Near Earth Objects, our Celestial Neighbors: Opportunity and Risk
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Some aspects of the statistics of Near-Earth Objects

J.R. Donnison

Astronomy Unit, School of Mathematical Sciences,
Queen Mary, University of London, Mile End Road

Abstract. The distribution of Near-Earth Objects, in particular Near-Earth asteroids is examined using maximum likelihood methods. These are analysed with respect magnitudes, taxonomic classes and to their orbital distances. Comparisons are made with the distributions of main-belt asteroids and short-period comets.

Keywords. Near-Earth objects, population

1. Introduction

The Near-Earth Object (NEO) population is defined as the group of small bodies with perihelion distance $q < 1.3$ AU and aphelion distance $Q > 0.983$ AU. These are composed of the 334 Atens that have semi-major axis $a < 1.0$ AU, the 1994 Apollos with perihelion $q < 1.0AU$ and $a > 1.0$ AU, and the 1748 Amors with $1.0 < q < 1.3$ AU (as of August 2006). These NEOs with semi-major axes smaller than that of Jupiter are thought to be mainly asteroids that have escaped from the main asteroid belt, although as we shall see some may be extinct cometary nuclei.

In order to estimate the size of an NEO from its measured absolute magnitude its reflectivity (that is its geometric albedo) must be known. The measurements of albedo are only available for less than one percent of the NEOs, and the values span a wide of values from 0.023 to 0.63 (Binzel *et al.* 2002).

Attempts to debias the albedo and taxonomic distribution of NEOs have been made by Luu & Jewitt (1989) using Monte-Carlo simulations of the distribution. Binzel *et al.* (2002) conducted a similar study. They attempted to define a reasonable albedo distribution for each of the main belt source regions previously identified as sources of asteroidal and cometary material to the NEO population by Bottke *et al.* (2002). Stuart & Binzel (2004) utilize the direct observations of a taxonomically well determined subset of NEOs to determine the albedo distribution of the NEOs for which albedos are not available.

In this work we use maximum likelihood techniques to help determine the size distribution from the observed population with well determined absolute magnitudes. We draw heavily on the debiasing work particularly of Stuart & Binzel (2004). The absolute magnitude data used is taken from the MPC data sites at http://www.cfa.harvard.edu/iau/lists/Atens.html, http://cfa- www.harvard.edu/cfa/ps/lists/Amors.html, andhttp://cfa-www.harvard.edu/cfa/ps/lists/Apollos.html. In Fig. 1 we show a plot of the semi-major axis versus eccentricity of the Atens, Amors and Apollos used in this analysis.

2. Tisserand parameter

The Tisserand parameter is a dynamical quantity that is approximately conserved during an encounter between a planet and an interplanetary body. It therefore provides a

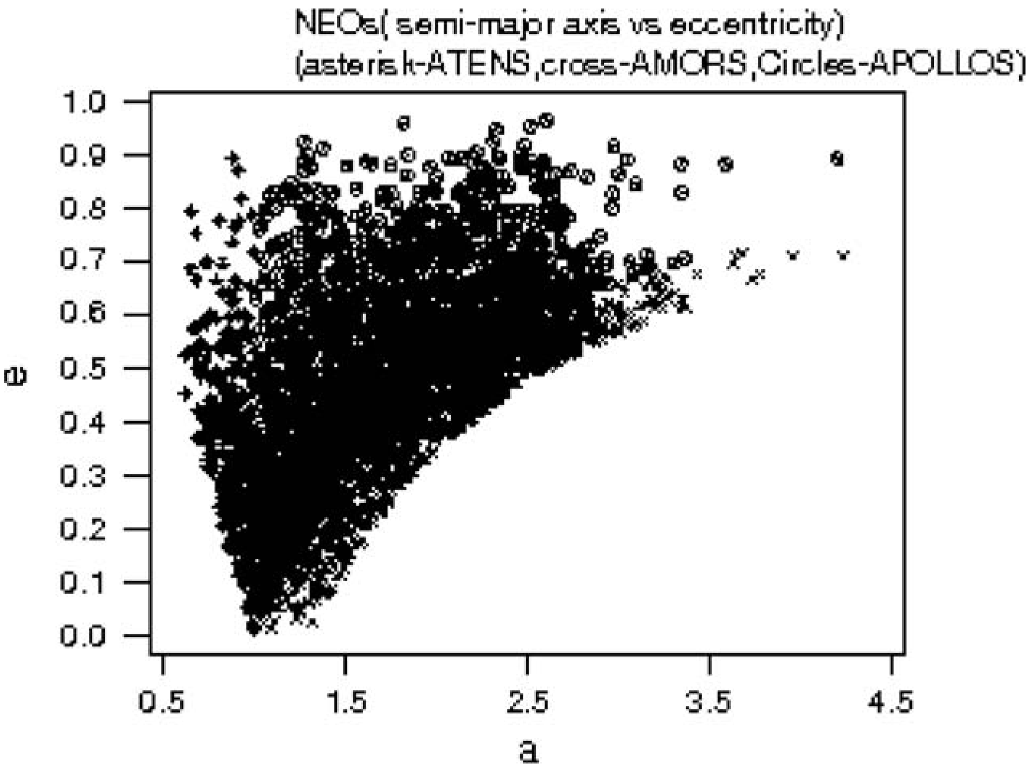

Figure 1. Plot of semi-major axis versus eccentricity of the Atens, Amors and Apollos used in this paper.

way to connect the post-encounter dynamical properties with the pre-encounter properties. The Tisserand parameter also provides a measure of the relative speed of an object when it crosses the orbit of a planet.

The Tisserand parameter T_J relative to Jupiter under the restricted circular three-body problem is given by

$$T_J = a_J/a + 2\sqrt{a/a_J\,(1-e^2)}\cos i, \tag{2.1}$$

where a_J is the semi-major axis of Jupiter, and a, e, and i are the semi-major axis, eccentricity, and inclination of the asteroid. Solar system bodies with $T_J \leqslant 3$ are dynamically coupled to Jupiter. A number of Near-Earth Asteroids have $2 < T_J < 3$. They tend to have low albedos and Fernandez *et al.* (2001) showed that there was a strong correlation between T_J and albedo which suggested that there is a significant cometary contribution to this asteroid population. The Jupiter family of comets have $2 \leqslant T_J \leqslant 3$ and the Halley and long-period comets tend to have $T_J \leqslant 2$. Bodies with $T_J > 3$ are generally decoupled from Jupiter and asteroids generally fall into this category. Binzel *et al.* (2004) did a similar analysis to Fernandez using taxonomic classes rather than albedo. They found a distinct taxonomic difference with respect to T_J, where C,D and X-type asteroids predominate for $T_J < 3$. This group is much more dominated by very dark objects than those with $T_J > 3$.

The Tisserand parameter will be incorporated here into the statistical determination of the distribution.

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3. Magnitudes

The apparent magnitude m, of a near-Earth body is given by (Russell 1916; Jewitt 1999)

$$m = m_{\odot} - 2.5 \log_{10} \left(\frac{p_V R_A^2 \phi(\alpha)}{2.25 \times 10^{16} R^2 \Delta^2} \right), \qquad (3.1)$$

where $m_{\odot}$ is the apparent magnitude of the Sun (-26.8 magnitude in the visible), p_V is the geometric albedo, R_A(km) is the radius of the asteroid, $R(AU)$ is the heliocentric distance, Δ(AU) is the geocentric distance, and $\phi(\alpha)$ is the phase function which at opposition with $\alpha = 0$ is $\phi(0) = 1$. This equation can be rewritten in terms of the absolute magnitude, H, as

$$m = H + 5 \log_{10} R + 5 \log_{10} \Delta - 2.5 \log_{10} \phi(\alpha), \qquad (3.2)$$

where H is defined as the apparent magnitude that a near-Earth body would have if it was observed at 1 AU from the Sun, 1 AU from the Earth and at zero phase angle. In this analysis we consider the cumulative magnitude distribution using the absolute magnitudes H as listed in the Minor Planet Center list mentioned earlier. The H values are accurate to 0.05 magnitudes and clearly leading to errors in the estimation of the diameter. Combining equations (3.1) and (3.2), the diameter of the asteroid D $(= 2R_A)$ is related to the absolute magnitude and can be written in the form

$$H = C - 5 \log_{10} D - 2.5 \log_{10} p_V, \qquad (3.3)$$

where $C = 15.618$ (Harris & Harris 1997; Stuart & Binzel 2004). The albedos of comet nuclei are typically $p_V \sim 0.04$, and those of the Centaurs, which probably originated in the Kuiper Belt, have a wide range of values from 0.04 to 0.17. The NEOs, as mentioned earlier, range from 0.023 to 0.63 (Binzel *et al.* 2002) giving a factor of 5 in possible diameter of an NEO for a given absolute magnitude. Stuart & Binzel (2004) found that for NEOs the correlation between albedo and absolute magnitude was not statistically significant and they assumed in their analysis that there was no correlation. We shall here make the same assumption. In order to assess the distribution more quantitatively, we consider the range of absolute magnitudes of the bodies. In Fig. 2 we have a plot of the cumulative number of the NEO population with absolute magnitude greater than H (as of August 2006). The bodies span the large magnitude range $9.45 \leqslant H \leqslant 30.01$.

The distribution shows a linear section for magnitudes up to $H \sim 20.0$ magnitudes. Above this magnitude the cumulative number no longer increases so steeply. For these higher magnitudes selection effects are presumed to be very important and many of the fainter and more distant bodies have yet to be discovered. Below this value the linear slope is not very sensitive to value of H chosen, though care must be taken in its assessment. In order to fit this linear section of the distribution we employ maximum likelihood estimation methods. The use of least-squares fits by many previous authors is not appropriate for cumulative plots as the data points are not independent of each other (Donnison & Sugden 1984; Gladman *et al.* 1998). We now proceed to the basic statistical model of this distribution.

4. Statistical model

We shall assume that the expected proportion of near-Earth bodies with diameters greater than D follows a Pareto power law of the form (that is a power law distribution

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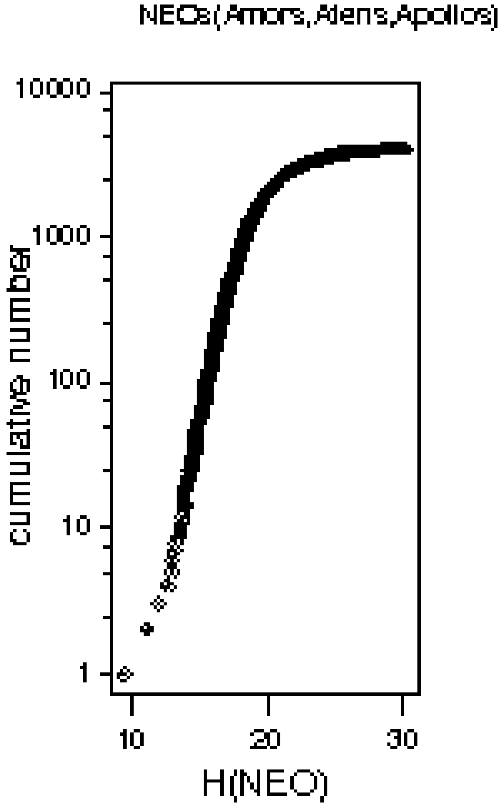

Figure 2. Plot of the cumulative number of the NEO population with absolute magnitude greater than H.

truncated at the lower end)

$$F(>D) = \left(\frac{D}{D_*}\right)^{-\alpha}. \tag{4.1}$$

Here α is the power law or cumulative size index and D_* is the lower limit in diameter that can be observed. Ideally we would like to work with the distribution of p_V but since the number of known albedos is very small we only have average values for taxonomic classes to work with (see Stuart & Binzel 2004). Therefore for a given p_V, in terms of absolute magnitude using Eq. (3.3), the expected proportion of bodies with magnitudes less than H is

$$F(<H) = \left(\frac{p_V}{p_{V_*}}\right)^{0.4\alpha} 10^{0.2\alpha(H-H_*)}, \tag{4.2}$$

where H_* is the critical upper limit in magnitude that is detectable corresponding to bodies with diameter D_*, and p_{V_*} is the corresponding albedo. The coefficient of the magnitudes is often denoted by β, so that $\alpha = 5\beta$.

An equivalent description of the diameter distribution is in terms of the probability density function given by

$$N\,(D) = \frac{\alpha}{D_*}\left(\frac{D}{D_*}\right)^{-(\alpha+1)}, \; D > D_*, \tag{4.3}$$

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where $N(D)\,dD$ is the expected proportion of bodies with diameters between D and $D+dD$. Eliminating the diameter using Eq. (3.3), we can write the distribution in terms of magnitude, so that we have $\Phi(H)$, the expected proportion of bodies with magnitudes between H and $H+dH$, for a given p_V is given by

$$\Phi(H) = \frac{\bar{C} p_{V_*} \alpha}{10^{-0.2H_*}} \left(\frac{p_V}{p_{V_*}}\right)^{\frac{1}{2}(\alpha+1)} 10^{0.2(\alpha+1)(H-H_*)}, \tag{4.4}$$

where $\bar{C}$ is a constant. To proceed further to determine α we use maximum likelihood estimation. The method of maximum likelihood is applicable to a random sample of observations taken from any given distribution. If we consider a set of n near-Earth bodies conditional on the number in the taxonomic class with absolute magnitudes $H_1, H_2, ...H_n$, then the corresponding likelihood function is given by the product of the joint probability density functions as

$$L(\alpha) = \prod_{i=1}^{n} \Phi_{c_i}(H_i), \tag{4.5}$$

where c_i is the taxonomic class for i^{th} observation. It is easily seen that the maximum unconditional likelihood over α is the same as the maximum conditional likelihood given the numbers in the taxonomic classes. This can be handled more easily in log form so that

$$\begin{aligned}\ell(\alpha) = \log_e L(\alpha) = n \log_e \alpha + 0.2(\alpha+1)\sum_{i=1}^{n}(H_i - H_*)\log_e 10 \\ + 0.2nH_* \log_e 10 + \sum_{i=1}^{n}\log_e \bar{C}_i + \frac{1}{2}(\alpha+1)\sum_{i=1}^{n}\log_e\left(\frac{p_V c_i}{p_{V*}}\right) \\ + n\log_e p_{V*}\end{aligned} \tag{4.6}$$

Maximum likelihood estimation of the index now proceeds by maximizing $\ell(\alpha)$ as a function of α. The solution of the likelihood equation

$$\frac{\partial \ell}{\partial \alpha} = 0, \tag{4.7}$$

is then given by

$$\frac{n}{\hat{\alpha}} + 0.2\log_e 10 \sum_{i=1}^{n}(H_i - H_*) + \frac{1}{2}\sum_{i=1}^{n}\log_e\left(\frac{p_V c_i}{p_{V*}}\right) = 0, \tag{4.8}$$

that is

$$\hat{\alpha} = \frac{n}{0.2\log_e 10 \sum_{i=1}^{n}(H_* - H_i) - \frac{1}{2}\sum_{i=1}^{n}\log_e\left(\frac{p_V c_i}{p_{V_*}}\right)}. \tag{4.9}$$

The term involving the albedos can be written in terms of the various taxonomic orbital classes. That is

$$\sum_{i=1}^{n}\log_e\left(\frac{p_V c_i}{p_{V*}}\right) = \sum_{\text{taxonomic classes c}} \log_e\left(\frac{p_V}{p_{V_*}}\right)^{class_c}. \tag{4.10}$$

$Class_c$ is the observed number in class c. We can estimate this quantity from the fractional abundances and debiased albedos derived from the ten taxonomic complexes A, C, D, O, Q, R, S, U, V, X by Stuart & Binzel (2004). This takes into account that $T_J \leqslant 3$ for 30% of the NEOs and that $T_J > 3$ for the remaining 70%. Here we also extrapolate

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the abundances from the large sample size used by Stuart & Binzel (2004) to the current sample size used here. For the over simplified case where all the asteroids have the same albedo, that is $p_V = p_{V_*}$ then we have

$$\hat{\alpha} = \frac{n}{0.2 \log_e 10 \sum_{i=1}^{n} (H_* - H_i)}. \tag{4.11}$$

This gives an expression similar to that of Donnison (2006) used for trans-Neptunian bodies. The sampling variance of $\hat{\alpha}$ for large n for both the general case given by equation (4.9) and the simple case given by equation (4.11) is then approximately given by

$$\left\langle -\left(\frac{\partial^2 \ell}{\partial \alpha^2}\right)^{-1} \right\rangle = \frac{\alpha^2}{n}, \tag{4.12}$$

so that the estimated standard error of $\hat{\alpha}$ in large samples is therefore

$$\frac{\hat{\alpha}}{\sqrt{n}}. \tag{4.13}$$

5. Results

From Fig. 2 the linear part ranges up to absolute magnitudes of around 20.0. The number of NEOs with magnitudes less than or equal to this value is 2079 (as of August 2006). These will form the data necessary for our determination. Before we proceed we can obtain a lower limit for the index if we assume the unrealistic situation that all the albedos are equal, that is $p_V = p_{V*}$, then $\hat{\alpha}$ given by Eq. (4.11) has a value of 1.16 ± 0.025. More realistically, estimating the albedo term using the fractional abundances and debiased albedos of Stuart & Binzel (2004) with a p_{V_*} of 0.05 (equivalent to D=0.6 km at magnitude 20.0) gives $\hat{\alpha}$ of 1.813 ± 0.040. This compares with the value of α of 1.95 found by Stuart & Binzel (2004) as the nearest power law fit and by Stuart (2001) who using a power law fit found a magnitude index β of 0.39. Bottke *et al.* (2002) by modelling derived an α of 1.75 (based on a magnitude index β of 0.35). Rabinowitz *et al.* (2000) also found a β of 0.35 using directly the debiased magnitude distribution observed by the NEAR survey. The result obtained does show some sensitivity to the value of p_{V_*} that is assumed.

6. Comparison with cometary distributions

The size distribution of cometary nuclei of long and short period comets has been investigated by a number of authors. Two approaches have been adopted. In the first approach, Donnison (1986, 1990, 1997, 1999), Hughes & Daniels (1980, 1982) and Hughes (2002) used the absolute magnitude, H_{10}, of the integrated dust and gas coma of *active comets* to estimate the magnitude, size and mass distributions of the cometary nuclei of both long and short period comets.

In the second approach the cometary diameters are measured directly. In the past only a few such size measurements were possible. However, the number of comet diameters accurately determined at large heliocentric distances has recently increased from ground based observations and the Hubble Space Telescope (HST) (Lamy *et al.* 2000; Licandro *et al.* 2000; Lowry *et al.* 2001; Lowry & Fitzsimmons 2003 and Weissman & Lowry 2003). The cometary nuclei at these distances are not obscured by the surrounding coma and dust and are able to be measured directly. This has enabled the cometary index to be estimated for short period comets.

The distribution of masses of such bodies as comets asteroids and trans-Neptunian bodies is usually characterized by a power law index s defined through the mass distribution function $\zeta(m)$, such that the number of bodies with masses between m and $m + dm$ is given by

$$\zeta(m)dm = Am^{-s}dm, \tag{6.1}$$

where A is constant over a specific mass range. The s for near-Earth asteroids is related to α derived earlier by

$$s = \frac{\alpha}{3} + 1. \tag{6.2}$$

Currently using the active H_{10} short period comet data, the mass index s is about 1.6 (Donnison 1990; Hughes 2002). For the direct determination Fernandez *et al.* (1999), Tancredi *et al.* (2006) found for the Jupiter family of comets an s of 1.88, while Weissman & Lowry (2003) found an s of 1.53. Recently Meech *et al.* (2004) found 1.48 for the value of s. Since not all the data used has been published and is not is readily available a full explanation for the differences has not been found. The larger value of Fernandez *et al.* (1999) may however reflect the fact that their sample includes many comets observed at very small heliocentric distances where activity was possible and finding the index is complex. The present author is currently working on a new assessment of this index. These values for comets indicate that for the distribution of comets the majority of the mass lies in a few large bodies suggesting if considered in isolation that planetesimal accretion as opposed to collisional fragmentation is the most likely mode of formation. However, since they probably have their origin in the Kuiper Belt, their small sizes indicate that they may be partly collisional remnants of the larger bodies. For the near-Earth asteroids investigated in this paper we find s of around 1.65, suggesting that this is a distribution derived from larger bodies and that collisional fragmentation could be significant in their evolution. Yoshida *et al.* (2003), Yoshida & Nakamura (2004) have found that the slopes of the cumulative size distribution of Subaru-detected main belt asteroids in the magnitude range $16.5 < H < 18.5$, vary with semi-major axis with an α range from 1.11 ± 0.06 for outer belt asteroids to 1.91 ± 0.008 for those near the 4:1 resonance gap ($2.0AU < a < 2.2AU$). This corresponds to s values of 1.37 to 1.64. This supports the dynamical theory that the inner gaps can convey asteroids efficiently into the near-Earth region.

7. Conclusions

The size distribution index of NEOs has been estimated by using maximum likelihood methods and the fractional abundances and debiased albedos of Stuart & Binzel (2004). The value found is in line with previous estimates and with the main-belt asteroid size distribution and some estimates of the cometary distribution.

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