point could be argued about point set topology, but in this case the author's chapter 2 is refreshing and vigorous enough to justify inclusion.

Chapters 3 to 9 form a well-written account of standard measure and integration theory. The concept of measure is taken as basic, and integration is introduced by means of simple functions. The concept of linear functional on the space of continuous functions is discussed later and its connection with the earlier ideas is established. In addition to routine material such as L_p-spaces, Fubini's theorem and the Radon-Nikodym theorem, the authors also include the mean ergodic theorem and prove the result often quoted but seldom seen in standard texts, that a function is Riemann integrable if and only if its set of discontinuities has measure zero. Thus the area covered is wide, by no means exclusively geared to the applications in probability. A disappointing exception, for this reviewer, is the rather sketchy account, confined to metric groups, of the Haar measure. Uniqueness apart from a constant multiple is not proved, and the wording of Ex. 7, page 260, though formally accurate, might mislead an incautious reader by giving him the impression that no uniqueness theorem for non-compact groups is known.

The probability half of the book again covers a variety of topics in some depth. The introductory chapter 10 - "What is probability?" - contains a good discussion of the relationship between measure and probability. It is made quite clear what assumptions are involved, and that it would be unreasonable to expect a proof of them. However, a thorough discussion of elementary examples makes the assumptions very plausible for beginners. The discussion of statistical independence and product measures is particularly lucid.

The final chapters - eleven to nineteen - headed, in order, "Random Variables", "Characteristic Functions", "Independence", "Finite Collections of Random Variables" and "Stochastic Processes" provide a sound groundwork in Probability Theory. Where space forbids a complete account, as with the law of the iterated logarithm, suitable references are given.

Altogether, this is an excellent book, particularly suited for graduate students or for undergraduates in their senior year.

A.M. Macbeath

Elementary Differential Geometry by Barret O'Neill. Academic Press, 1966.

Geometry has been advancing very rapidly in research level; by contrast the traditional undergraduate course has changed very little in the last few decades. There should be general agreement that the undergraduate course needs to be brought up to date, and this book may be said to be one that meets well this demand in presenting both materials and the mathematical style. After introducing the notations and symbols to be used throughout the book, Chapter II is devoted to the geometry of curves in E^3, where Frenet's formulas are emphasized as those indicating the rate of changes of frame constituting vectors along a curve. The notion of the rate of change leads readers to accept the covariant derivatives of vector fields by pointing out the fact that the connection forms ω^i_j are nothing but the curvature and torsion of a curve appearing in Frenet's formula, if the vector fields in general sense are to constitute the frame fields along a curve. In Chapter III the concept of congruence in Euclidean geometry is shown quite applicable by dealing with the notion of rigid motion and Bonnet's theorem on the congruency of curves with each other is proved when the isometry is subject to the motion. Chapter IV is occupied by rather classical explanation of surfaces in E^3; geometrical quantities such as Gaussian and mean curvatures are derived by issuing lines of curvature, and topics include special curves and familiar surfaces in almost the same classical calculus. But these classical items are now formulated in terms of differential forms ω^i_j and θ^i, and in Chapter V readers begin to know how to interpret the Gaussian curvature in terms of the differential forms (ω) and (θ) composing the structural equations due to E. Cartan. Gauss and Codazzi equations are formulated as the exterior differentials of ω_{12} and ω_{13}, ω_{23}, respectively. Chapter VI is concerned with the intrinsic properties, shape problems of surfaces in E^3 and the Euclidean geometry of E^3 itself. The intrinsic geometry of them is based on the dot product applied to the tangent vectors, and it is pointed out here that the shape problems can be harmonized only by means of such intrinsic geometry and the geometry of the enveloping space. This being done, the author brings readers to the intrinsic geometry, that is, Riemannian geometry, in Chapter VI. Abandoning the definition of Gaussian curvature for surfaces in E^3, the theorema egregium is stated to find a satisfactory generalization. It is defined as a unique real function K such that for any frame field on a manifold the second structural equation holds. The last chapter is the approach to the Riemannian geometry itself. Parallelism of a vector field along a curve is given by the notion of covariant differentials, and geodesic is dealt with by means of the first and second variation; through the latter the relation between curvature and conjugate points is explained by setting up the Jacobi equations. Gauss-Bonnet theorem is discussed in such a way as to emphasize the influence of Gaussian curvature upon the topological conformation of a Riemann manifold of two-dimension. Thus, this book is highly recommendable to undergraduates who aspire for the advanced study of differential geometry in the sense that it sheds light far towards the places the reader should reach, and also it serves the professional ones as their elbow book in their attempt to keep one's way of notations and symbols in a well-ordered shape.

Tanjiro Okubo, McGill University

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