REFERENCES

1. J. P. Boyd, *Chebyshev and Fourier spectral methods* (Lecture Notes in Engineering No. 49, Springer-Verlag, Berlin, 1989).

2. C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang, *Spectral methods in fluid dynamics* (Springer-Verlag, Berlin, 1988).

Falconer, K. J. *Techniques in fractal geometry* (Wiley, Chichester–New York–Weinheim–Brisbane–Singapore–Toronto, 1997), xvii+256 pp., 0 471 95724 0, £24.95.

This book has proved surprisingly hard to write a review for. This is not due to problems with the book (indeed it is excellent) but more due to the reviewer's concerns with his own bias: he was involved in the proof-reading.

This book is a sequel to the author's earlier book, *Fractal Geometry* [1] (recently published by Wiley in paperback). It follows closely the style of its ancestor and details in a clear and elegant manner developments in fractal geometry since the earlier book was published. The author concentrates on presenting the key ideas in the subject rather than obtaining the most general results. This makes the book a useful primer before reading more technical papers/works.

The book begins by reviewing some of the more useful results from *Fractal Geometry*, obviating the need for the reader to possess this book as well (although it would not do any harm).

The first part of the book discusses general methods for calculating the dimension (be it Hausdorff, packing or Box) of a set and introduces the family of examples which the book is primarily concerned with: cookie-cutter sets. These are essentially non-linear versions of the more familiar self-similar sets and many of the results for self-similar sets hold for this more general class. The thermodynamic formalism is introduced and it is shown how this enables dimensions of these non-linear sets to be (theoretically) found. The author explains clearly the analogies with thermodynamics and makes the methods used appear very natural.

The middle part of the book describes how probabilistic ideas may be used in the study of fractals. In particular, the author states and proves versions of the Ergodic theorem, Renewal theorem and the Martingale Convergence theorem. He gives a nice application of the Martingale Convergence theorem to the study of Random cut-out sets (a set which is formed by randomly cutting out pieces of decreasing size from some initial set, such as a square). If you know how the size of the cut-out pieces varies, then with positive probability you know the dimension of the cut-out set.

The latter part of the book is a survey of several topics from geometric measure theory and fractal geometry, discussing some of the developments of recent years. There is an introduction to the theory of tangent measures and an explanation of their rôle in studying the geometry of measures (and sets) in Euclidean spaces. This chapter serves as a useful primer to the more technical account given by Mattila in [2].

There is a chapter discussing dimensions of measures; this involves, essentially, looking for "typical" values of

$$\underline{\alpha}(\mu, x) = \liminf_{r \searrow 0} \frac{\mu(B(x,r))}{\log r}, \quad \overline{\alpha}(\mu, x) = \limsup_{r \searrow 0} \frac{\mu(B(x,r))}{\log r}$$

for a measure μ (here $B(x, r)$ denotes the closed ball of centre x and radius r). When $\underline{\alpha}$ (or $\overline{\alpha}$) may take many different values we are led naturally to the idea of decomposing measures into pieces where they do behave similarly. This in turn leads, ultimately, to investigating the multifractal decomposition of a measure, which involves looking at the dimensions of sets of the form

$$\{x : \underline{\alpha}(\mu, x) = \alpha\} \quad \text{or} \quad \{x : \overline{\alpha}(\mu, x) = \alpha\}.$$

The final chapter of the book gives a brief but lively introduction to the rôle of fractals in the study of differential equations, including discussions of how to estimate dimensions of attractors and how to solve the heat equation on domains with a nice self-similar boundary. This involves a nice application of the Renewal theorem discussed earlier in the book.

To summarise: this is a lovely book and a worthy successor to *Fractal Geometry*. It presents concisely many of the techniques currently used in the study of fractals and is a good survey of results which have appeared since the publication of the last book.

The book, unfortunately, does possess many minor errors which may be occasionally distracting (this is especially embarrassing to this reviewer since he was involved with proof-reading . . .) but even the most serious error I found (in the chapter on tangent measures, where the set of tangent measures of log 2/log 3-dimensional Hausdorff measure restricted to the usual $\frac{1}{3}$-Cantor set is misdescribed) is not particularly bad.

The author's clear, lucid style of writing is a pleasure to read and his decision to concentrate on the essential ideas rather than to obtain the greatest generality pays off handsomely. Anyone who reads this book will, by the end of it, be able to pick up a current research paper on fractals and have a good chance of understanding it.

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1. K. J. Falconer, *Fractal geometry* (Wiley, 1989).

2. P. Mattila, *Geometry of sets and measures in euclidean spaces* (Cambridge, 1995).

Aschbacher, M. *3-Transposition Groups* (Cambridge Tracts in Mathematics Vol. 124, Cambridge, 1997), vii + 260pp., 0 521 57196 0 (hardback), £35.00 (US$49.95).

A 3-transposition group is a finite group G which is generated by a normal set D of elements of order 2 such that any two elements of D either commute or have a product of order 3. The symmetric groups S_n $(n > 2)$ are 3-transposition groups. Less obviously, several classical groups in characteristics 2 and 3 are 3-transposition groups. Part I of the book under review is devoted to a proof of B. Fischer's beautiful theorem, proved around 1970, which characterized finite 3-transposition groups G in which $Z(G) = 1$, G' is simple and D is a single conjugacy class. This characterization led directly to the discovery of three new "sporadic" simple groups, and Fischer groups also turn out to be closely related to several other sporadic simple groups. Parts of the proof of Fischer's theorem had remained unpublished prior to the appearance of this book.

Fischer's work was one of the most imaginative chapters in the classification of the finite simple groups, and the author of this book was and is the foremost exponent of the use of geometries and graphs derived from internal group-theoretic structure in characterization theorems which may be viewed (at least in part) as a natural development of that work. The author gives the reader the benefit of his own deep insight as he develops the material in an assured manner.

In the second and third parts of the book the author considers the questions of existence and uniqueness of the Fischer groups and the local structure of the Fischer groups. Both of these are viewed in the wider context of developing a reasonably uniform theory of sporadic simple groups.