

## Introduction

The structure of a quantum field theory often simplifies when one considers processes involving large momenta or short distances. These simplifications are important in improving one's ability to calculate predictions from the theory, and in essence form the subject of this book.

The first simplification to be considered involves the very existence of the theory. The problem is that there are usually ultra-violet divergences caused by large fluctuations of the field(s) on short distance scales. These manifest themselves in Feynman graphs as divergences when loop momenta go to infinity with the external momenta fixed. The simplification is that the divergences can be cancelled by renormalizations of the parameters of the action. Consequently our first task will be to treat the ultra-violet renormalizations. Renormalization is essential, for otherwise most field theories do not exist.

We will then expose the methods needed to handle high-energy/short-distance problems. The aim is to be able to make testable predictions from a strong interaction theory, or to improve the rate of convergence of the perturbation expansion in a weakly coupled theory. The simplifications generally take the form of a factorization of a cross-section or of an amplitude, each factor containing the dependence of the process on phenomena that happen on one particular distance scale. Such a factorization is useful, because the coefficients of the perturbation expansion for a process are large when the process involves widely different distance scales.

The industry called 'perturbative QCD' consists of deriving such factorization theorems for strong interactions (Mueller (1981)) and exploring their phenomenological consequences. We will only study the earliest of these factorizations, the operator product expansion of Wilson (1969). We will also discuss the theorems that describe the behavior of a theory when the masses of its fields get large (Appelquist & Carazzone (1975) and Witten (1976)). These large-mass theorems have their main uses in weak interaction theories.

The presence of ultra-violet divergences, even though they are cancelled by renormalization counterterms, means that in any process there are

contributions from quantum fluctuations on every distance scale. This is both a complication and an opportunity to find interesting physics. The complication is that the derivation of factorization theorems is made difficult. The opportunity is given by the observable phenomena that directly result from the existence of the divergences. A standard example is given by the scaling violations in deep-inelastic scattering.

It is the renormalization group (Stueckelberg & Petermann (1953) and Gell-Mann & Low (1954)) that is the key technique in disentangling the complications. The infinite parts of the counterterms are determined by the requirement that they cancel the divergences, but the finite parts are not so determined. In fact, the partition of a bare coupling  $g_0$  into the sum of a finite renormalized coupling  $g_R$  and a singular counterterm  $\Delta g$  is arbitrary. One can reparametrize the theory by transferring a finite amount from  $g_R$  to  $\Delta g$  without changing the physics: the theory is renormalization-group invariant.

This trivial-sounding observation is in practice very useful, and far from trivial. Suppose one has some graph whose renormalized value is large (so that it is inadequate to use a few low orders of the perturbation theory to compute the corresponding quantity). Then in appropriate circumstances it is possible to adjust the partition of  $g_0$  (viz.,  $g_0 = g_R + \Delta g$ ) so that the counterterm  $\Delta g$  cancels not only the divergence but also the excessively large piece of the graph's finite part. The large piece is now in the lowest order instead of higher orders. Construction of factorization theorems of the sort reviewed by Mueller (1981) provides many circumstances where this trick is applicable. Without it the factorization theorems would be almost powerless.

We see that the subjects of renormalization, the renormalization group, and the operator product expansion are intimately linked, and we will treat them all in this book. The aim will be to explain the general methods that are applicable not only to the examples we will examine but in many other situations. We will not aim at complete rigor. However there are many pitfalls and traps ready to ensnare an unwary physicist. Thus a precise set of concepts and notations is necessary, for many of the dangers are essentially combinatorial. The appropriate basis is then that of Zimmermann (1969, 1970, 1973a, 1973b).

One other problem is that of choice of an ultra-violet cut off. From a fundamental point of view, the lattice cut-off seems best as it appears in non-perturbative treatments using the functional integral (e.g., Glimm & Jaffe (1981)). In perturbation theory one can arrange to use no regulator whatsoever (e.g., Piguet & Rouet (1981)). In practice, dimensional reg-

ularization has deservedly become very popular. This consists of replacing the physical space-time dimensionality 4 by an arbitrary complex number  $d$ . The main attraction of this method is that virtually no violence is done to the structure of a Feynman graph; a second attraction is that it also regulates infra-red divergences. The disadvantage is that the method has not been formulated outside of perturbation theory (at least not yet). Much of the treatment in this book, especially the examples, will be based on the use of dimensional regularization. However it cannot be emphasized too strongly that none of the fundamental results depend on this choice.