrm{R}y$ implies $y\mathrm{R}x$. In Greek, 'being symmetric' in general usage is synonymous with 'being harmonious', and in technical usage, as in Euclid's *Elements*, it is synonymous with 'commensurable'. Due to the second meaning, which is close to the etymology of *σύμμετρος*, 'with measure' has likewise to be read as 'being [in] rational [ratios]' and displays the origin of the concept of rationality of establishing a proportion. Heraclitus can be read as a master of such connections. Exercising rationality is a case of simultaneously finding and inventing symmetries. On that basis a proposal is made of how to relate the modern logical notion of symmetry, a second-order concept, on the one hand with modern first-order usages of the term symmetric in geometry and other fields, and on the other hand with the notion of balance that derives from the ancient usage of symmetric. It is argued that symmetries as states of balance exist only in theory, in practice they function as norms vis-à-vis broken symmetries.

The concept of symmetry is almost omnipresent. We have learned about its descriptive as well as explanatory power in a variety of sciences, and this includes facts concerning the relevance of observing asymmetries. Likewise, we have been made acquainted with samples of important symmetric patterns and of violating symmetries in the arts.

I want to direct your attention first to the logic of being symmetric and away from the wealth of empirical issues, that is, to symmetry as a conceptual tool and not to symmetries as objects of investigation.

Let us start with a reminder that it is rash to speak of just one concept of symmetry. A short look back at history, at the use of the original terms *σύμμετρος* and *συμμετρία* by our Greek ancestors, reveals a meaning of these terms that is

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different from the modern logical notion of symmetry, i.e. from symmetry being a property of a binary (or many-placed) relation: a binary relation is symmetric, if, and only if, it coincides with its converse, like 'being identical with' or 'being similar to', but unlike 'being heavier than' or 'being colder than'. The relation, 'being the mirror-image of', between geometric figures counts as the paradigm of a symmetric relation. Different from this, we observe the original meaning of the Greek terms as being rather close to their etymology: 'with measure'. This is quite succinct in the first definition of the tenth Book of Euclid's *Elements*, where (any two) magnitudes – straight lines or areas – are called symmetric (with each other) if, and only if, they have a common measure, i.e. are commensurable with each other. As a consequence, any magnitude that is commensurable with an assigned one – chosen as a unit, we would say – is called rational, or expressible (*ξρητόν*) in Euclid's terminology. The meaning of *σύμμετρος* in general, i.e. its meaning beyond mathematics, and at the same time apparently more specific than just rational ratios may be conveyed by the phrase 'being harmonious' as it is applied to a well-balanced relationship between parts and whole of some entity that is treated as a whole out of parts. This is a meaning that is more or less covered by the corresponding terms in our modern languages, too. When such a harmony obtains, according to a widespread – but not uncontroversial – conviction, the entity is called beautiful. Is beauty, then, a display of some outstanding rationality? Or is it rather, a balanced interplay of rationality with irrationality, a harmony of logically higher order? Remember a quotation from Dagobert Frey, contained in Hermann Weyl's famous book on *Symmetry*: 'Symmetry signifies rest and binding, asymmetry motion and loosening, the one order and law, the other arbitrariness and accident, the one formal rigidity and constraint, the other life, play, and freedom'.[1]

We all know how the golden section – *sectio aurea* or *divina proportio* – has been held in high esteem as a sign of balance from its very discovery by the Pythagoreans: with respect to their magnitudes the side of a regular pentagon turns out to be the mean proportional of the *whole* diagonal and the *part* that you get by subtracting the side from the diagonal. Later on, in the Renaissance period, the golden section was regarded as testifying to the aesthetic beauty of objects that are internally structured by golden sections. Of course, such a treatment of beauty, Apollonian rather than Dionysian, was challenged in various ways in antiquity and later on, as well. I cannot deal here with the ancient discussions, although they would be illuminating, especially in view of the curious fact that there is no Latin equivalent to the Greek *συμμετρία* a fact that had been noted by ancient authors. For instance, Pliny the Elder wrote in his *Natural History* [34.65] of AD 77: *non habet Latinum nomen symmetria.*

Instead, I want to stress something that escaped attention until the logical structure of language became better understood. It will lead us to an appreciation

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of how the ancient and the modern notions of symmetry relate to each other. Nowadays, we know that predicative expressions, and 'symmetric' is one of them, may sometimes occur meaningfully both in the language about objects and in the language about our verbal tools. Furthermore, in ordinary language they often play a descriptive and an evaluative role, where evaluations involve both logical layers of language. Take as a simple example the colour-term 'red'. We basically apply it when seeing something red or when looking for something red. In the first case it is said that something red is given, out there in the world around us, in the second case it is said that something red has to be produced in either of two ways: it is out there but elsewhere, not in the spatio-temporal presence, and will have to be made present, possibly just by waiting, or it might be elsewhere or even nowhere and is, disregarding prior existence, brought into existence anew or even for the first time. In the latter case we encounter the well known difficulty of identifying something as being red without knowing in advance what it is to be red, hence, of being dependent on some other characterization, e.g. by the wavelength of the emitted or reflected light if it reaches the retina unchanged and, after proper processing, is registered by the visual centre of the brain. Whether we state that something is red or expect that something will be red, the term 'red' is used descriptively, although embedded in two different speech act[-type]s, and quite often no further analysis is offered.

It is neglected that, in addition to the descriptive use of a term in an utterance, we normally encounter an evaluation as well. Besides stating that something is red, the statement is evaluated as being true; besides expecting that something will be red, the expectation is evaluated as being warranted. Being-true or being-warranted are not just predicative expressions within the meta-language only, they are the outcome of judging what has been said by thematizing the difference between a claim and its satisfaction, and hence they are concerned with the peculiar bond between object- and language-level, world and language. In ordinary language, a proposition and a judgment are rarely differentiated; the same words perform both services. Their difference often remains unconscious. This, besides historical reasons like the fight against psychologism in logic some hundred years ago, has certainly contributed to the fact that even in the modern philosophies of logic and of language, both terms, 'proposition' and 'judgement', are treated as referring, essentially, to the same type of entities, 'proposition' logically and 'judgement' psychologically.

It is in scientific language where we rely on identifying the reflective use of language within the use of ordinary language, since scientifically, besides saying something, we also account for why it has been said and for what reasons. Giving reasons signals the presence of judgements and not just statements, expectations, wishes, and so on. In science and as well in art – although in art the notion of language includes the realm of arbitrary sign-languages – it is essential to become aware of language at the reflection-level as a distinct mode of speech beyond object-language and meta-language, yet without forgetting that in ordinary language we are regularly confronted with an amalgamation of all three levels.

Reflective use of language as something discernible within the use of ordinary language was a decisive discovery in Greek antiquity, a discovery that may as well be called an invention, because it does not really exist prior to its explicit identification. Reflection marks the advent of philosophy in its original sense where philosophy is understood as tantamount to science. It was unfortunate, and has had unhappy consequences in Western intellectual history, that the arts were not included in such an understanding of philosophy, although this restriction, which I have added in order not to fall prey to deplorable generalizations, remained a much disputed topic even in antiquity.

As the reason for the schism between the sciences and the arts, of which the still lively opposition between the natural sciences and the cultural sciences is a somewhat weaker afterimage, we may identify the widespread Greek conviction that only from things that do not owe their existence to human activity, i.e. only of nature and not of culture, real knowledge can be gained. From the Renaissance onwards, such a conviction lost currency, and it was Kant who radically concluded that in the case of real knowledge it is the other way round: 'die Vernunft sieht nur das ein, was sie selbst nach ihrem Entwurfe hervorbringt' (*Critique of Pure Reason*, B XIII). Reason, according to his simile, is a judge who puts conceptually devised experimental questions to nature and judges her answers. In present-day post-Kantian science, such a radical culturalization of nature is usually considered to be unconvincing; rather, we are confronted now with powerful moves towards a naturalization of culture, using the neuro-sciences as a Trojan Horse. I think it should be uncontroversial that either way of looking at knowledge is far too simple. We all know that our knowledge, whether of nature or of culture, whether perceptual as in art or conceptual as in science, is essentially bound to human activity, be it on the object-level with all kinds of successively invented technical tools, or be it on the language-level with our theory-building, foremost by the development of mathematical tools. After the discovery of reflection, tantamount to its invention, as I said, we can never get rid of the inherent duality between objects (of procedures) and procedures (of making objects available), a duality that was first observed by the French mathematician, Jean Cavaillès as one of *objet* and *opération*, and, hence, as a case of symmetry both in the old sense of being well-balanced and in the modern sense of being a mirror-symmetry. We owe the realization of the importance of Cavaillès' discovery to Gilles-Gaston Granger who took great pains to develop it further and to follow up on some of its various ramifications.[2] Not yet treated, to my knowledge at least, as a conspicuous case of such a duality is the relation between the objectival *fermions* and the force-mediating, hence, operational *bosons* in the discussions about super-

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symmetry in theoretical physics – for clarification I should add: when referring to bosons, they are, as usual in mathematics, operations turned into objects, although, properly speaking, into objects of logically second order.

Logic and mathematics are the prominent areas where, within the context of the duality of object and procedure, the interplay of something made and something found, can be studied most effectively, provided the role of the linguistic tools is properly taken into account. So, returning to the notions of symmetry under discussion: what is the difference between looking at symmetry, on the one hand, as a first- or second-order property of certain entities such that among descriptions of these entities 'being symmetric' may occur, and, on the other hand, as the result of an evaluation of something said or done such that 'being symmetric' will be close in meaning to 'well-balanced' or 'located [metaphorically] in the centre of gravity'?

To find an answer to this question it is useful to return to antiquity again, to an author who puts balancing, in fact the balancing of extremes that fight with each other, in the centre of his reflections on rationality or the *λόγος*. It was Heraclitus, who uses the construction of mean proportionals to show how rationality works such that a mean proportional may even serve to characterize rationality. In fragment B 83 Heraclitus states: 'Compared with god, the wisest/most knowledgeable [*σοφώτατος*] of men will appear an ape, in wisdom/knowledge [*σοφίη*], beauty and all else.' Instead of treating this and related fragments as a sign that Heraclitus had a 'religious sense of the worthlessness of human knowledge in comparison with divine'[3], it should be read as an explicit presentation of his method of rational thinking and, thus, of the peculiar worth he ascribes to human knowledge. In B 83, man is the mean proportional between god and ape with respect to the binary predicate 'wiser/more rational than'. Heraclitus does not simply want to compare two entities like god and man: both are wise, but god more so than the wisest of men. He wants to measure the difference in wisdom/knowledge between the two, yet without having a unit of wisdom at his disposal that is public and not private. His solution is a definition of equality between two differences (presented multiplicatively as two ratios: god to man and man to ape with respect to 'wiser than'), thus, and this is quite modern, in fact it is an anticipation of definition by abstraction avoiding the definition of a necessarily arbitrary unit of wisdom. Thus, man as the mean proportional between god and ape unites contraries. Man is at the same time wise/rational (with respect to ape) and unwise/irrational (with respect to god). Heraclitus suggests proceeding in the same way with respect to any other pair of polar-contraries, i.e. contrary predicates that belong to scales of comparison, like 'beautiful–ugly' to the comparison 'more beautiful than'. He identifies man as the central point of reference with respect to a world that is governed by extremes that fight with each other, by the 'ever-living fire' in his words (B 30). The measuring rod with which to grasp nature (*φύσις*) is not out there to be found – *φύσις κρύπτεσθαι φιλεῖ* (B 123 ['nature loves concealment', in the excellent reconstruction by Kirk:[4] 'The real constitution of things is accustomed to hide'], but it is the *λόγος* as exhibited ambiguously by the rational activity of man, an activity that has been identified as a specific way to relate man to nature.

With the tool of mean proportionals, Heraclitus may indeed feel justified in pleading for his thesis of a coincidence of opposites against Pythagoras. Contrary forces, if brought into balance by some mean proportional between them should not be treated as having disappeared by mutual neutralization. He denounces Pythagoras as a cheater and mere polymath. To search for simple, symmetric and harmonious, i.e. rational proportions as the basis of cosmic order, i.e. of the *κόσμος* (the term *κόσμος* implies being well-ordered, already), as the Pythagoreans do, is due, so we may infer, to a bias towards mathematical theories, to what is limited by applying the limit (*πέρας*) to the unlimited (*ἄπειρον*); it neglects making judgements on how theories relate to the ever-changing, although unitary, world around us. Aristotle develops a far more advanced argumentation along these lines in his critique of Plato's Pythagorean inclinations. Yet, his contempt for Heraclitus' apparent disregard of logical laws we may now identify as being based on a misunderstanding. In B 83, Heraclitus exercises rationality and at the same time characterizes it as something balanced: There is symmetry between the relation 'more rational than' and its converse 'less rational than', in fact the symmetry of the meta-relation $\mathrm{R}(\rho,\tilde{\rho})$ between a relation ρ and its converse $\tilde{\rho}$. We have thus found an instance where the gap between the ancient and the modern meaning of being symmetric is, in principle, closed.

In addition, we have learned that exercising rationality is not searching for a simple comparison of two objects by establishing a (rational) proportion between them, but an attempt to relate such proportions with each other, that is to look whether they are analogous, if I may use Euclid's terminology for an equality $a : b = c : d$. I should add that to speak of a rational proportion would originally be a pleonasm, since proportion, in Greek *λόγος*, used to imply being rational (irrational proportions have been called *ἄλογοι*), but after the discovery of irrational proportions in geometry, most probably at first by the Pythagoreans on the occasion of comparing the magnitudes of side and diagonal in a regular pentagon, the terminology was changed. In Euclid's *Elements*, irrational proportions that on being squared become rational, were introduced to be an additional species of *δυνάμει*, i.e. potentially, rational ones. Modern terminology with respect to exponentiation still keeps these ancient roots.

We are prepared now to take this example of Heraclitus as a case of how generally to relate the first- and second-order use of the predicate 'symmetric': A set *A* of objects, structured by operations and relations, is symmetrically structured with respect to some special structure S (for simplicity's sake I assume

S to be just a binary relation), if and only if there is an idempotent one-to-one mapping T of A onto itself, i.e. $TTx = x$ for all $x \in A$, that is compatible with S, i.e. $S(x,y)$ implies $S(Tx,Ty)$; in case S is defined by a function f such that $S(x,y) \leftrightharpoons fx = y$, this is equivalent to the commutability of T and f [$fTx = Tfx$ being equivalent to $S(x,y) \prec S(Tx,Ty)$]. From the idempotency of T it follows that the associated relation $R(x,y) \leftrightharpoons Tx = y$ is a symmetric relation in the modern logical sense, i.e. $R(x,y) \prec R(y,x)$. Under the conditions mentioned, the first-order symmetry of x and y is based on the existence of a second-order symmetric relation R between x and y.

If you start with the set of all binary relations where structure S is just the relation of identity you come back to the example of Heraclitus, then conversion establishes a simple symmetry among binary relations. Another well-known example is the set of all logically compound formulae in systems of classical logic with the relation S being the logical equivalence between them. Then, mapping T that transforms any formula into its dual [structural changes!], fills all the requirements that had been set: duality in two-valued logic is a case of symmetry, a symmetry of Boolean lattices in the language of lattice theory. The cases of duality in geometry are covered in the same way by taking as initial set A the set of sentences, e.g. of an axiomatized two-dimensional Euclidean geometry, and as structure S again just logical equivalence between such sentences. In order to arrive at ordinary symmetries of plane geometric figures such as, for example, given by relations S in a two-dimensional Euclidean space A, any mapping T compatible with S that produces mirror-images satisfies the necessary requirements: S and TS are symmetric with each other – the natural extension of mirroring to automorphisms that are not idempotent leads to a generalization of the notion of symmetry, unless, of course, non-idempotent elements can be expressed by iteration of idempotent ones (as is the case, for example, with rotation-symmetry in two-dimensional Euclidean geometry).

I will now review the examples covered and look at what precisely has happened when first-order symmetry was based on second-order symmetry. Symmetries are structural properties of objects and differ from ordinary ones in being dependent on already available predicates about the objects. You do not simply observe symmetries in nature, rather you use the logical concept of symmetry as being one of invariance under commutation of means of reference, be they characterizing predicates or spatio-temporal coordinates, to measure the difference of observed relations with ideal ones. Symmetry is a feature of theories such that asymmetries, be they produced as by artists or observed by scientists, are not just events to be registered but the result of a judgement – in deeds or with words – of how our picture of the world fits or is at odds with the world of culture and of nature around us, i.e. with the world we actually live in by taking part in a process of uninterrupted interference. To look for symmetries, or, more accurately, to judge deviations from symmetries, being the most conspicuous case of regularities beyond equalities that account for mere classifications, is a peculiar way to look for reasons. I may just remind you once again of the almost mythical kinship between beauty and truth that we come across quite often in the work of leading mathematicians. In antiquity, it was Harmonia, daughter of Ares and Aphrodite, who represented order and balance of the universe.

In judging the fit of what we suffer by being exposed to objects, with what we do in order to get acquainted with those very objects, we have returned to the very general duality of object and procedure – *objet et opération* – I have already referred to, and which Nelson Goodman has identified as the mutual dependency of matter and manner. We choose the facts as much as the frameworks, he says,[5] although this statement is better split into two complementary – that is, again symmetric – ones. We produce the facts as much as the frameworks and we experience the frameworks as much as the facts. If one pursues this line of reasoning a bit further (which cannot be done here) we end up with a dialogical duality inherent in any human activity. In the process of acquiring an action competence there are two roles, the I-role of performing the action in producing an action token, i.e. the role of agent, and the You-Role of recognizing the action in witnessing an action type, i.e. the role of patient. The dialogical duality amounts to two different approaches, i.e. manners, to an action, a practical and a theoretical one. Transformed into matter, we speak of empirical action-tokens and rational action-types. It is this derived duality that can be shown to exist among arbitrary objects, which, after a number of additional steps reminiscent of Leibnizian constructions,[6] makes it possible to speak of the duality of body and mind for humans and other living beings. If we switch back from the duality among objects, empirical tokens and rational types, to the duality of procedures, practical performance and theoretical recognition, it is not difficult to arrive at the following conclusion: humans become persons by mutual recognition that comes about by a process of developing the ability to play both dialogical roles at the same time. It is a process of both individuation and socialization that includes a growing consciousness that any one of us is bound to continually losing and recovering the balance between the two dialogical roles.

Symmetries that are states of balance that exist in theory only, in practice they are norms vis-à-vis broken symmetries. And it is they that keep things going, whether in nature or in culture.

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