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OF  
SYMBOLIC LOGIC

EDITED BY

ALONZO CHURCH    S. C. KLEENE    ALICE A. LAZEROWITZ

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## ERRATA

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- Page 36, line 14. For «variagles», read «variables».
- Page 49, after the last displayed formula, insert a «I» at the right margin.
- Page 51, line 5 from below. For «PRESSBURGER», read «PRESBURGER».
- Page 69, line 15 from below. Insert a left-hand paranthesis immediately after the left-hand bracket.
- Page 70, line 14, For «ca», read «ca».
- Page 73, line 8. For « $a = b$ », read « $\alpha = \beta$ ».
- Page 74, line 19. For « $\alpha = \beta$ », read « $\alpha \equiv \beta$ ».
- Page 77, line 14. Insert an additional parenthesis after «XXI 79(4)».
- Page 98, line 10 from below. For «suchthat», read «such that».
- Page 98, line 12 from below. For «generalre cursive», read «general recursive».
- Page 100, line 11. For «contra productive», read «contraproductive».
- Page 102, Line 8, For «as equence», read «a sequence».
- Page 102, line 12. For «hyper-simple», read «hypersimple».
- Page 105, line 6 from below. For «112», read «109».
- Page 107, line 1. For «clasica», read «clásica».
- Page 107, line 11. Insert a comma after 0.
- Page 107, line 17. For «1951», read «1952».
- Page 110, Line 11. For «principal», read «principle».
- Page 206, line 3 from below. For  $(x\exists)$ , read  $(\exists x)$ .
- Page 211, line 21. For «ceruits», read «circuits».
- Page 213, line 22. For «Re- printed», read «Reprinted».
- Page 219, line 5 from below. For «““yellow””», read «“ “yellow” ”».
- Page 315, line 16 from below. For «intitation», read «initiation».
- Page 319, line 2. For  $(\exists R)/(S)$ , read  $(\exists R)(S)$ .
- Page 323, line 21. Before «1949», insert «Paris».
- Page 329, line 14 from below. For «precedure», read «procedure».
- Page 330. At the end of line 2 from the bottom. For «clauses», read «clause».
- Page 331, line 2. For «latter», read «letter».
- Page 335, line 3 from the bottom. For «is», read «in».
- Page 336, line 16 from below. Insert a period after “Taxonomy”.