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How to estimate distance and velocity from parallax and proper motion

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Abstract. If the observed parallax ϖ' has a gaussian measurement error σ, there is a 68% probability that the actual parallax ϖ is in the range $\varpi' - \sigma < \varpi < \varpi' + \sigma$ (the frequentist approach). The probability distribution within this range is not known from ϖ' and σ alone, and in particular, we cannot state that the most probable distance D is given by $D = 1/\varpi'$. To obtain a probability distribution, we need to know or assume a distribution of pulsar distances. Similar assumptions are also required to estimate the velocity distribution of radio pulsars.

Keywords. methods: statistical, stars: distances

1. Conditional probability

For a detailed discussion of the conversion of the parallax to distance and of the role of priors, we refer to an excellent paper by Bailer-Jones (2015). Here we briefly summarize the articles of Igoshev *et al.* (2016) and of Verbunt *et al.* (2017).

If an object with a real parallax ϖ and distance $D = 1/\varpi$ is measured with a gaussian measurement error σ, the probability of measuring ϖ' when the real value is ϖ is

$$p(\varpi'|\varpi)d\varpi' = p(\varpi'|1/D)d\varpi' = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(1/D - \varpi')^2}{2\sigma^2}\right] d\varpi'. \tag{1.1}$$

In this equation ϖ' varies and ϖ is fixed. There is an approximately 68% probability that the measured parallax ϖ' lies in the range $|\varpi' - \varpi| < \sigma$, or equivalently that $\varpi' - \sigma < \varpi < \varpi' + \sigma$. However, a measurement ϖ' may also result from a different $\varpi_2 \neq \varpi$. In that case there is still a probability of 68% that $\varpi' - \sigma < \varpi_2 < \varpi' + \sigma$. Therefore, from ϖ' and σ we can indicate an interval for the actual parallax ϖ with a corresponding probability, but not the probability distribution within or outside the interval. This is the frequentist approach.

To obtain a probability distribution, we need to know or assume a prior probability distribution of the parallaxes; in practice a prior of the distance distribution, $f(D)$, is used. This prior acts as a weighting, and for the probability of a real distance D for a fixed measurement ϖ' we have

$$p(D|\varpi')dD = Cf(D)\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(1/D - \varpi')^2}{2\sigma^2}\right] dD \tag{1.2}$$

where C is a normalization constant.

Various authors, e.g. Verbiest *et al.*(2012), following Faucher-Giguère & Kaspi (2006), erroneously replace dD in Eq.(1.2) with $d\varpi = (1/D^2)dD$, which leads to a wrong weighting of D.

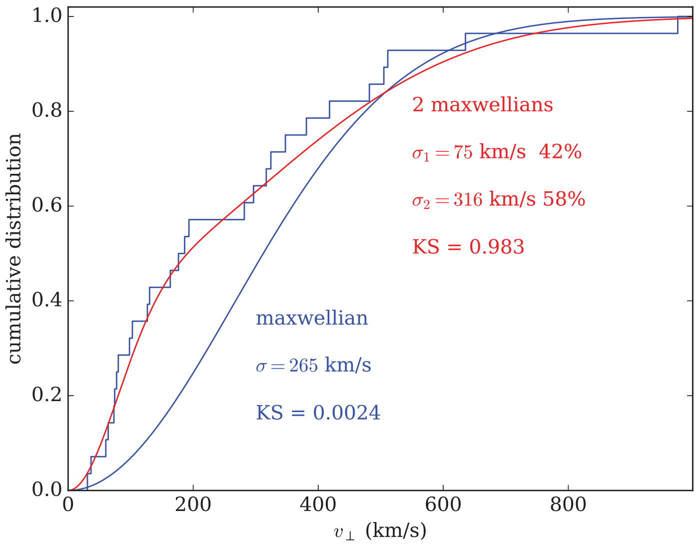

Figure 1. Our best model velocity distribution, the sum of two Maxwellians, converts to a good description (red line) of the observed cumulative distribution of nominal projected pulsar velocities $v'_\perp \equiv \mu'/\varpi'$ (histogram), in contrast to the single Maxwellian found by Hobbs *et al.* (2005, blue line). The p-values of one-sided Kolmogorov-Smirnov tests confirm this.

2. Velocities of young radio pulsars

The measured velocity projected on the sky is found by combining a measured parallax and a measured proper motion: $v'_\perp = \mu'/\varpi'$. Thus each model velocity must be converted into a parallax and proper motion to properly take into account the measurement errors in the model fitting. This requires a (known or assumed) distance distribution $f(D)$.

We have found that the bimodal distribution which consists of two maxwellians with $\sigma_1 = 75^{+20}_{-15}$ km/s $\sigma_2 = 316^{+58}_{-40}$ km/s and $w = 0.42^{+0.10}_{-0.12}$ describes the young isolated radio pulsar velocity distribution much better than the single maxwellian with $\sigma = 265$ km/s that describes the result from a non-parametric analysis by Hobbs et al. (2005). A direct comparison of the velocity distribution with the nominal pulsar velocities $v'_\perp = \mu'/\varpi'$, illustrates this well (Figure 1).

References

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