7.4 THE NON-THERMAL CONTINUUM FROM THE CRAB NEBULA: THE 'SYNCHRO-COMPTON' INTERPRETATION

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Abstract. The continuum emission from the Crab Nebula may be radiation from relativistic electrons moving in the electromagnetic wave field radiated by the rotating magnetic dipole of the pulsar. This radiation, called Synchro-Compton radiation, would show ordering over the whole nebula, as is observed in measurements of polarisation. The properties of this radiation are described in an Appendix.

Studies of the 'oblique rotator' model, by Ostriker and Gunn (1969) and others, suggest that the rotational braking of pulsars may be primarily caused by emission of electromagnetic radiation at the rotation frequency. This paper will explore the possibility that such very low frequency (30 Hz) radiation emanating from NP 0532 plays the role conventionally ascribed to a large scale magnetic field, and that the continuum emission from the Crab Nebula is due to relativistic electrons moving in this wave. This suggestion was briefly discussed by Gunn (1970).

The rate of emission by an isotropic distribution of relativistic electrons moving in an electromagnetic field is determined by the electromagnetic energy density. It is therefore convenient to express the intensity L of the 30 Hz wave in terms of the magnetic field H_{eq} for which the wave energy density equals $H_{eq}^2/8\pi$. We find

$$H_{eq} = 1.8 \times 10^{-4} \left(\frac{L\,(30\ \mathrm{Hz})}{5 \times 10^{38}\ \mathrm{erg\ sec^{-1}}} \right)^{1/2} \left(\frac{r}{10^{18}\ \mathrm{cm}} \right)^{-1} q\ \mathrm{G}\,. \quad (1)$$

This assumes that the wave propagates with velocity c, and ignores the fact that dipole emission would be twice as intense along the rotation axis as in the equatorial plane. The factor q (which is $\geqslant 1$) allows for possible reflection back into the nebula. For the Crab we expect the terms in brackets to be ~ 1. Thus, if $q \simeq 1$, the wave energy density is comparable with that of the weakest magnetic field ($\sim 10^{-4}$ G) permitted by energetic and dynamical considerations.

It is also useful to define an 'equivalent gyrofrequency' $\Omega/2\pi \simeq 3 \times 10^6 H_{eq}$ Hz, since the parameter $f = \Omega/\omega$, where ω is the wave frequency, determines the character of the relativistic particle orbits and of the radiation which these particles emit. Throughout the Crab Nebula we would expect $f \gtrsim 10$. In this situation a relativistic electron radiates at frequencies $\sim \gamma^2 \Omega$, as in the case of synchrotron radiation, and *not* $\sim \gamma^2 \omega$ as for inverse Compton emission. Further details of this radiation mechanism, – which we shall call 'synchro-Compton' emission – are given in the *Appendix*, but for the moment it is sufficient to know that the usual synchrotron formulae still, in general terms, apply, so that the standard inferences of the electron density and spectrum in the Nebula remain applicable.

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Self-consistency demands that the plasma density within the Nebula should be low enough to allow the 30 Hz radiation to propagate. At first sight one might suspect that the formal plasma frequency $9 \times 10^3 n_e^{1/2}$ Hz would have to be below 30 Hz, which would lead to the exceedingly stringent condition that the electron density throughout the nebula be $\lesssim 10^{-5}$ cm^{-3}. However in the case of a 'strong' wave $(f > 1)$ this condition can be relaxed somewhat (Ostriker and Gunn, 1969), so that a sufficient condition for the wave to propagate is

$$n_e \lesssim 10^{-5} f \quad \text{cm}^{-3}. \tag{2}$$

(The extra factor f occurs because, in order to reflect the wave, the electrons (moving at speeds $< c$) must be numerous enough to carry the induced current, and the latter is proportional to f. It is also easy to see that all particles exposed to a wave with $f > 1$ must be relativistic with $\gamma \gtrsim f$). The relativistic particles required to produce the observed continuum from the nebula all have $\gamma \gg f$. The propagation condition sets a limit on their density of

$$\int n(\gamma) \frac{\log \gamma}{\gamma} \, d\gamma \lesssim 10^{-5} \text{ cm}^{-3}. \tag{3}$$

(Zheleznyakov 1967), where $n(\gamma)$ is the differential electron spectrum. The main contribution is made by the particles of the lowest γ, but (3) is satisfied, with a factor ~ 10 to spare, by the particles with $\gamma \gtrsim 100$ whose density is directly inferred from observations above a few MHz. Since (2) is obviously not fulfilled by the general interstellar medium, the 30 Hz waves cannot penetrate beyond the boundary of the nebula – indeed the observable nebula would, in this picture, be delineated by the region which has been evacuated sufficiently for the wave to propagate. Also, the waves would not be able to penetrate the filaments in the nebula.

Before considering the synchro-Compton radiation mechanism in more detail, we shall briefly discuss how the general viewpoint suggested here affects current ideas on the acceleration and confinement of relativistic particles. Several other speakers at this symposium have argued that relativistic particles must be ejected continuously from the pulsar (and indeed Ostriker and Gunn have shown that the very strong wave in the vicinity of the speed of light cylinder constitutes an embarrassingly potent accelerator). For our considerations here, it is of course essential that *not all* the rotational energy of the pulsar should go directly into fast particles, but that a substantial fraction (~ 10 per cent at the very least) should escape as 30 Hz radiation. It is interesting to investigate the eventual fate of this wave energy. When the wave reaches the boundary of the nebula, only a fraction $\sim v_{\text{exp}}/c$ of its energy is used in pushing against the external medium, v_{exp} being the expansion velocity of the boundary. The bulk must be either reflected or absorbed. We shall show below that the high polarisation of the continuum from the nebula implies that the low frequency radiation must be ordered rather than random, and this precludes more than ~ 50 per cent reflection (so that, in (1), $q \lesssim 2$). This means that the energy must all be deposited in a thin 'skin' at the boundary. The densities are so low that there is no possibility of this energy being radiated thermally, so there seems no alternative to the view that it generates relativistic particles, probably mainly electrons. Thus the pulsar would be almost 100 per cent efficient in accelerating particles: whatever fraction escapes into the wave zone in the form of 30 Hz emission will produce particles at the boundary, or at the inner edges of filaments.

Even though the wave field simulates a stationary magnetic field as regards the radiation (except, as we shall see, in the important respect of circular polarisation) it is much less efficient for confining particles, since the orbits are basically straight lines. However even a very weak magnetic field, which is negligible as regards the emission mechanism, could confine the particles adequately if it were sufficiently tangled. Alternatively, the particles could be 'mirrored' at the boundary by the external interstellar field, and by the filaments if these contain a magnetic field.

The electromagnetic radiation emitted from an ideal spinning magnetic dipole would, in the equatorial plane, be completely linearly polarised, the electric vector lying perpendicular to the plane. At higher latitudes, the wave would be elliptically polarised, and along the rotation axis the polarisation would be purely circular. As discussed in the *Appendix*, the polarization and propagation direction of the 30 Hz wave would determine the polarisation properties of the synchro-Compton radiation that is actually observed. We cannot, however, compare the model with the polarisation data on the Crab without assuming the orientation of the dipole. In certain pulsar models the existence of an interpulse indicates that the observer is located close to the equatorial plane. Guided by this, let us suppose that the rotation axis of NP 0532 is precisely in the plane of the sky. Then, provided that (3) is satisfied by a large enough margin that the effects of the medium are negligible, the synchro-Compton radiation from the equatorial plane should be linearly polarised parallel to the rotation axis. The direction of polarisation will be similar at other latitudes (even along a line of sight intercepting the rotation axis there will be a linearly polarised contribution). Both the optical and the radio observations show that the direction of polarization is in fact fairly constant over the inner part of the Crab Nebula. It is therefore tempting to take this as supporting evidence for the synchro-Compton model. The observed polarization angle would then imply that the rotation axis is aligned in a NW–SE direction. It will be interesting to compare this with the orientation predicted by various pulsar models. Regions close to the rotation axis would contribute *circularly polarized* radiation (see the Appendix) and we would expect a circular polarization of a few per cent at radio, optical and X-ray wavelengths. However the polarization will have opposite senses in the two halves of the nebula, and so the net circular polarisation from the whole nebula may be very low.

Near the edges of the nebula, we may plausibly expect the evacuation to be less effective, and the effects of the medium more important. In this situation, the 30 Hz wave would suffer refraction, and, if the density increases outward, there will be a tendency for it to be deflected tangentially. Since, as is shown in the appendix, the synchro-Compton emission is polarised at right angles to the propagation direction of the 30 Hz wave when the refractive index is significantly less than 1, one may thus be able to account for the observed polarisation vectors normal to the boundary (especially around the bays). The reduced group velocity when $\mu < 1$ increases the wave energy density, and this tends to enhance the synchro-Compton emission from regions where (2) and (3) are only marginally satisfied.

The gross features of the linear polarisation thus support the synchro-Compton interpretation of the radiation. When one recalls the difficulties of accounting for a large scale ordered magnetic field in the Crab Nebula*, the attractiveness of a theory which removes the need for such a field altogether becomes even greater. Perhaps the most crucial test of the general scheme would be the detection of circular polarisation (of both senses) from regions in the nebula. This would be inexplicable in a standard synchrotron picture. On the other hand, the fact that circular polarisation is less easily smeared out than linear polarisation means that the *absence* of circular polarisation at, say, the one per cent level would pose a severe problem for the attractive and widely-held view that much of the pulsar's energy is radiated at the rotation frequency (or low harmonics thereof). Further high-resolution observations of the continuum will obviously help to test the model further. Also, the details of the process whereby the 30 Hz wave is absorbed at the edge of the nebula deserve greater theoretical study, with a view to determining the likely energy spectrum of the resulting relativistic electrons.

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Appendix

We summarize here some useful results pertaining to the motion of charged particles in low frequency electromagnetic waves and the properties of the resulting radiation. Although detailed derivations will not be given, and some results merely quoted, anyone familiar with the usual theory of synchrotron and inverse Compton emission should be able to confirm them without difficulty.

We consider particles with various energies (characterised by the Lorentz factor γ) in an electromagnetic wave of frequency ω propagating in the direction $\mathbf{k}$. As in the text, we define Ω as the gyrofrequency in a magnetic field with the same energy density as the wave. The nature of the orbits, and of the emitted radiation, depends on the parameter $f = \Omega/\omega$. First we discuss the behaviour of a test particle in a wave propagating through a vacuum, and then (in (II)) the modifications arising from the presence of a 'cold' plasma which causes the refractive index at frequency ω to depart appreciably from unity.

* If, as was suggested first by Piddington (1957), the field were amplified as a result of being tightly wound, the scale of the field reversals would be so low that (a) the associated current densities would be too high to be carried by the available particles; and (b) the emission mechanism would not be standard synchrotron radiation, because of the rapidly-reversing field.

I. TEST PARTICLE

(a) When $f \ll 1$ we have ordinary Compton (or inverse Compton) scattering. A particle released from rest into the wave oscillates *non*-relativistically ($v/c \lesssim f$) in a plane perpendicular to the propagation direction $\mathbf{k}$. In a linearly polarised wave the particle oscillates along a line in the direction of the $\mathbf{E}$ vector; in a circularly polarised wave it executes circular motion. Particles scatter the wave in accordance with the standard Thomson formula. Relativistic particles – or indeed any particle for which $v/c \gg f$ – move basically in straight lines, but the wave induces transverse oscillations with wavelength $2\pi c/\omega\ (1-\mathbf{v}\cdot\mathbf{k}/c)^{-1}$ around the mean path (and in a frame sharing the mean velocity, particles would execute *non*-relativistic transverse oscillations). The scattered radiation due to relativistic particles is beamed in the direction of v, and its spectrum peaks at a frequency $\sim\gamma^2\omega(1-\mathbf{v}\cdot\mathbf{k}/c)$. The spectrum of the scattered radiation cuts off sharply above this frequency, but there is a low-frequency tail $\propto \nu^1$ contributed by photons which, in the moving frame, are scattered almost into the backward direction. This is the standard case of 'inverse' Compton scattering, in which both linear and circular polarisation are largely preserved (Bonometto *et al.*, 1970).

(b) When $f \gtrsim 1$ the wave is strong enough to impart relativistic speeds to a charge released from rest. The $\mathbf{v}\times\mathbf{B}$ term then cannot be neglected, so the motion is not (as in (a) above) restricted to a plane perpendicular to $\mathbf{k}$. A particle would acquire a typical γ of up to $\sim f^2$ (Jory and Trivelpiece, 1958). If the low frequency wave were plane polarised, particles would move in the plane defined by $\mathbf{k}$ and $\mathbf{E}$. The radiation from these particles would typically be at frequencies $\sim\gamma^2\Omega$, analogously to synchrotron radiation. The low frequency tail of the spectrum would be roughly proportional to $\nu^{1/3}$ (again as in the synchrotron case) though the exact spectrum would depend on the particular orbit. Even in a strong electromagnetic wave with $f \gg 1$, particles with $\gamma \gg f$ move in wavy lines, the angular excursions from the direction of the mean v being $\sim f/\gamma$. The peak frequency of the radiation emitted by such a particle, however, will be $\sim\gamma^2\Omega$, and *not* $\sim\gamma^2\omega$ as in the standard inverse Compton case. The reason for the close analogy with synchrotron emission despite the very different character of the orbits is that the radiation is always beamed in a cone of angle $\sim\gamma^{-1}$, and the particle turns through this angle in a distance small compared with $2\pi c/\omega$, consequently 'seeing' a quasi-static field over the relevant period. Another way to understand this result is to transform to a frame moving with the particle's mean velocity. The Lorentz factor of this frame is $\sim\gamma f^{-1}$, and with respect to it the particle moves *relativistically* with Lorentz factor $\sim f$ (in contrast to the situation for $f \ll 1$, when the mean velocity has Lorentz factor γ and the oscillatory component of the motion is non-relativistic). Below the frequency $\sim\gamma^2\Omega$ where the spectrum peaks, the slope is $\frac{1}{3}$ over a frequency range $\sim f^3$, but below $\sim f^{-3}\gamma^2\Omega$ the slope steepens to ~ 1 as for inverse Compton emission.

The polarization of synchro-Compton radiation can be visualised qualitatively if one bears in mind that, when $\gamma \gg 1$, the only relevant electrons are those moving almost directly towards us, and that the **E** and $\mathbf{v} \times \mathbf{B}$ contributions to the transverse acceleration are of comparable importance, As regards *linear* polarisation the results closely resemble those of Bonometto *et al.* (1970) for the standard inverse Compton case. If the low frequency wave is linearly polarised, the synchro-Compton emission will be linearly polarised in the direction of the projected E-vector. (In the frame sharing its mean motion, each particle traces out a figure-of-eight orbit lying in a plane). When the low frequency wave is *circularly* polarised, the non-uniform part of the motion of a particle with $\gamma \gg f$ is relativistic circular motion, with Lorentz factor $\sim f$, in a plane perpendicular to v. This motion would give rise to synchrotron-type radiation concentrated in a 'fan' at angles $\pi/2 \pm (\sim f^{-1})$ to **v**. This radiation would be circularly polarised in opposite senses on the two sides of the plane of the orbit. In the transformation to the rest frame, the factor $(1 - v/c \cos\theta)$ in the Doppler formula favours the emission from the *forward* hemisphere by a factor $\sim(1+f^{-1})/(1-f^{-1})$, which leads to a net circular polarisation $\sim f^{-1}$. Therefore synchro-Compton emission by electrons of *all* energies can possess circular polarisation of order f^{-1}. This contrasts with the γ^{-1} dependence found by Legg and Westfold (1968) for synchrotron radiation, which leads to an undetectably small degree of circular polarization in most astronomical contexts.

II. INFLUENCE OF MEDIUM

The above remarks have all referred to a test particle. However if the density of 'cold' particles (by which is meant merely those less energetic than the one under consideration) is such that (3) is satisfied by less than, say, an order of magnitude, then the refractive index would be significantly less than unity. This situation allows an important new effect: namely that the synchro-Compton radiation can be linearly polarised even when the low frequency wave is itself *un*polarised. To understand this effect, consider the situation when **k** is at right angles to the line of sight. The only electrons which concern us are, of course, those whose velocities **v** are directed almost towards us. The transverse acceleration of these electrons is caused by the components of the E and B field *perpendicular* to the **k**–**v** plane; the E field yields synchro-Compton emission polarised perpendicular to the plane, and the B-field gives radiation polarised *in* the plane. For an unpolarised low frequency wave propagating in a vacuum, these contributions are equal, so the synchro-Compton emission is unpolarised. However the E/B ratio varies as μ^{-1}, when μ is the refractive index. Therefore, when $\mu < 1$, as it is for a plasma, the synchro-Compton emission will be polarised in a direction *perpendicular* to **k**. The possible degree of polarization is $\sim(E^2 - B^2)/(E^2 + B^2) \simeq (1-\mu^2)/(1+\mu^2)$. Thus if (3) is only satisfied by a factor of 10, one might expect ~ 10 per cent linear polarisation perpendicular to the propagation direction of the low-frequency wave.

It is also possible to derive this result by transforming to a frame sharing the mean motion of the particle, as we did in I – one finds that when $\mu \neq 1$ the wave appears polarised in this frame even when it is unpolarised in the rest frame.

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Discussion

P. Stewart: It is not obvious that permittivity has any meaning in intense radiation fields.

M. Rees: I agree that the problem is complicated when the intensity of the wave is high, and it was mainly for that reason that I avoided being too quantitative. However, all that is necessary in order for the 'synchrotron' radiation to be polarised perpendicular to the propagation direction of the 30 Hz radiation is that E/B should exceed its vacuum ratio, and I believe that this will be true even for a *strong* low frequency wave.

J. E. Felten: I did not understand your diagram of the 'bay' – what happens to the polarisation of the radiation scattered from a ray which is travelling outward in a direction normal to the boundary of the bay?

M. Rees: There will indeed be some 30 Hz radiation that is incident precisely normally on the boundary. However, there will be a tendency for most of it to be curved away towards the tangential direction as it moves into a region of increasing density (and increasing refractive index), and a consequent tendency for the observed radiation to be polarised perpendicular to the boundary.

J. A. Roberts: Do you envision the optical line emitting filaments as being outside the volume swept to electron densities $< 10^{-5}\,\gamma\,\mathrm{cm}^{-3}$.

M. Rees: The 30 Hz radiation would be absorbed on the inner edge of filaments, and would not propagate through them. If the absorbed energy goes into relativistic electrons, this may be relevant to the enhanced radio emission from the region of some filaments which was reported by Mr. Wilson on Wednesday.

Note added in proof. In response to the suggestion made in this paper, Landstreet and Angel (*Nature* **230**, 103 (1971)) searched optically for circular polarization from the Crab Nebula. Their reported upper limit of $\sim 0.05\%$ (which refers to two regions along the major axis of the nebula) is hard to reconcile with the predictions of the synchro-Compton model. It may imply that the 30 Hz waves from NP 0531 do not penetrate beyond the wisps in the nebula.

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