3355. A biquadratic 6-cycle

Will someone please provide a proof of the following?

We are given that $\alpha_1, \alpha_2, \alpha_3$ are angles of a triangle, and we define $\alpha_{r+3} = \alpha_r$. For all integral r, $T_r = (1+k)t_r^2 - 2kt_r \cot\alpha_r + (1-k)$ and $T_r T_{r+1} = (t_r + t_{r+1})^2$. Thus t_0 gives two values of t_1. Each t_1 gives two t_2, and each t_2 gives two t_3. Hence t_0 generates a set of eight values of t_3. Similarly t_0 gives two t_{-1}, four t_{-2} and a set of eight values of t_{-3}. Prove that these two sets of eight are equal. I have a proof which is complicated, long-winded and does not seem to go to the heart of the matter. For particular values of α_r, k and t_0, a numerical verification has been made by Graham Batty on the computer at Blackpool Grammar-Collegiate School Sixth Form Centre.

This biquadratic 6-cycle arose in the attempt to get an elementary proof of a discovery which G. B. Money-Coutts made as a result of careful drawings. A simple version of his discovery is this. We are given three congruent circles K_1, K_2, K_3 in a plane. An arbitrary circle S_0 touches K_1 and K_2 *directly*. [A circle touches a pair of circles directly (or transversely) if and only if its chord of contact passes through the external (or internal) centre of similitude of the pair.] Circles $S_r (r = 1$ to $6)$ are drawn so that S_r touches S_{r-1} and directly touches those two of K_1, K_2, K_3 whose suffixes are not congruent to r (mod 3). Then a chain of circles exists such that S_6 coincides with S_0.

A clear statement of Money-Coutts' problem and a proof using 3-D complex geometry and elliptic functions has been given by J. A. Tyrrell and M. T. Powell in the *Bulletin of the London Mathematical Society* **3**, 70–74 (1971). An elementary proof of the special case when K_1, K_2, K_3 are straight lines appears in a note by E. A. Maxwell in the *Mathematical Gazette* for February 1971, p.61.

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The Klein four-group and the Post Office

DEAR MR. QUADLING,

One answer to the suggestion in James Ridley's letter (October 1973) is the automatic letter facing machine, such as is used in modern postal sorting offices. Each letter travels along a conveyor belt resting on one of its long edges. There are four possible positions:

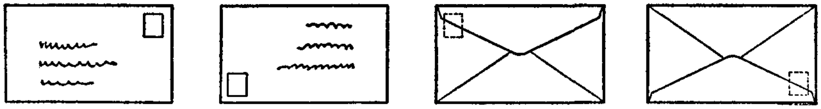

FIGURE 1.

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It is the task of the machine to give all letters a common orientation so that the addresses can be easily read and the stamps cancelled.

We would expect such a machine to be able to perform four basic operations:

I: Pass the letter through unchanged;
H: Twist the letter through 180° about a horizontal axis in the plane of the letter;
V: Rotate the letter through 180° about a vertical axis;
R: Roll the letter end for end about an axis perpendicular to the plane of the letter.

Since these operations just describe the symmetries of a rectangle, I, H, V and R determine a Klein four-group.

Now in the construction of an automatic facing machine, it is easy to design a moving belt system which will perform H. It is rather more difficult to effect V and R by mechanical means for a fast moving stream of letters. Since the structure of the four-group tells us that $HV = R$, and $HR = V$, we see that in practice only one of the operations V, R needs to be built into the machine.

The diagram below shows the design of an early letter facing machine. At each 'scan', the lower leading edge of the envelope is scanned by a sensor. If the stamp appears at that corner, the letter is directed to the lower route and passes on through the machine unchanged. We observe that this design makes use of the operations H, $HV = R$ and $HVH = RH = V$.

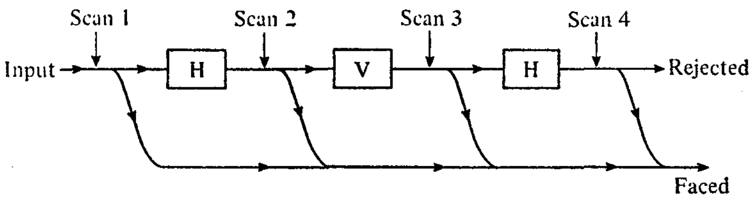

FIGURE 2.

It is an interesting and instructive classroom exercise to experiment with different possible designs for a letter facing machine, as any economical layout must make use of the four-group. Information on machines currently in use can be found in a paper on automatic letter facing by G. P. Copping in "British Postal Engineering", *Proceedings of the Institution of Mechanical Engineers* **184** (1969–70).

Yours sincerely,
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Minimum-MSE estimators

DEAR SIR,

In a recent article [1] Mr. B. J. R. Bailey draws attention to some of the problems which arise in point estimation. The article is very welcome, and one hopes it will be read by some of those who regard the Principle of Maximum Likelihood as an article of religious faith.

Mr. Bailey gives the impression, however, that the theory of minimum-MSE estimators is somewhat nebulous, and it would be a pity to leave this impression uncorrected. The classic paper by Pitman [2] deals with the subject in fiducial terms, and possibly for this