7. COMMISSION DE LA MECANIQUE CELESTE

PRÉSIDENT: D. Brouwer.

MEMBRES: MM. Armellini†, Belorizky, Bucerius†, Chebotarev, Clemence, Cox, Cunningham, Davis, Duboshin, Eckert, Fabre, Hagihara, Hamid, Heinrich, Herget, Herrick, Jeffreys, Lemaître, Milankovitch, Moiseiev†, Plavec, Sadler, Sconzo, Slavenas, Stumpff, Subbotin, van Woerkom, Vernič, von Zeipel, Wilkens, Zagar.

With regret the deaths are noted of three members of the Commission, G. Armellini of Rome, H. Bucerius of Munich and N. D. Moiseiev of the Sternberg Astronomical Institute, Moscow.

MATTERS FOR DISCUSSION

Reports received from members of the Commission contain a number of items recommended for discussion at the forthcoming meeting of the Commission.

D. Belorizky remarks that, since celestial mechanics provides ephemerides for celestial bodies, 'Je trouve que la séparation en commission 7 et 4 est artificielle; les membres de la commission 7 doivent pouvoir faire parti, s'ils le désirent, de la commission 4'.

H. Fabre writes that, when the International Astronomical Union was created in 1922, Commission 1 on Relativity was formed. This commission was discontinued by the action of the Executive Committee [1]. Fabre proposes that a satisfactory solution would be to enlarge the domain of Commission 7, which could have the name 'Celestial Mechanics and Relativity'.

Applications of high-speed computers

In the past, accomplishments in celestial mechanics were as a rule the results of efforts of scientists working alone or with assistants in a single institution. The spectacular recent advances in the design and construction of high-speed computers have raised questions concerning the assistance that centres with high-speed computing facilities may render to scientists working in celestial mechanics who have no such facilities at their disposal.

S. Hamid inquires whether financial aid might be made available by the I.A.U. to enable astronomers without high-speed computing facilities to get research done in a computing centre.

As an example of a useful undertaking he mentions the work by G. W. Hill on Gauss' method of computing secular perturbations [2]. The object would be retabulation in natural values instead of logarithms, and extension to cover a wider range.

Hamid further remarks that established computing centres possess much information on punched cards that would be useful to research workers in celestial mechanics elsewhere. He suggests that the Commission study what might be done to make such data more generally available.

D. Belorizky remarks that theoretical considerations show that it is advantageous to use in the numerical integration of the Moon's orbit in rectangular co-ordinates the independent variable u, related to t by $dt = cr\,du$, c being a constant that may be chosen at will. He suggests that at least the beginning of a numerical integration in this form be attempted with an electronic computer.

Problem of three bodies

A treatise by C. L. Siegel [3] and the publication *Stability in Celestial Mechanics* by Y. Hagihara [4], the latter in commemoration of Hagihara's sixtieth birthday, are two recent volumes on the mathematical aspects of celestial mechanics.

Activity in this field at the Institute of Theoretical Astronomy at Leningrad includes investigations by G. A. Merman [5] on hyperbolic motion in the problem of three bodies; by J. W. Batrakov [6] on periodic orbits in the restricted problem and in the general problem of three bodies; by V. A. Brumberg on permanent configurations in the problem of four bodies and their stability.

Z. Hitotuyanagi published Matukuma's posthumous work on periodic orbits [7]; Y. Kozai [8] published a study of the stability of orbits in the neighbourhood of commensurabilities by the method of Hagihara [9].

J. Moser [10], extending investigations by Levi-Civita, discussed the stability in the restricted problem of periodic orbits in the vicinity of commensurabilities. For orbits associated with circular orbits at the commensurability $(p+q)/p$, he finds that for $q>4$ the orbits are 'almost stable'; the departures from periodic orbits proceed very slowly, weaker than any power of the time. For $q=2$ and 3 the orbits are unstable; for $q=1$ and 4 the stability is undecided.

R. Vernić has developed further the methods of Sundman. In papers published and to be published by the Yugoslav Academy of Sciences and Arts he presents a numerical procedure for solving any given example of the problem of several bodies. The author describes the procedure as straightforward, without approximation or iteration, and apparently satisfactorily convergent. He recommends that the Commission stimulate an undertaking of treating by this method the motions of the nine planets and the Moon with a large-scale digital computer in order to obtain a representation of the motions over a long interval of time.

Planetary theory

R. L. Duncombe [11] completed an exhaustive discussion of the observations of Venus, which he compared with Newcomb's tables. The observed motion of the node is in good agreement with the tabular value. This result removes what had heretofore been the outstanding discordance between theory and observation among the secular variations of the orbits of the inner planets. It appears that Newcomb's result had been affected by systematic errors in the older observations. The addition in Duncombe's discussion of extensive new series of observations with high weight accounts for the significant change in the observed value of the motion of the node.

Duncombe's discussion also confirmed the results of discussions of the Sun, Moon and planets by H. R. Morgan [12] and of Mercury by G. M. Clemence [13] that the obliquity of the ecliptic is diminishing more rapidly than Newcomb's tables indicate. The origin of this discordance was traced by Duncombe and Clemence [14] to inadequacies of the theory. They compared a numerical integration of the motion of the Earth-Moon system about the Sun for the years 1920–2000 with Newcomb's tables. The numerical integration had been performed by P. Herget on the Naval Ordnance Research Calculator. Both Newcomb's and Leverrier's expressions for the Sun's latitude disagree with the numerical integration, while the latter is in essential agreement with the observations. The cause appears to be the omission of terms of higher order in the theories of both Leverrier and Newcomb.

Clemence has continued his construction of a new general theory of Mars by Hansen's method, the first-order part of which was published in 1949 [15]. The theory is now essentially complete, but the comparison with a numerical integration of the orbit brought to light that certain second-order perturbations in the latitude must be included in order to attain the desired accuracy for the theory. The calculation of these new portions of the theory is now in progress.

The two investigations at the U.S. Naval Observatory referred to in the preceding paragraphs illustrate the effectiveness of comparisons between general theories and numerical integrations as a test of the accuracy of a general theory.

At the Yale Observatory some further progress was made with the construction of new theories of the planets Uranus, Neptune and Pluto. The calculation of the first-order perturbations is well advanced. Work was halted pending the recent installation of an IBM-650 calculator.

The theory of asteroids of the Trojan Group was the subject of investigations by G. A. Chebotarev [16] and by Y. A. Riabov [17] with applications to (588) Achilles and (884) Priamus, respectively. An investigation by S. Aoki [18] deals with the stability of the motion of a Trojan asteroid.

K. A. Steins [19] published an investigation of Delaunay's method and considered its application to the motions of minor planets of the Hecuba type.

N. B. Yelenevskaya [20] has continued her work on the development of the disturbing function for large mutual inclinations of the orbits.

Progress is reported with calculations by Hill's method of second-order perturbations of Ceres by V. F. Proskurin and with second-order perturbations of Pluto by Sh. G. Sharaf.

Satellite theory

The motions of the inner satellites of Saturn were the subject of a study by Y. Kozai [21]. The values that he finds for the masses of the satellites and the constants for the figure of Saturn are, on the whole, in good agreement with the earlier results by H. Jeffreys [22].

P. J. Message has constructed a new theory of Hyperion, taking into account terms neglected by Woltjer. This work was done at Cambridge, England, and will be published in the near future. The mass of Titan found by Message $1/(4061 \cdot 3 \pm 2 \cdot 3) = (2 \cdot 4622 \pm 0 \cdot 0013)\ 10^{-4}$, but as the residuals in the equations of conditions are rather large the standard error should probably be multiplied by 5 or so.

G. A. Wilkins has in progress an exhaustive discussion of the observations of the satellites of Mars which he hopes to complete within the next few months.

G. van Biesbroeck [23] discussed the observations of Nereid, the second satellite of Neptune, and derived for the mass of Neptune $1/(18889 \pm 62)$.

T. E. Sterne [24] developed a theory of the motion of a body with negligible mass under the gravitational attraction of a spheroid by integrating the Hamilton-Jacobi differential equation for a modified Hamiltonian that permits exact integration. He thus obtains an intermediate orbit that possesses some of the characteristic features of the motion. B. Garfinkel [25] introduced certain changes into the procedure adopted by Sterne.

J. W. Batrakov [26] and V. K. Abalakin [27] deal with periodic orbits and the stability of libration points, respectively, of a particle moving under the gravitational attraction of a uniformly rotating tri-axial ellipsoid.

A paper by V. G. Fesenkov [28] treats properties of the motion about a gravitating centre surrounded by a resisting medium of uniform density, rotating uniformly.

Chebotarev [29] reports on the calculation of a symmetrical trajectory of a rocket around the Moon with return to the Earth and without propulsion during the motion.

Planetary rotations, constants, etc.

A further contribution to the problem of the fluctuations in the Earth's rate of rotation and the secular accelerations of the Moon and the Sun is contained in a paper by N. Sekiguchi [30].

P. J. Message [31] has worked out a second-order theory of the figure of Jupiter and finds that the data agree with a model for Jupiter nearer to Ramsey's J 3 (5% of mass in central particle) than with J 4 (no central particle).

H. Jeffreys and R. O. Vicente [32] have produced a dynamical theory of nutation and the variation of latitude, taking into account elasticity of the Earth's shell and fluidity of the core. The 19-yearly nutation in obliquity comes out in good agreement with observation. The differences from the rigid-body theory for the short-period nutations are smaller than for a rigid shell and fluid core. The free period, in the absence of the ocean, would be about 390 days, but the ocean would probably increase it to about 430 days.

Jeffreys notes that Yakovkin's data for a free libration of the Moon in longitude would be consistent in both period and phase with the forced motion of argument $2(\varpi - \Omega)$. On either interpretation f is very close to 0·67.

Jeffreys has found that the damping of the variation of latitude can be explained by elastic after-working in the Earth's shell on two types of hypothesis, according as the time needed to approximate to the field yield under long-continued stress is of the order

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of 20 or 400 days. The former is found to give satisfactory explanations of the rotations of the Moon and Mercury. The latter is doubtfully adequate for the Moon and definitely inadequate for Mercury. Application to the satellites of Mars shows that bodily tidal friction in Mars cannot account for the secular accelerations.

The properties of the Earth cannot be adopted for the outer planets, but it appears that the only one of their satellites that can have its orbit appreciably modified by tidal friction in the past is Jupiter's first satellite.

DIRK BROUWER
President of the Commission

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