

26. A Possible Mechanism for the Capture of Microparticles by the Earth and Other Planets of the Solar System

F. DI BENEDETTO
Rome University
Rome, Italy

By application of Lyttleton's theory for the formation of comets, it is shown that a possible mechanism for the origin and formation of a concentration of cosmic particles around the Earth and the other planets of the solar system exists.

In the vicinity of the neutral point, where the velocity of colliding particles is not greater than 6 km/s, it is found that if the solid particles after collision must remain in a solid state, there can be no possibility of accretion for Mercury, Mars, and the Moon, where the maximum value of the "closing-in parameter" p (distance of the center of the planet to the asymptotic trajectory) is less than the radius of the planet.

On the other hand, the capture radii of microparticles in solid form varies from a minimum of 2.95 planetary radii for Venus and 3.47 for the Earth, to about 986 for Jupiter.

IN THIS PAPER, we shall apply Lyttleton's theory on the formation of comets to explain a possible mechanism concerning the origin and formation of a concentration of cosmic microparticles around the Earth and the other planets of our solar system.

This process of formation and concentration of particles can be conceived as bringing about an increment of particles along an axis, i.e., along a certain direction, deriving from, say, a meteor swarm of cometary origin, under the action of the most important force that is believed to act upon these particles—the planetary gravitational force.

MECHANISM FOR THE POSSIBLE INCREASE OF PARTICLES

Suppose that a cloud of meteor particles encounters a planet from any arbitrary direction and that, in the limited time in which this phenomenon occurs, the planet's trajectory can be

considered a straight line. The single particles forming the cloud, on the other hand, can all be considered to have the same (vector) velocity \mathbf{V} .

Then we take the x axis through the center of the planet parallel to the direction from which the single particles of the swarm are approaching the planet from outside the gravitational field of influence of that planet.

We now follow the movement of a single meteor particle. We can assume that, starting from the surface of the planetary gravitational field of attraction, which we consider to be finite, the trajectory of this particle with respect to the center of the planet is hyperbolic.

The basic consequence, then, is that all the hyperbolic trajectories described by the single meteor particles belonging to the cloud must intersect the x axis at points on that part on the opposite side of the planet.

The action of the planet is evidently that of a convergent lens: Particles that form a meteor

cloud with an initial density are obliged to converge toward the x axis.

This process is similar to that indicated by Lyttleton (1953) for the formation of comets under the gravitational attraction of the Sun.

Let us now see what happens to all particles found on the surface of a hyperboloid. In this case, it is sufficient to consider a cross section of the hyperboloid with a plane passing through the x axis. We then obtain two hyperbolic trajectories of two particles of the meteor cloud, which are symmetrical with respect to the x axis, contained in a plane passing through the center T of the planet (fig. 1). It is immediately evident that two such particles are on "collision trajectories." The collision point C is situated on the x axis at a distance $l/2$ from the center T of the planet.

Before dealing with this subject mathematically, we shall give a brief description of the physics of the impact phenomena. Just before impact at point C , the two particles A and B have the same radial velocity V (along the x axis); the transverse components of V of both particles previously in A and B , perpendicular to the x axis, have the same magnitude but opposite directions. These two components at point C make possible a relative collision velocity of both particles.

By this process, the action of planetary gravity comes into play and leads to a change of the relative velocity of particle A with respect to the velocity of B . The relative velocity goes from zero at points A and B , nearly on the surface of the planetary sphere of attraction, to finite values. These relative values assume a certain magnitude at point C and increase toward the planet; they decrease rapidly immediately outward from point C , since the action of the planetary sphere of attraction is practically nil for moving particles arriving at a distance greater than the radius of this sphere.

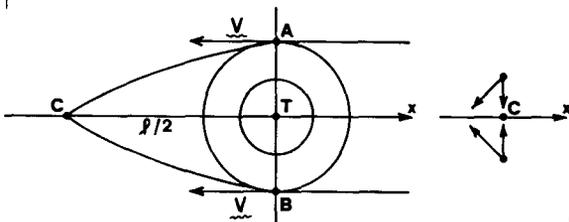


FIGURE 1.—Dynamics of the impact phenomena of a planet with center at T .

If the masses of two opposite particles are equal and if the collision of the particles is inelastic, the total relative kinetic energy would be lost. The velocity of a particle thus formed after the collision would obviously be equal to the radial component, whose direction is along the negative part of the x axis.

On the other hand, the velocity component normal to the x axis before the collision takes place is not negligible. Therefore, such a collision, by eliminating transverse motion, is capable of reducing the energy of the cosmic particle, originally in a hyperbolic orbit, to values of energy that may correspond to an elliptic orbit. Cosmic particles can thus be captured by the planet after collision.

We therefore have available in the solar system a mechanism that can possibly change hyperbolic velocities of particles to elliptic velocities and bring about the capture of these particles by a planet.

The above qualitative description of the capture mechanism is also valid even when the two particles that collide have neither the same velocity nor the same mass.

The essential part of the concept is to admit that this process exists, which would permit dust particles to be accumulated along a certain portion of the x axis.

At the beginning of the process, the presence of meteoroid material along the x axis greatly enhances the probability of repetition of particle collisions that follow the first, in the initial phase.

It should be noted that not all particles are captured by the planet. Those particles that are located farther away from the center, before and after collision, may have a radial component of velocity that is greater than the escape velocity. In this case, the particles can never be captured by the planet.

On the other hand, particles that cross the x axis nearer the planet can, after collision, have a radial velocity that is less than the escape velocity and, therefore, can be captured.

We can thus admit that along the x axis there is a portion characterized by the latter. This part can be called the "concentration segment," whose farther end from the planet defines the so-called "neutral point," introduced by Lyttleton for

comets. This point represents the separation point; i.e., for distances from the center of the planet greater than that determined by the neutral point, all particles that arrive along the x axis will continue with velocities greater than the appropriate escape velocity, while for distances less than that of the neutral point, all particles lying on the concentration segment will definitely be captured by the planet.

THEORETICAL DETERMINATION OF THE NEUTRAL POINT

The position of the neutral point along the x axis can be determined by the following considerations, due mainly to Lyttleton.

With reference to figure 1, let \mathbf{V} represent the (vector) velocity of one of the many particles of a cloud. This vector is parallel to the x axis; the trajectory of the particle is therefore parallel to the x axis and is at a distance p from the x axis.

Owing to the gravitational attraction of the planet, all particles with velocity \mathbf{V} describe hyperbolic trajectories until point C , on the x axis, is reached.

In polar coordinates r, θ , with the center of the coordinate system coincident with the center of the planet, the equation of the trajectory, which is hyperbolic, is given by

$$r = \frac{\ell}{e \cos \theta + 1} \tag{1}$$

where $e = c/a = \sqrt{a^2 + b^2}/a$ is the eccentricity ($e > 1$), $\ell/2$ is the distance of point C from the center T of the planet (origin) along the transverse axis, and ib is the semi-minor axis (imaginary for a hyperbola). The distance CT , which corresponds to $e \cos \theta = 1$, is related to the parameter ℓ by

$$2CT = \ell \tag{2}$$

By applying the area integral to the motion of the particle, which is subject only to a central force, we have

$$r^2 \dot{\theta} = \text{constant} = h \tag{3}$$

Therefore,

$$r^2 d\theta = h dt \tag{4}$$

where $h = \sqrt{\mu l} = V_\infty p$ (moment of momentum per unit mass). The quantity $\mu = GM$, with G the gravitational constant and M the mass of the

planet; V_∞ is the velocity of the particle at infinity, i.e., before it reaches the sphere of attraction of the planet. We then have

$$\frac{1}{2} r^2 d\theta = \frac{1}{2} p ds \tag{5}$$

where ds is the differential of the arc of the hyperbolic trajectory of the particle. Therefore,

$$\frac{1}{2} p ds = \frac{1}{2} h dt \tag{6}$$

and

$$p \frac{ds}{dt} = p V_\infty = h \tag{7}$$

From equation (7), we have

$$V_\infty = \frac{h}{p} \tag{8}$$

On the other hand, it can be shown that the radial component of velocity dr/dt at point C on the x axis is equal to the velocity at infinity V_∞ . Furthermore, we know that the velocity of a particle on a parabolic trajectory around the planet is given by

$$V_p^2 = \frac{2GM}{r} \tag{9}$$

so that, if after collision of two conjugate particles A and B , $V \leq V_p$ at $r = \ell/2$, then the two particles involved in the collision will be captured by the planet. This relationship leads to

$$V_\infty^2 \leq \frac{2GM}{p} \tag{10}$$

i.e.,

$$p \leq \frac{2GM}{V_\infty^2} \tag{11}$$

The parameter p , therefore, depends on the mass M of the planet and the velocity V_∞ of the particle outside the sphere of influence of the planet itself. It is obvious that p must be greater than the radius R of the planet, so that

$$R \leq p \leq \frac{2GM}{V_\infty^2} \tag{12}$$

An analysis carried out by Lyttleton shows that particles can remain in solid form if the kinetic energy converted into heat by the collision does not cause complete evaporation of the colliding particles.

TABLE 1.—Maximum Values of p for the Planets and the Moon

Planet	M ($g \times 10^{27}$)	R ($cm \times 10^6$)	p ($cm \times 10^6$)	Accretion
Mercury	0.328	2420	121.52	no
Venus	4.871	6100	1804.97	yes
Earth	5.977	6378	2214.72	yes
Mars	0.645	3380	239	no
Jupiter	1899.5	71350	703870	yes
Saturn	569.01	60400	210849.5	yes
Uranus	86.666	23800	32114	yes
Neptune	202.804	2200	75150	yes
Pluto	4.786	3000	1773.47	yes
Moon	0.074	1738	27.421	no

In the vicinity of the neutral point, V_∞ must not be greater than about 6 km/s. By taking this approximate value for V_∞ , we can roughly determine the radius p of capture of the planets, provided that equation (12) is valid. We adopt cgs units so that $G = 6.67 \times 10^{-8}$ and $V_\infty = 10^5 V$, where V is the radial velocity at point C expressed in kilometers per second. The values of p and other important parameters are listed in table 1.

CONCLUSIONS

The planets for which accretion of solid particles is possible are those indicated in the last column of table 1. The capture radii for these planets are listed in table 2.

As the value of p must be greater than the

TABLE 2.—Capture Radii of Microparticles in Solid Form

Planet	Radii (\times radius of the planet)
Venus	2.95
Earth	3.47
Jupiter	985.8
Saturn	34.9
Uranus	13.5
Neptune	341.1
Pluto	5.9

radius R of the related planet, we reach the very important conclusion that not all the planets of the solar system can accrete dust by collision processes that conserve dust in a solid state.

REFERENCE

- LYTLETON, R. A., 1953. *The Comets and Their Origin*, Cambridge Univ. Press.