A Course in Mathematical Analysis, I Introduction to Analysis, by Norman B. Haaser, Joseph P. LaSalle and Joseph A. Sullivan. Ginn, Boston, 1959. xlv + 688 pages. $8.50.

In the preface to this book we read "... We recognize that analysis is a rigorous subject that is applicable to science and engineering and that today the only practical course of study is one that emphasizes understanding and ideas. The mere acquisition of certain manipulative skills, while necessary, does not prepare anyone to make effective use of mathematics, certainly no one at the professional level of the scientist and the engineer. It is not to be overlooked that this type of presentation is moreover the only way in which mathematics can make its full contribution to the liberal education of our students."

These principles at the beginning, expressing what many of us feel, distinguish this book from the great number of texts on calculus and college mathematics where the stress lies on routine and manipulation. Whether the authors have actually succeeded in their method, only experience can show. How they develop their system may be seen from the following survey.

Chapter 1, Real Numbers, after some simple facts on sets, introduces the real number system R axiomatically; abstract field axioms + order axioms + the "least upper bound axiom". Naturally it cannot be proved at this stage that these axioms actually lead to the field of the real numbers; but reference is made to Chapter 9, "The Least Upper Bound Axiom", where, apart from a proof of the Archimedean property of R and some inclusion theorems, it is shown that such symbols as $\sqrt{2}$ actually represent real numbers. Assuming that the beginner is able to swallow (a) an axiomatic introduction of the number system and (b) a postponement of one of its essential characteristics, this is certainly a simple method; indeed, until limits enter the stage the algebraic and order axioms are sufficient, although in examples and elsewhere symbols such as $\sqrt{3}$, π, ... occur which so far are real numbers only "unofficially".

Chapter 2, Plane Analytical Geometry, including vectors in a cartesian coordinate plane, up to the equation of a straight line. Chapter 3, Functions, introduced as sets of ordered pairs of abstract elements. The identity function (x,x) is denoted by I; it has the "rule of correspondence" $I(x) = x$. Functional composition is discussed, illustrated by graphs, and such notions as univalent function and inverse function are introduced. The reviewer feels that the "algebra of functions" as proposed here involves an unnecessary duplication: a polynomial $f(x) = x^2 + 2x + 3$ appears also as $f = I^2 + 2I + 3$. The

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symbol f^n should have been reserved for functional iteration. Chapter 4, Rigid Transformations, has the general idea of a mapping subordinated to the notion of function. The idea of an abstract group is introduced and immediately applied to the group of all non-singular transformations; this has hardly any bearing on later developments and only a few details are given on groups of rotations and displacements.

Chapter 5, Graphs of Equations, extends the analytical geometry of chapter 2 to the quadratic domain, including a reduction of quadratic forms in two variables. Chapter 6, Analytic Trigonometry, introduces sine and cosine based on an intuitive definition of the length of a circle arc. Chapter 7, Mathematical Induction. Chapter 8, Limits and Derivatives, begins with the notion of slope of the tangent to a curve at a point; there follows limit of a function at x_o, continuity and derivative. The clumsiness of the function notation proposed by the authors becomes evident in the definition

$$f'(x_o) = \lim_{x_o} \frac{f - f(x_o)}{I - x_o}$$

which indeed is followed immediately by the definition of $f'(x_o)$ in the usual form, after which we find at least one other unnecessary form displaying some symbolism of the "algebra of functions". Chapter 9, apart from matters mentioned above, discusses some properties of R (e.g. the Heine-Borel theorem) and uniform continutiy. Chapter 10, Applications. Chapter 11, Solution of Equations. Complex numbers are introduced as pairs of real numbers and the field axioms are established. The fundamental theorem of algebra without proof. The "regula falsi" and Newton's iteration method.

Chapter 12, The Definite Integral, begins with the axioms of area of a plane set and shows that the definite integral (after Riemann) provides a reasonably general definition of area. The double notation of function entails a two-fold notation of the integral, viz. $\int_a^b f$ and $\int_a^b f(x)\,dx$. Since in all examples only the latter is used, it is not easy to see why the first one is carried along; indeed it might lead the beginner to a wrong application of the substitution rule. Elementary classes of Riemann integrable functions are established (monotonic functions and continuous functions) and the fundamental theorems of calculus are proved. An elementary discussion of improper integrals concludes the chapter. Chapter 13, Applications, deals with further examples of area in cartesian and polar coordinates and examples from mechanics. The indefinite integral is defined in accordance with the second fundamental theorem and there is a section on methods of integration. Chapter 14, Elementary Functions with a brief discussion of Taylor's theorem. Chapter 15, Methods of Integration, including numerical integration.

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Each section is concluded by a set of relevant examples and exercises. The text of the book is clear, only in some places burdened by notational innovations leading to duplications which, in the reviewer's opinion, cannot be helpful to the average reader. Further experiments with this book in undergraduate courses will be interesting for students and instructors and should be recommended.

H. Schwerdtfeger, McGill University

Elementary Mathematical Programming, by Robert W. Metzger. Wiley, New York, 1958. 246 pages. $5.95.

As implied by the title, this book is devoted to a detailed and elementary exposition of a number of methods of mathematical programming, including the simplex method and the "stepping-stone" method of Cooper and Charres. Applications covered include the transportation problem, production planning, stock slitting, scheduling, and job and salary evaluation.

A minimum of mathematical background is required, and proofs are omitted. The mathematically mature reader will find the spelled-out detail somewhat tedious. The author's attempt to avoid the term "vector" leads to such peculiarities of language as "c = the objective coefficients of the variables.", "x = the variables of the problem", (p. 111), etc.

However, within the self-imposed limitations, the author has achieved his aims. The book can be recommended to management analysts or industrial engineers who require some knowledge of the techniques for solving the subject problems.

H. Kaufman, McGill University

Mathematical Programming and Electrical Networks, by Jack B. Dennis. Wiley, New York, 1959. 186 pages. $4.50.

This book represents the author's research for his doctoral thesis at M.I.T. The work is an outgrowth of the observation that simple linear programming problems can be solved by equivalent electrical networks. This equivalence is fully exploited for both the linear and quadratic programming problems, and leads to an algorithm for solving network flow problems.

Additional chapters are devoted to a breakpoint tracing procedure, which is applied to the solution of general linear and quadratic programming problems. Two algorithms are presented, one similar to the simplex method, and the second equivalent to the primal-dual method of Dantzig, Ford and

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