Magnetic Fields throughout Stellar Evolution
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Coronal influence on dynamos

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Abstract. We report on turbulent dynamo simulations in a spherical wedge with an outer coronal layer. We apply a two-layer model where the lower layer represents the convection zone and the upper layer the solar corona. This setup is used to study the coronal influence on the dynamo action beneath the surface. Increasing the radial coronal extent gradually to three times the solar radius and changing the magnetic Reynolds number, we find that dynamo action benefits from the additional coronal extent in terms of higher magnetic energy in the saturated stage. The flux of magnetic helicity can play an important role in this context.

Keywords. MHD, Sun: magnetic fields, Sun: activity, Sun: rotation, turbulence, Sun: corona

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1. Introduction

The solar magnetic field is produced by a dynamo operating beneath the solar surface. In the convection zone, the turbulent motions driven by convection and shear from the differential rotation are able to amplify and organize the magnetic field. These fields manifest themselves at the solar surface in form of sunspots, in which the field is so strong that the heat transported by convection is suppressed, leading to dark spots on the solar disk. One important feature of these sunspots is their latitudinally dependent occurrence. Averaging over longitude, one finds the typical behavior of equatorward migration of the underlying mean magnetic field. This behavior gives clear evidence for the existence of a dynamo mechanism in the Sun. In dynamo theory the α-effect plays an important role, because this effect describes the amplification of large-scale magnetic field in the absent of shear. In the Sun, it is believed that the α-effect produces new poloidal field from the toroidal field. How strong its contribution for the production of toroidal field is, is currently under debate (see e.g. Käpylä *et al.* 2013).

Numerical simulations of turbulent dynamos have shown that the α-effect can be catastrophically quenched at high magnetic Reynolds numbers (see Brandenburg & Subramanian 2005, for a detailed discussion). One possible loophole to alleviate the quenching is to allow for magnetic helicity fluxes (Blackman & Field 2000; Subramanian & Brandenburg 2006; Brandenburg *et al.* 2009). In this context, it is very important to choose a realistic boundary condition for the dynamo, which allows for magnetic helicity fluxes. Preventing a transport of helicity out of the simulation domain may influence the dynamo solution and the strength of the amplified magnetic field. Besides the magnetic helicity fluxes, commonly used boundary conditions for the magnetic field such as vertical field or perfect conductor restrict the dynamo and the magnetic field to certain solutions. This led to the development of the so-called "two-layer model", where we combine the lower layer, in which the magnetic field is generated by dynamo action, representing the solar convection zone with a upper, force-free layer, representing the solar corona.

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Our first application of the two-layer model led to the formation of structures reminiscent of plasmoid- and CME-like ejections, driven by a forced turbulent dynamo (Warnecke & Brandenburg 2010; Warnecke *et al.* 2011, 2012a) and, subsequently, by a self-consistently driven convective dynamo (Warnecke *et al.* 2012b) in the lower layer. This indicates that the dynamo can be directly responsible for producing coronal ejections and form structures in the solar corona. But in the recent work of Warnecke *et al.* (2013a), the authors find that differential rotation and the migration of the mean magnetic field can be also influenced by the presence of a coronal layer. In this paper, we investigate how the corona influences the dynamo action.

2. Model

We use spherical polar coordinates, (r, θ, ϕ). The setup is the same as that of Warnecke *et al.* (2011), where we use a spherical wedge with $0.7R_\odot \leqslant r \leqslant R_\mathrm{C}$, $\pi/3 \leqslant \theta \leqslant 2\pi/3$, corresponding to $\pm 30°$ latitude, and $0 < \phi < 0.3$, corresponding to a longitudinal extent of $17°$. $R_\odot$ is the radius of the Sun and R_C is the outer radius of the coronal layer. At $r = R$ the domain is divided at into two parts. The lower layer mimics the convection zone, where a magnetic field gets generated by turbulent dynamo action. The upper layer is a nearly force-free part, which mimics the solar corona. We solve the following equations of compressible magnetohydrodynamics,

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \boldsymbol{\nabla}^2 \boldsymbol{A}, \tag{2.1}$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -\boldsymbol{\nabla} h + \boldsymbol{g} + \frac{1}{\rho}\left(\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla} \cdot 2\nu\rho\boldsymbol{\mathsf{S}}\right) + \boldsymbol{F}_\mathrm{for}, \tag{2.2}$$

$$\frac{\mathrm{D}h}{\mathrm{D}t} = -c_s^2 \boldsymbol{\nabla} \cdot \boldsymbol{U}, \tag{2.3}$$

where $\boldsymbol{A}$ is the magnetic vector potential and the magnetic field is defined by $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, which makes Equation (2.2) obey $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ at all times. η and ν are the magnetic diffusivity and the kinematic viscosity, respectively. $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U} \cdot \boldsymbol{\nabla}$ is the advective derivative, $\boldsymbol{g} = GM\boldsymbol{r}/r^3$ is the gravitational acceleration, G is Newton's gravitational constant, and M is the mass of the Sun. We choose $GM/R_\odot c_\mathrm{s}^2 = 3$. $\boldsymbol{J} \times \boldsymbol{B}$ is the Lorentz force and $\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}/\mu_0$ is the current density, where μ_0 is the vacuum permeability. The traceless rate-of-strain tensor is defined as $\mathsf{S}_{ij} = \frac{1}{2}(U_{i;j} + U_{j;i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla} \cdot \boldsymbol{U}$, where the semi-colons denote covariant differentiation, $h = c_\mathrm{s}^2 \ln \rho$ is the specific pseudo-enthalpy, with $c_\mathrm{s} = \mathrm{const}$ is the isothermal sound speed. As in the work of Warnecke *et al.* (2012a), the forcing function is only present in the lower layer of the domain. This means that the forcing function goes smoothly to zero in the upper layer of the domain ($r \gg R_\odot$). The function $\boldsymbol{f}$ consists of random plane helical transverse waves with relative helicity $\sigma = (\boldsymbol{f} \cdot \boldsymbol{\nabla} \times \boldsymbol{f})/k_\mathrm{f}\boldsymbol{f}^2$ and wavenumbers that lie in a band around an average forcing wavenumber of $k_\mathrm{f} R_\odot \approx 63$. The forcing function also has a dependence on the helicity which is here chosen to be $\sigma = -\cos\theta$ such that the kinetic helicity of the turbulence is negative in the northern hemisphere and positive in the southern. More detailed descriptions can be found in Warnecke *et al.* (2011) and Haugen, Brandenburg, and Dobler (2003). The magnetic field is expressed in units of the equipartition value, $B_\mathrm{eq} = \mu_0 u_\mathrm{rms}\overline{\rho}$, where $\overline{\rho} = \langle\rho\rangle_{r \leqslant R_\odot,\theta,\phi}$, $u_\mathrm{rms} = \langle u_r^2 + u_\theta^2 + u_\phi^2\rangle_{r \leqslant R_\odot,\theta,\phi}^{1/2}$, and $\langle\cdot\rangle_{r \leqslant R_\odot,\theta,\phi}$ denotes an average over θ, ϕ and $r \leqslant R_\odot$, i.e., over the whole dynamo in region. The fluid and the magnetic Reynolds numbers are defined as,

$$\mathrm{Re} = u_\mathrm{rms}/\nu k_\mathrm{f}, \quad \mathrm{Re}_M = u_\mathrm{rms}/\eta k_\mathrm{f}. \tag{2.4}$$

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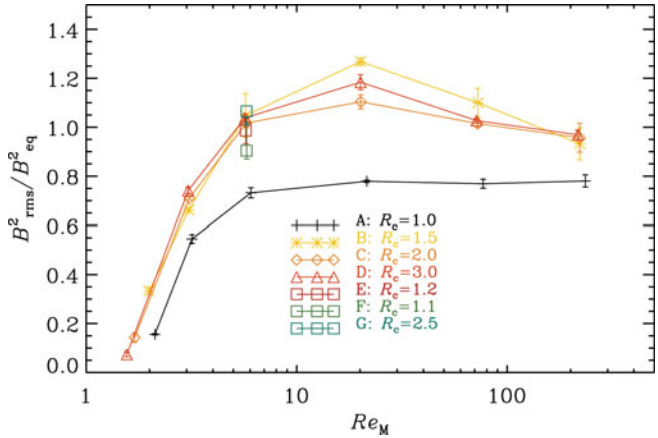

Figure 1. Dependence of magnetic field energy normalized by the equipartition value $B^2_{\rm rms}/B^2_{\rm eq}$ with coronal radial extent $R_{\rm C}$ and magnetic Reynolds number ${\rm Re}_M$. The solid black line indicates the dynamo region without corona.

Their ratio is expressed by the magnetic Prandtl number ${\rm Pr}_M = {\rm Re}_M/{\rm Re}$.

As an initial condition we use Gaussian noise as seed magnetic field in the dynamo region. Our domain is periodic in the azimuthal direction. For the velocity field we use a stress-free boundary condition on all other boundaries. For the magnetic field we apply a perfect conductor conditions in both θ boundaries and the lower radial boundary ($r = 0.7\,R_\odot$). On the outer radial boundary ($r = R_{\rm C}$), we employ vertical field conditions. We use the Pencil Code† with sixth-order centered finite differences in space and a third-order accurate Runge-Kutta scheme in time; see Mitra *et al.* (2009) for the extension of the Pencil Code to spherical coordinates.

3. Dynamo action

We perform 27 runs where we change $R_{\rm C}$ and ${\rm Re}_M$, but keep ${\rm Pr}_M$ constant. The letters for different sets indicate the coronal extents: $R_{\rm C}/R_\odot = 1$, 1.5, 2, 3, 1.2, 1.1, and 2.5 for Sets A–F. In the first four sets, we vary ${\rm Re}_M$ from 1.5 to 220, for the last three sets we use ${\rm Re}_M = 6$.

For all runs the turbulent motion in the lower layer of the domain drives dynamo action, which amplifies the magnetic field. After exponential growth, the field saturates and shows cycles. The field shows an equatorward migration of the all three magnetic field components, as described in Warnecke *et al.* (2011). This is caused by an α^2 dynamo, where α changes sign over the equator (Mitra *et al.* 2010a). In Figure 1, we show for all the 27 runs the normalized magnetic field energy $B^2_{\rm rms}/B^2_{\rm eq}$ as function of magnetic Reynolds number ${\rm Re}_M$. The value for $B^2_{\rm rms}/B^2_{\rm eq}$ is obtained by averaging in space over the lower layer of the domain $r \leqslant R_\odot$ and in time over many hundred turnover times in the saturated stage. The error bars in Figure 1 reflect the quality of the temporal averaging. From Figure 1, we can deduct two important results. First, for runs with a corona the magnetic energy peaks at ${\rm Re}_M \approx 20$. This seems to be not the case for runs without a corona. On the other hand, the magnetic energy declines for larger ${\rm Re}_M$, as was also found by Käpylä *et al.* (2010), which could be related to a change in the onset conditions for the different cases. Second, the magnetic energies for all runs with a coronal extent are larger by a factor of ≈ 1.5. It seems that the actual radial size of the coronal

† `http://pencil-code.googlecode.com`

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extension is not that important as long as there exists a coronal layer. The run of Set F has just a coronal extent of $R_\mathrm{C} = 1.1\,R_\odot$, but the magnetic energy is closer to runs with larger coronal extent than to the one without corona.

Magnetic helicity fluxes might be a key to solving this riddle. However, the outer radial boundary condition in the runs without a corona also allow for magnetic helicity fluxes. We recall that the simulation with a corona generates large ejections of magnetic helicity (Warnecke *et al.* 2011, 2012a). Without possessing a coronal extent the dynamo might be not able to produce ejection of magnetic helicity and therefore has a much lower magnetic helicity flux through the boundary. Studies on the nature of helicity fluxes in these runs are already on the way (Warnecke *et al.* 2013c, in preparation). However, magnetic helicity fluxes might be important only much larger magnetic Reynolds numbers (Del Sordo, *et al.* 2013).

4. Conclusions

We have shown that a coronal layer on the top of a dynamo region can support dynamo action. This is visible through an increase in magnetic energy by adding a corona at the top of the domain and leaving all other parameters the same. However, it will be necessary to study magnetic helicity fluxes through the surface of the lower layer for these cases to derive any further conclusions. The two-layer model has been used before to show the impact of a dynamo on coronal properties and generating CME-like ejections (Warnecke & Brandenburg 2010; Warnecke *et al.* 2011, 2012a). With this model is was also possible to generate spoke-like differential rotation and equatorward migration in global convective dynamo simulations (Warnecke *et al.* 2013a), whereas models without a corona have not been able to reproduce these features in the same parameter regime (Käpylä *et al.* 2013). Besides dynamo models, this two-layer approach is successful in combination with stratified turbulence in producing a bipolar magnetic region (Warnecke *et al.* 2013b) as a possible mechanism of sunspot formation.

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