

## A mathematical analysis of reaction diffusion systems in chemical and biological reactors with macro and micro structures

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This thesis is concerned with generalised models for biological and chemical reactors such as the tubular, fluidised, fixed, packed, continuously stirred and trickle bed reactors.

Suppose  $n$  chemical components at concentrations  $C_i$  ( $i = 1, 2, \dots, n$ ) are “diffusing” and reacting in a homogeneous incompressible fluid with a known velocity profile  $u(z)$  independent of  $C_i$  so that in the reactor region  $\Lambda$ ,  $\text{div } u$  is zero. Immersed in the fluid may be a uniformly distributed population of particles which absorb these chemicals and act as local sites for reaction-diffusion phenomena. The particles are sources and sinks for the chemicals  $C_i$  in the fluid and these fluid concentrations govern the boundary conditions for the particle or local behaviour.

A system of equations is set up as a general model for these complex interactions. The principle limitations of this model are firstly that  $u(t, z)$ , the velocity profile in  $\Lambda$  is known and not coupled with the concentrations  $C_i$  in any way, and secondly the particles are assumed to be fixed relative to the coordinate system of  $z$  in  $\Lambda$  and sufficiently small so that a representative sample of them can be taken to be in a spatially constant concentration environment in  $\Lambda$ .

The objectives of this thesis are generalised comparison theorems for these systems which are used to prove uniqueness, existence, stability and other general qualitative features of such models. A number of examples from the literature are examined.

Models conforming to the system described in this thesis have applications in biological wastewater treatment, biochemical manufacture, urea removal by the compact artificial kidney and industrial fermentation processes. Other potential modelling areas concern fertiliser or pollutants diffusing in soil moisture and reacting with soils, oxidation with product formation in waste deposits and industrial ore reduction processes. There are many other industrial and environmental problems with similar interacting

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Received 4th February 1993.

Thesis submitted to Massey University, March 1993. Degree approved August 1992. Chief supervisor: Professor G. Wake. Second supervisors: Dr A. McNabb and Associate Professor R. Bhamidimarri.

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macro and micro structures. These include the catalytic cracking and synthesis processes in chemical industries ranging from the making of synthesis gas from coal to oil refining.

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