Proof of a Fundamental Relation in the Theory of Bending between the Bending Moment and Load Curves, or between the Deflection and Bending Moment Curves.

By E. M. Horsburgh.

(*Read and received 8th March 1912.*)

The important relation in the Theory of Bending between the curves of Bending Moment (B.M.), Shearing Force (S.F.), and Load, or between those of Deflection, Slope, and Bending Moment, viz., that the tangents to the first of either set intersect in a vertical line through the centroid of the corresponding area of the last, under the usual convention of drawing, is usually not proved in Engineering Treatises, or else is established in simple cases by the polygon of loads.

An analytical proof might be given with advantage for students of engineering. Let the B.M. or the Deflection curve be represented by $y = F(x)$ (i), then the corresponding Load or B.M. curve is $y = F''(x)$ (ii). If the origin be taken at the foot of one bounding ordinate, and if the other bounding ordinate be chosen where x has the value x_1, the x co-ordinate of the centroid of the area of (ii) is

$$\bar{x} = \frac{\int_0^{x_1} dx\, x\, F''(x)}{\int_0^{x_1} dx F''(x)} = \frac{\Big[xF'(x) - F(x)\Big]_0^{x_1}}{\Big[F'(x)\Big]_0^{x_1}}$$

$$= \frac{x_1 F'(x_1) - F(x_1) + F(0)}{F'(x_1) - F'(0)}.$$

The corresponding tangents to (i) are

$$y - F(0) = F'(0)x$$
$$y - F(x_1) = F'(x_1)(x - x_1).$$

On subtracting, the x co-ordinate of the point of intersection is seen to be

$$x = \frac{x_1 F'(x_1) - F(x_1) + F(0)}{F'(x_1) - F'(0)}.$$

This proof being general includes the Engineering Theorems as particular cases.