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#### Abstract

Consider the possible astrometric use of the following concept: If the image of a field of stars is moved past a high-speed detector array at a precisely known velocity, relative positions can be determined by measuring time intervals. Attractive features of this approach are: times can be measured very accurately with photoelectric detectors; a relatively large area of the sky may be swept for reference stars; the images are measured essentially on-axis; and seeing effects can be partially frozen out. The only available detector with spatial and temporal resolution and stability adequate for this technique is the MAMA. A rudimentary observational test with this detector has yielded promising results. Further observations are planned to determine whether milliarcsecond accuracy can be obtained.


## I. THE BASIC IDEA

Imagine the following: a telescope with a narrow slit in the focal plane; a fast photoelectric detector recording, with high time resolution, the amount of light entering the slit; and a mechanism for slewing the telescope at a precisely known rate. A sharp pulse in the detector output would signal the passage of each star through the slit. The angular separation of two stars, projected in the direction perpendicular to the slit, is the time interval between their transits across the slit, multiplied by the known angular rate at which the image is moving. Stellar positions can thus be measured in terms of time measurements alone; no angles or spatial displacements need be measured.

This concept, of course, is the basis of the time-honored meridian transit telescope (e.g. Høg, 1974). Implementation of the concept in exactly the form stated above is difficult. But the following two technological developments help: (1) Photoelectric detectors, coupled with high-speed digital data recording techniques, can time the arrival
H. K. Eichhorn and R.J. Leacock (eds.), Astrometric Techniques, 353-368.
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of individual photons with great accuracy. (2) Detector arrays can be used--in place of a slit/detector arrangement--to largely eliminate the enormous inefficiency resulting from the very short time a star spends in the narrow slit.

Both of these developments are implemented in the Multi-Anode Microchannel Array (Timothy, Joseph and Wolff, 1982). In this device, each detected photon is assigned a precise $X$ and $Y$ coordinate (resolution $\sim 25 \mu \mathrm{~m}$ ) and a precise arrival time (resolution $\sim 10 \mu-\mathrm{sec}$ ). Each pixel, then, acts as a slit in the above sense.

In the present scheme the rigid metric against which star positions are measured is provided by a combination of the controlled motion of the image and accurate timing of the detected photons. In principle, time-of-arrival measurements entirely replace the more difficult spatial measurements.

However, this goal is not completely attainable. For unless the target and reference star happen to lie along a straight line parallel to the scan direction, different stars will traverse entirely different pixels and simple timing is not enough: the inter-pixel spacing must be precisely known, at least as projected along the scan direction. It is anticipated that variations in the pixel geometry can be mapped with sufficient accuracy that they will not be a real limitation on the method.

The technique proposed here is one of several based on timing of events. Frank Drake's comment that time is easier to measure than position has inspired a number of workers (Drake, 1961; see also comments by Drake and by G. Gatewood in the Minutes of the 1975 "First Workshop on Planetary Detection").
II. THE IMAGE MOTION; FIELD SIZE

The displacement of the star field must be accurately known as a function of time, either (a) because the motion is very uniform with an accurately known velocity, or (b) because the motion is accurately monitored.
(a) General Considerations

Many ways of moving the image are possible. A simple scheme is to introduce a rotating mirror into the optical train. There are a number of difficulties with this idea, mostly connected with maintaining good image focus across the locus of motion. A better technique is to move the entire telescope in such a way as to sweep the star field of interest across the detector.

A big advantage of this approach is that all stars are measured close to the optical axis of the telescope, where the images are best.

Hence the nonlinearity of the mapping from the sky to the image plane, and distortions of the star images, are minimized.

In particular, the instantaneous field-of-view is set by the relatively small size of existing detectors, typically 25 mm . If the image scale is $101 / \mathrm{mm}$, the field diameter is about ' '. Hence large $^{\prime}$ later high-quality fields-of-view are not needed. There is then great freedom in the choice of telescope design. This freedom may be important in space applications, where weight, size, cost, and gravity gradients are important considerations which grow rapidly with focal length.

On the other hand, a major objection to a narrow field-of-view is the small number of reference stars available for a given target star. [E.g., Monet (1981) cites the small detector size as an important limit on the Kitt Peak CCD astrometry program.] This problem has a natural solution in the moving image method, since the effective field-of-view is not the area of the detector, but the amount of sky swept past the detector. This is a rectangular area which can be very long.
(b) Using the Earth as a Clock

For the preliminary tests reported here, the simplest possible source of image motion was employed: turning off the telescope drive, thereby using the earth's rotation to establish the image velocity (shades of the meridian transit!) The siderial rate is perhaps a factor of 4 less than the optimum rate for seeing compensation (see Section III), but it is near enough that the basic concept can be tested in this way.

Refraction fluctuations and environmentally induced telescope motion (e.g., due to wind) will produce departures, on various time scales, from ideally uniform motion. Any of such effects that are not small will have to be averaged out or otherwise corrected for.

This scanning is in right ascension only. Although some declination information is available, because the detector has good resolution in this direction too, a technique for moving the telescope in two coordinates will eventually be necessary. The major technical problem is moving the telescope with precision but without image degradation. One could monitor the telescope position with a laser interferometer, using a light path through the telescope itself. Another idea is to use the detected positions of the reference stars themselves in a kind of self-calibration of the motion.

## III. SEEING COMPENSATION?

Sufficiently rapid scanning "freezes" the seeing fluctuations in the sense that target and reference stars are compared at times sufficiently close that the atmospheric displacements of the images do not change significantly.

Gatewood et al. (1980) found that the degree of correlation between the fluctuations in the apparent positions of two stars separated by angle $\theta$ falls off gradually from $100 \%$ at $\theta=0$ to about $50 \%$ at $\theta=1000^{\prime \prime}$, leveling off by that time to an essentially constant value. Hence rapid scanning should allow partial seeing compensation, even when the images are separated by distances on the order of a degree or more.

But how fast should the images be scanned? Unfortunately, there are very limited data on the time scales for seeing effects as a function of angular separation. Young (1974) suggests that there is a minimum fluctuation time scale associated with a given angular scale that is proportional to the angular separation:

$$
\begin{equation*}
t(\sec )=\theta(\text { arc minutes }) ; \tag{1}
\end{equation*}
$$

stars separated by angle $\theta$ need not be compared closer together than time $t$, because there are no coherent (and therefore cancellable) fluctuations on shorter time scales. This directly implies that the sweep rate need not be faster than about 1 degree per minute.

Note that the siderial rate used in the present tests is only four times slower than this rate. The possibility that faster image motion will improve the measurements will be investgated.

## IV. DETECTOR CHARACTERISTICS

The above scheme clearly imposes stiff requirements on the detector. The multi-anode microchannel array (Timothy, Joseph and Wolff, 1982; Timothy, 1982) appears to be an ideal detector for this application--indeed, the only one!

Briefly, the "MAMA" is a photoelectric, photon counting array detector, the basic parts of which are shown in Figure 1. Spatial resolution is provided by a very fine gridwork of many separate electrodes deposited on a ceramic header in proximity focus with the output face of a microchannel plate. Figure 2 is a magnified photograph of this gridwork. The MAMA used in the observation reported below has a $1024 \times 256$ grid of $25 \mu \mathrm{~m}$ anode strips. Each detected photon is ingeniously assigned to one, and only one, of the resulting 262144 square pixels.

In a recent version of the MAMA (Butcher, Joseph and Timothy 1982) each photon's arrival time is individually recorded to the nearest 10 $\mu-s e c$. The remainder of this section outlines how such "time-tagging" and other features of the MAMA are appropriate to moving-image astrometry.
(a) Timing precision

The allowed timing error is simply the desired angular error (say 1 milliarcsecond $=1$ mas), divided by the scan rate [60"/sec from eq.(1)], or about $16 \mu-\mathrm{sec}$. This means that "video rates" are not fast enough, and that CCD's can probably be ruled out because read-out noise dictates relatively slow read-out rates. On the other hand, the $10 \mu-\mathrm{sec}$ timing resolution of the MAMA leaves a handy margin in case, e.g., faster scan rates are desired for seeing compensation. Improvement of the MAMA's resolution to $0.1 \mu-\mathrm{sec}$ would require only minor modifications.

## (b) Spatial Resolution

The basic constraint on detector pixel size is that it be on the same order as the effective star image size, or smaller. With a typical image scale of $10 \mathrm{k} / \mathrm{mm}$, and a minimum image size of 0.5 l , we conclude that the pixel size should be .05 mm or so. The pixels of the existing coincidence-anode MAMA's are typically .025 mm square, again providing a comfortable margin.

## (c) Stability

Since the detector geometry inevitably influences the angular metric (Section I), it must be very stable. The desired angular precision of .001" , at an image scale of $10 \mathrm{"} / \mathrm{mm}$, implies a linear precision of $0.1 \mu \mathrm{~m}$. Since the absolute spatial accuracy of the MAMA detector is $\pm 2 \mu \mathrm{~m}$ (Timothy et al. 1982), the required relative stability over short times is almost certainly attainable.

The importance of photometric stability (both from pixel to pixel, which is relatively poor in the MAMA, and over time) for these measurments is not known.

## V. INTEGRATION TIMES AND MAGNITUDE LIMITS

As with any astrometric scheme, a fundamental precision limit is set by the number of photons received from the stars. A simple argument shows that the number of photons that must be collected is equal to the square of the ratio of the effective angular size of the stellar image to the desired astrometric precision.

The relevant image size is that given by the seeing disk, or perhaps a bit smaller if seeing compensation is effective. (Unfortunately the photon rate per speckle is too small to allow use of the small angular size of the speckles.) Taking $0.5^{\prime \prime}$ as a best-case number, $2.5 \times 10^{5}$ photons must be collected to achieve . 001" precision.

We next estimate typical counting rates for the comparison stars. (Assume that the target star is at least as bright as the reference
stars. If this is not true, the integration times will be accordingly longer.) For definiteness, we use a typical field of view of $5^{\prime}$ by 1.25'. At galactic latitude zero (where the star density is the highest, and where there are many interesting target stars) there is on the average one star of V-magnitude 13.1 in such an area. For a star of this brightness, with a 1 square meter aperture, a bandpass of 3000 Angstroms at 5500 Angstroms, and an overall system efficiency of $10 \%$, the counting rate is $1.7 \times 10^{4}$ photons $/ \mathrm{sec}$.

It follows that 15 seconds are required to attain the desired precision. Since the passage of each star across the detector takes about 6 seconds, several passages will be necessary. How long this will require in total observing time depends on how long the scans are. A scan of 1 degree, about the largest that will be necessary, would take one minute of time. Including lost time between scans, it will thus take approximately 5 minutes to obtain a precision of .001" for a target star brighter than 13 and lying near the galactic plane.

On the average 10 reference stars would appear in these scans. This should be more than adequate in view of the fact that only the central few arc minutes of the field are used, so that very few "plate constants" need be determined.

One of the practical problems will be the management of the enormous data stream that these photon rates correspond to. In the MAMA data system in the "time-tag" mode, each detected photon produces 48 -bit bytes of data, so that the above-quoted rate of some 20,000 photons per second corresponds to 0.1 megabyte/sec. Since several stars will of ten observed simultaneously, the total data rate may approach 1 megabyte per second.

In summary: with modest observing times, say one hour, a precision of 0.3 mas is theoretically attainable for 13 -th magnitude stars.

## VI. DATA ANALYSIS

Many fascinating possibilities for new analysis techniques are opened up by the availability of precise time and space information for each photon. A few ideas of this kind will be included here. It is clear that much more work needs to be done if the full information content of the data is to be extracted.

The first step is to read and decode the data tape. Decoding means translating the compacted format, in which the data are originally written, into a stream of three numbers for each photon: $X$-coordinate, $Y$-coordinate, and time. $X$ and $Y$ are integers, in the ranges $(0,1023)$ and $(0,255)$, respectively; time is denoted by an integer, with a very large range, giving the arrival time in units of 10 $\mu$-sec.

## MULTI-ANODE MICROCHANNEL ARRAY <br> four-fold coincidence-anode array



Fig. 1: Schematic of a two-dimensional Multi-Anode Microchannel Array. The inset shows how electrons are ejected from the cathode, accelerated in the curved channels, and collected by the anode grid.

The next task is to make sense out of this vast stream of events. An image display system is very convenient for this step. A picture is constructed, for a given short time interval, by accumulating a coordinate histogram of all the photons arriving during the interval. Such pictures (hereafter called frames) are constructed for a sequence of time intervals spanning the time of each scan. (A scan means one transit of the field-of-view across the detector.) The resulting series of frames is like a motion picture, and is useful for identifying the sequence of moving star images. Figure 3 shows a single scan, consisting of 9 frames, obtained in the test observations described in the next section.

Next, each scan can be transformed into a single picture by undoing the effect of the motion. To do this, transform the spatial coordinates of each event with these formulas:

$$
\begin{align*}
& X_{i}^{\prime}=X_{i}-R\left(t_{i}-T_{o}\right),  \tag{2a}\\
& Y_{i}^{\prime}=Y_{i}, \tag{2b}
\end{align*}
$$

where $R$ is the scan rate (pixels per unit time), $t_{i}$ is the arrival time of photon $i$, and $T$ is an arbitrary time origin. We here assume perfect alignment, so that only $X$ is affected; the more general case would include motion in both $X$ and $Y$, to allow for possible misalignment of the axes of the detector.

These transformed coordinates are those which would have obtained for the given event at time $T$ if the motion were not present. A two-dimensional histogram of the transformed coordinates thus constitutes a picture with the effects of the image motion removed-and with a field-of-view larger than the detector area. The photons for the individual stars are co-added, irrespective of arrival time, improving the signal-to-noise over the "snapshot" frames. Figure 4 shows such a picture, obtained from the data of Figure 3. Note the similarity of this procedure to tracking and guiding of the telescope.

The scan rate can either be calculated from the known telescope velocity and image scale, or can be determined from the data itself. At least with these data, experimentation has shown that the latter is more satisfactory, and that the most sensitive way to fix the rate is to adjust it to maximize the roundness of the deduced star images.

The most fundamental task is the determination of the centroid of a star image. This is a large problem (see, e.g., King, 1983), and much work needs to be done. The following procedure, although clearly rudimentary, appears to be adequate for the present purposes: First, a rough guess is made for the star position. Second, all photons which (in transformed coordinates, of course) lie within a prescribed radius of this guess are assigned to that star. Third, the mean values of the coordinates of these photons are computed, defining a new centroid position. This procedure is repeated until the centroids have converged to some desired tolerance.


Fig. 2: Microphotograph showing a corner of the MAMA anode array. The pixels are approximately $25 \mu$ square. Each detected photon is assigned to the correct pixel by means of special decoding circuitry (not shown), which interprets signals transmitted by the leads shown extending from the edges of the array.

The convergence of this iteration is usually rapid. The final centroid position is somewhat dependent on the initially guessed position, on the prescribed radius, and on the convergence tolerance. Work is in progress to find an improved algorithm for centroid determination, taking into account a slowly varying background.

I have carried out a few trials of algorithms which implement the seeing compensation idea. The basic approach is to do the coordinate subtraction which yields the relative star positions on a photon-by-photon basis, preferentially weighting photon pairs close together in time. The point is that, if the random error in $X$ is correlated between the two star images, then $\left\langle X_{i}-X_{j}\right\rangle$ has a smaller variance than does $\left\langle X_{i}\right\rangle-\left\langle X_{j}\right\rangle$.

A straightforward application of this idea to the present data did not yield significant improvement in the internal consistency of the relative positions. Better methods for seeing compensation will be developed and tested.

Other questions to be investigated include: What is the optimum relation between the pixel size and the star image size? How precisely can the pixel locations be mapped, to allow correction for departures from an ideal evenly spaced orthogonal grid? How uniform and stable is the pixel geometry? What is the actual accuracy of the recorded times, as opposed to the nominal time resolution? What advantages accrue if the stars are made to transit the same part of the detector each time they are measured (i.e., over the months and years that measurements will be made)? Should the seeing compensation be carried out differently for pairs of stars that are on the detector simultaneously than for those which are not?

## VII. TEST OBSERVATIONS

Gethyn Timothy and the author are collaborating on test observations, using the time-tag configuration of his $256 \times 1024$ MAMA detector.

The data obtained so far are from an engineering run with the speckle imager/spectrometer (Butcher, Joseph and Timothy, 1982) at the Kitt Peak National Observatory 4-meter Mayall Telescope, on November 2, 1982. The observations consisted of a series of 13 drift scans in which the 4 stars of the Orion "Trapezium" were allowed to drift across the detector by stopping the telescope drive. Also recorded were the return scans, for which the image velocity was determined by the telescope slew rate.

Three factors limited the usefulness of the data: (a) The seeing was unusually poor (image diameters 6.4" [ $\pm 20$ ]) ; (b) Several rows of


Fig. 3: "Snapshot" frames constructed f'rom a scan of the Orion Trapezium stars, using a rectangular 256 X 1024 MAMA detector. Each frame displays all photons detected within a subinterval of 0.143 second. Note the progress of three stars downward in the figure: In frame 1 a single bright star is on the upper edge; in successive frames this star moves downward and exits by frame 6. A second star enters near the right-hand end starting at frame 4, and exits at frame 7. A third star, located horizontally between the first two, enters at frame 6 and exits on the last frame.


Fig. 4: A Picture constructed from the data in Fig. 3, with the image motion removed using Eq. (2a). Each dot represents one photon in a $4 \times 4$ pixel block (only a few blocks had more than one photon). In the original picture, color was used to indicate the photon arrival times.
pixels were essentially dead; (c) Reduction of the detector gain was necessary for safety reasons, because of the extreme brightness of the Trapezium stars; the resulting photon rates were quite low. Nevertheless, useful results have been obtained with these data.

The procedures given in Section VI for reading, decoding, displaying, and reducing the data were followed for each of the 13 drift scans and 12 return scans. The internal consistency of the method was tested by intercomparing the results for the different scans. Figure 5 shows the standard deviations of the star separations, averaged over the three star-pairs considered, plotted against the number of photons. The solid line shows, for reference, the relationship for ideal normal statistics:

$$
\begin{equation*}
0=\sigma_{0} / \sqrt{ } \mathrm{N}, \tag{3}
\end{equation*}
$$

where o gives the image size, $\sigma$ is the uncertainty in the measurement, and $N$ is the number of photons. More precisely, since the distance between two normally distributed star profiles is being determined, we have that:

$$
\begin{equation*}
0=\sqrt{ }\left\{0_{1}^{2} / N_{1}+\sigma_{2}^{2} / N_{2}\right\} \tag{4}
\end{equation*}
$$

where the two stars correspond to the indices $1,2$.
Equations (3) and (4) are actually not limited to Gaussian image shapes, but hold for any shape as long as the second moment of the distribution is finite (Papoulis 1965, Section 8-3). It is interesting to note that the actual distribution of light in the star image probably has a divergent second moment [basically due to long tails of scattered light (King, 1971; Brown, 1982)]. Monte Carlo experiments (Bill Burke, private communication) indicate that for such distributions the centroid uncertainty may be independent of $N$ ! This important matter is under study.

In the figure the variances for the drift and return scans are plotted separately, in order to check whether the telescope slew rate is irregular enough to show up as an increased variance. The evidence from these data indicates that the slew rate is uniform-to the level of accuracy reached here.

The variances for declination and right ascension are also plotted separately, to check whether the image motion (in R.A. only) is actually providing increased accuracy. That the R.A. variances are smaller than those in declination indicates that there may indeed be an increased accuracy in the scan direction, in accord with the concepts advanced in Section I.

A second accuracy test, also shown in Figure 5, compares the coordinate displacements determined from the current measurements, averaged over the scans, with existing astrometric data. These points

## MOVING IMAGE ASTROMETRY

## ERRORS vs NUMBER OF PHOTONS



Fig. 5: Plot of accuracy (in seconds of arc) against number of detected photons. The theoretical limit, set by photon statistics alone, is shown as a straight line. The points labeled "individual scans" are the variances over the various scans; the other points are derived by comparing averages over all the scans with existing astrometric data. Circles are drift scans (siderial rate) and squares are return scans (slewing rate). That the filled symbols (right ascension) indicate somewhat better results than the open symbols (declination...no use of timing) may be evidence that the timing concept is actually providing improved accuracy.
(shown at logN just above 3) are based on data privately communicated by George Gatewood, Ron Probst, and Burt Jones. The result of this comparison is that the method appears good to the level of $\sim 0.1$ ", but this is far short of the goal of 1 mas.

## VIII. CONCLUSIONS AND NEXT STEPS

This report is the first step in the evaluation of a new approach to astrometry, which may circumvent many of the problems of conventional astrometry, and is theoretically capable of reaching milliarcsecond accuracy on selected stars. In a simple test the method performed nearly ideally, to a level roughly half-way (in the logarithm) from the seeing image size to the target accuracy of 1 mas.

If a planned series of test observations verifies the feasibility of the technique, further steps will include: consideration of ways to move the image at rates of $10 / \mathrm{min}$, or faster, in both coordinates, and investigation of how best to carry out "production run" observations. Both adaptation to existing telescopes and the construction of a dedicated "rocking primary" reflecting telescope will be considered.

## ACKNOWLEDGEMENTS

Many people have provided important assistance in this work. I am grateful to Dave Black, George Gatewood, Burt Jones, and Ron Probst for suggestions and comments on various aspects of this project. Hans Mark and Fred Scarf provided advice concerning detectors. I am especially grateful to Gethyn Timothy for obtaining the test data, and to the many people involved in transmittal and other aspects of the data, including Harvey Butcher (KPNO), Steve Colley and Richard Bybee (Ball Brothers), Gary Villere, Glen Boozer, Don Wiedman, and Robert Mecklenburg (Ames). Paul Swan developed the image display system used in this work.

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Discussion:
Is the MAMA refrigerated?
SCARGLE:
No, but there is some improvement if it is cooled a little. The pixels are 25 microns square.

