THE DETERMINATION OF TEMPERATURE FROM SPECTRAL LINES

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1. Introduction

Some stellar spectral lines are quite sensitive to temperature. With current techniques, we can, for example, measure temperature modulation of a few degrees. But high temperature sensitivity can also be a detriment when it masks variations in other physical parameters of interest. In such cases, we must have a way of determining the temperature precisely so that the residual differences or variations can be seen and interpreted. The situation with stellar surface features is particularly challenging because several physical variables interact simultaneously, each impressing their signature on the spectral lines. It is frequently a good plan to compare a highly temperature sensitive line to one that is insensitive or to one that has an inverse sensitivity. A good way to find suitable lines is to compare exposures of hotter and cooler stars, as illustrated in Fig. 1. Here we see lines that hardly vary (Si I), some that show modest changes (Fe I), some that show large changes (V I), and one that shows a reverse change (Fe II). Some caution is needed because differences in surface gravity and metallicity can also produce changes in line strengths. So the general idea is to play off the differences of one line compared to another and avoid any direct dependence on absolute line strengths. Yes, this is indeed nothing more than a careful refinement of determining a spectral type.

It is worth noting that temperatures deduced from lines compared to those deduced from color indices show only a small amount of unexplained scatter (for dwarfs, at least). In other words, normal lines are strongly

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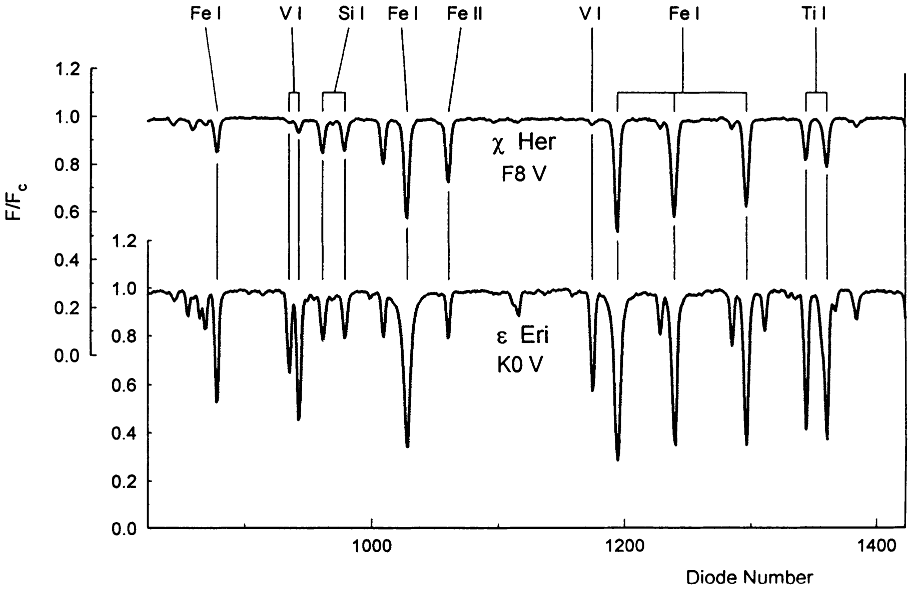

Figure 1. Here is a portion of spectrum near 6250 Å for two stars of different temperature. Different lines show different temperature sensitivity.

coupled to the continuum and therfore we should find the same temperature differences with either of these tools.

2. Calibration

Although one can conceive of several possible ways to calibrate changes of spectral lines with temperature, one of the most basic is to compare the spectral-line parameters to color indices. Color indices are related fundamentally to the energy distribution and the actual definition of effective temperature. Of course, a separate calibration of the color indices against true effective temperatures is also needed, but this is a separate step which we assume has been done. In favorable cases, spectral lines allow us to resolve temperature differences ≈ 1 K. A more typical case yields 3-4 K, i.e., two orders of magnitude more precision than the fundamental effective temperature calibration, and about one order more precision than color photometry. However, most calibration curve are approximately exponential (see Fig. 4 below), and spectral lines normally loose their leverage toward higher temperatures. Then one is forced to select other lines, and this is a disadvantage compared to color indices which generally have a broader working range.

3. Other variables

Naturally, other variables come into the picture, notably

- surface gravity
- metallicity
- line broadening by rotation and macroturbulence
- line broadening by the spectrograph

Consequently, we must find some combination of line parameters that largely eliminates all except temperature differences. This is best accomplished using the ratios of weak spectral lines. Three possibilities spring to mind: ratios of equivalent widths, of core line depths, i.e., continuum-to-core, and of core residual flux, i.e., zero-level-to-core. This third one is less good since their ratios do not scale with metallicity nor with shifts in zero level arising with scattered light or composite spectra. When the spectral resolution is high, core depth has the advantage over equivalent widths. By using the core depth, blends are less of a problem. How often have you anticipated using a line only to find a serious blend on one side or the other. In many such cases, the core is still usable, but only if the spectral resolution is high enough to isolate the blend from the core. Several of the lines in Fig. 1 illustrate the point.

Furthermore, small errors in setting the continuum cause much less deviation from the true central depth than from the true equivalent width.

However, if the broadening caused by the spectrograph is variable from one exposure to the next, or from one portion of the exposure to another (camera focus errors, for example), then the advantage swings more toward the ratio of equivalent widths. And if the broadening from the spectrograph is large, then one is using equivalent widths like-it-or-not. If there is any question as to the preference of equivalent widths versus line depths, one can always plot each ratio against color index and see which gives the smaller scatter, indicating which is the better one to use.

The effects of gravity are relatively easy to accommodate since they can be relatively small. Different surface gravities result in differing ionization equilibria and pressure broadening. Therefore we should be wary of comparing lines from different ionization stages or in using any characteristics of a line affected by pressure broadening such as the wings of strong lines. When we use a set of stars along the main sequence to gain a temperature calibration, we have built into it the change in surface gravity that occurs in concert with the temperature. Fortunately gravity changes slowly along the main sequence, and one can therefore reasonably expect to use such a calibration to interpret temperature modulation of the type found for magnetic cycles or the rotation of a star with surface features.

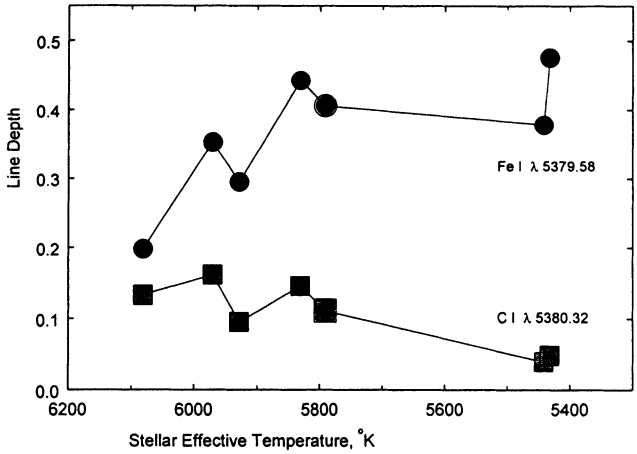

Figure 2. Spectral line depths of two lines for seven stars are plotted here against the effective temperature of the star. The symbol with the border is for the sun.

Larger changes in gravity can be more dangerous. The calibration is likely to be different for dwarfs compared to giants, for example. Not only do we have direct gravity effects on ionization and pressure broadening, but things like macroturbulence increase with luminosity class. The differences in line-depth ratios might be small for truly weak lines, but this should be carefully investigated before any calibration is used across different luminosity classes.

Metallicity differences can be compensated reasonably well by using only weak lines. When the lines are on the linear portions of their curves of growth, their strengths scale with the metallicity; ratioing cancels the scale factor. One is on the safest ground ratioing lines of the same element since then differential abundance effects cannot enter (see Gray 1994).

Consider Figure 2 which shows simple line depths of two reasonably weak lines for seven stars as a function of effective temperature. The iron line shows a general increase with declining temperature; the carbon line a decrease, but the changes are far from smooth. That's because each star has its own metallicity and rotation broadening. One would be hard pressed to use these as calibration curves. Now look at Fig. 3, where the ratios are shown in place of the actual line depths. We learn from this 1) ratios of line depths are what we are looking for, 2) the variation of the line-depth ratio is smooth but variable in slope, i.e., sensitivity, and 3) not all problems are eliminated, as indicated by the point well below the curve. But recall, that this ratio is between carbon and iron, two sufficiently dissimilar elements that differential abundance differences can occur and are even likely.

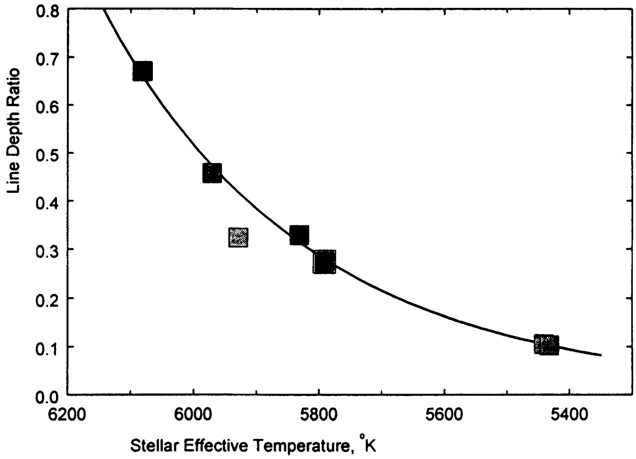

Figure 3. The ratio of spectral line depths, λ5380.32 to λ5379.58, is shown as a function of effective temperature.

Finally, consider rotational broadening. It impresses the same shape on all weak lines, and therefore ratioing of line depths cancels its effect. For stronger lines, those where the rotational broadening does not completely dominate the line broadening, the ratioing will be imperfect and a dependence on rotation will remain.

4. Considerations of errors and sensitivity

The signal-to-noise ratio in the spectrum, along with sampling errors, usually determine the limit of accuracy in determining a line depth (or equivalent width). The fractional error of the ratio of two line depths is given by

$$\frac{\delta r}{r} = \left(\left(\frac{\delta d_2}{d_2}\right)^2 + \left(\frac{\delta d_1}{d_1}\right)^2\right)^{\frac{1}{2}} \tag{1}$$

where the core depths of the two lines are d_1 and d_2 and δd_1 and δd_2 are their corresponding errors. Obviously we pay a price here for using weak lines, i.e., small d_1 and/or d_2 produce larger errors. How ratio errors translate into temperature errors depends on the slope of the calibration curve for each ratio. Consider the calibration curve in Fig. 4. A fixed error in ratio translates into a much smaller temperature error at 4500 K than at 6000 K. In fact, the relation becomes useless toward the higher temperature, and different spectral lines have to be used.

On the other hand, the ratio error is not usually constant with effective temperature, as we can deduce from Eq. (1). Since the expression is for a

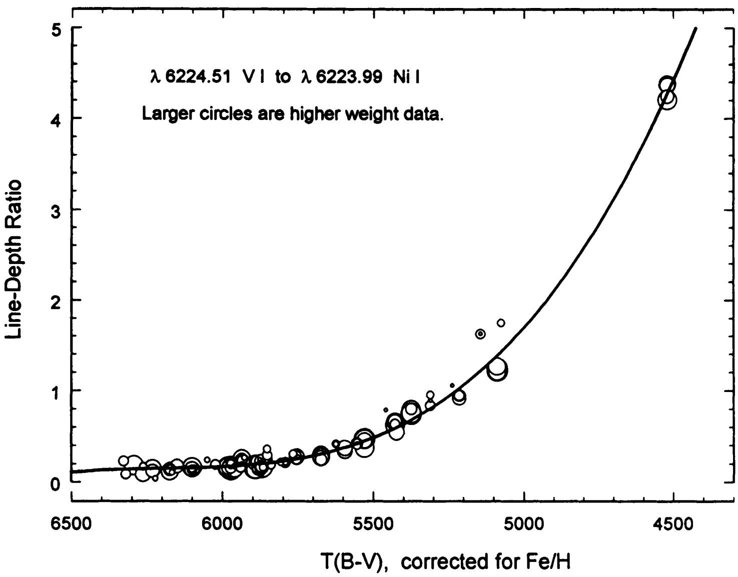

Figure 4. A typical calibration curve shows a steep rise with changing temperature. Here the line-depth ratio is plotted against stellar temperature based on the B-V color index.

fractional error, a logarithmic plot is more appropriate, as shown in Fig. 5. Here we can see several things. First, from ≈5800 K and cooler the slope changes slowly so that the error in temperature is

$$\delta T \approx const. \times \frac{\delta r}{r}, \tag{2}$$

where $\frac{\delta r}{r}$ is given by Eq. (1). Second, the scatter of the points around the line is roughly constant from 5800 K to cooler temperatures, but increases rapidly for star hotter than 5800 K as the line depths become small. Third, the slope leverage becomes rapidly worse for temperatures above 5800 K. We conclude that this particular line-depth ratio is suitable only for star cooler than 5800 K.

On a good exposure, with a suitable pair of lines, the line-depth ratio can be measured to $\frac{\delta r}{r} \approx 0.01$. We then need to know the slope of the calibration curve to get δT. Typical values range from a fraction of one degree to a few degrees, and specific examples are given in Gray (1994). Naturally, one gains precision by increasing the signal-to-noise ratio of the spectroscopy (smaller δr's) and by combining the results of several line-depth ratios. In favorable cases, temperatures can be measured with a precision of less than one degree. Please remember that the effective temperature scale has errors about two orders higher than this, so we have a very precise tool

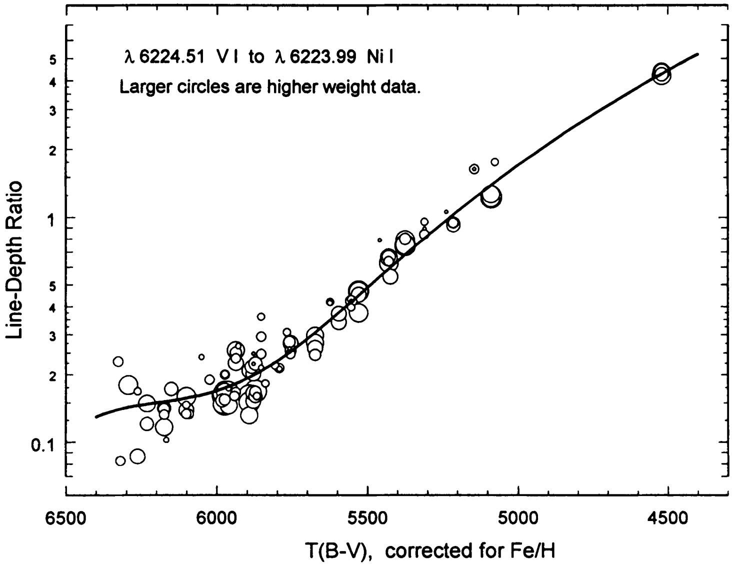

Figure 5. Same as Fig. 4, but with a logarithmic ordinate.

for measuring temperature changes of the type occurring during magnetic cycles or rotational modulation, and for measuring temperature differences between stars.

5. Messy cases – surface features

Surface features on stars produce messy cases, and disrupt our carefully thought-out procedures. What do we do when a bump migrates through our line profile? What is the connection between the bump and the temperature of the surface feature? What meaning does the line depth carry when the bump is in the core of the line?

Toward answering such questions, we must first review why surface features cause bumps in a profile. The simplest and most basic way to see what is happening is to go back to the discussion of the shape of a rotation profile. What we do there is divide the disk of the star into strips parallel to the rotation axis, assign the Doppler shift of rotation to each strip, and sum. Strips near the center of the disk are long and give the most light, but zero or small Doppler shifts, so the center of the rotationally-broadened profile has the greatest weight. Further from the center of the disk (along the equator) the strips are shorter, giving less light, and have a greater Doppler shift. When there is a dark spot on the disk, the strips containing the spot contribute less than their normal share of light, and so the Doppler shifts associated with that strip are under-represented, i.e., the Doppler-shift distribution has a notch in it (ref. Gray 1988). Spectral line profiles are, approximately, the convolution of this Doppler-shift distribution with the profile the star would have without rotational broadening. In effect, the convolution moves the Doppler-shift distribution to the position of the line, scales it to the equivalent width of the line, and turns it upside-down so that the notch now looks like an emission feature in the profile. If you are dealing with an emission line, you will see the notch right-side-up instead.

Let me digress a moment to review the expected behavior of such bumps arising from surface features. If we first consider a feature that spans a narrow range in longitude, or equivalently a narrow range in Doppler shift, then it is truly a narrow feature in the spectroscopic sense, and its shape will be the shape of the instrumental profile of the spectrograph. The bump will migrate through the line profile as rotation carries the spot through the range of projections of its rotation rate onto the line of sight. Naturally, the same feature seen at higher latitude would span a narrower range of Doppler shifts. And if the axis of rotation is inclined toward us, circumpolar features may be seen to migrate to and fro across a narrow Doppler-shift span, rather than disappearing from view the way lower latitude features would.

If we then consider a feature with considerable longitude extent, and if the rotation rate is large enough compared to the resolution of the spectrograph, then the leading and trailing edges of the feature will have different projections, going as the sine of the angle to the line of sight, and the bump in the profile will sharpen up as it crosses the center of the profile.

Enough digression. We need to ask now when and how the notion of a spectral line entered this discussion of surface features. Basically only in one way, as a sharp structure in the spectrum to act as a marker of Doppler shifts. In particular, the strength of the notch in the Doppler-shift distribution depends on the loss of light from the disk, i.e., the loss of continuum, albeit with its spectral lines. For example, if the light contributed from a strip on the disk is diminished by 10% because of a dark spot, then the bump will have a height of 10% of the depth of the line at that position. But if the depth of a spectral line is changed by 10% in the light from the spot, then the change in line depth integrated over the strip is only 1%. So, changes in the strength of the spectral absorption line between surface feature and surrounding continuum is of second order. In short, the spectral line features arising from surface spots are mainly effects of the continuum not the lines!

We can, therefore, conclude the following about messy cases.

– If the rotation rate is large enough to fully resolve the surface features in longitude, then the core depths of the lines can be used to determine the star's temperature by choosing times when the bump is away from the line core.

- The size of the bump, to first order, is determined by the loss of continuum light at the Doppler position of the bump. Any change in line strength enters as second order, and will probably be lost in uncertainty of the first-order term.
- As we consider cases of smaller $v \sin i$, those approaching the macroturbulence broadening of the line, the surface features no longer produce resolved bumps in the profile, but only rotationally-modulated distortions. This may well be the messiest situation to deal with. In the case of negligible rotational Doppler shifts, the first order effect is no longer distinguishable, and one is left with only the second-order change in line strength in the light from the spot. In that case, the apparent temperature differences we deduce for the star using line-depth ratios is reduced from the no-spot case by the true temperature deficiency of the spot multiplied by the fraction of light it contributes to the integrated spectrum we measure.

The case where rotation is small but not negligible is probably best handled by proper modeling through disk integration.

6. Summary

Ratios of spectral line depths, or equivalent widths when using low spectral resolution, are powerful indicators of temperature changes and temperature differences. Errors as small as 1 K can sometimes be attained. Surface features complicate and compromise this tool whenever they distort the line.

References

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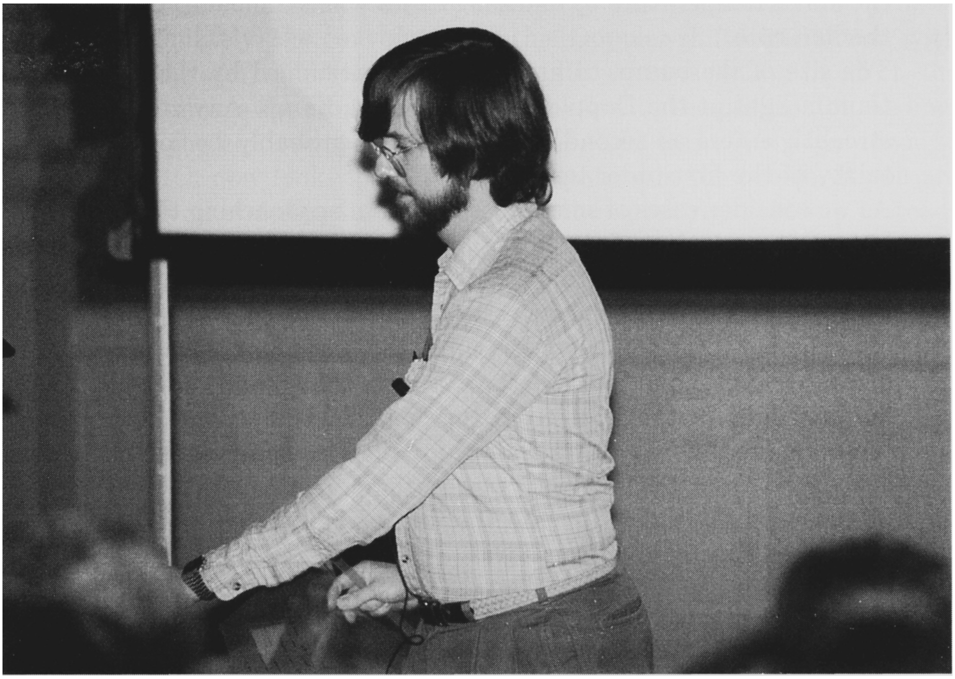

Steve Saar is getting ready for his talk ...

... but is probably still thinking about the reception by the mayor of Vienna from the previous evening (gee, notice the piano player!).

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