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ANALYSIS OF APPROACHES OF TOLERANCE ALLOCATION REGARDING TO ECONOMIC EFFICIENCY

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ABSTRACT

The aim of this paper is to examine the current state of research on tolerance-induced costs in Germany. Through a literature research already existing approaches for the determination of costs related to tolerances during the specification of technical components are pointed out and possible approaches for the reduction of these costs are presented. In addition, the actuality of these approaches will be considered. One question that is supposed to be answered here is to what state of standard for the specification of components these approaches can be assigned to. On the other hand, it should be clarified whether the existing approaches are applicable to the currently valid standard system of the Geometrical Product Specification (GPS).
Can the economic efficiency of the specifications selected for tolerancing be determined in a technical drawing during the product development process in accordance with GPS on the basis of the current state of research?

Keywords: Design engineering, Design to X, Tolerance representation and management, Geometrical Product Specifications (GPS), Economic efficiency

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1 INTRODUCTION

During the manufacturing of components, various influences and inaccuracies lead to parts that deviate from the ideal design. In order to limit these potential deviations, tolerances are allocated at an early stage of the product development process. However, the choice of tolerances has a variety of effects.

The main purpose of tolerance definition is to ensure the functionality of the component. It should be tolerated in such a way that all the requirements set for the fully assembled technical product can be met throughout its lifecycle. This is the only way to prevent functional limitations and quality defects in the product.

The choice of tolerances can also influence the use of natural resources. If the permissible deviations for products or semi-finished products cannot be met, sustainability suffers as a result of rejects during production or faulty functioning when the product is used.

Tolerances thus have a significant influence on the economic efficiency of products and processes. In addition to their primary task of ensuring functionality, tolerances also influence the manufacturing process (Ehrlenspiel *et al.*, 2014). It is obvious that machining effort increases with narrow tolerancing. As well as additional machining steps, more precise machines and processes are required, leading to an increase in manufacturing costs. The designer is often not sufficiently aware of this connection. Moreover, without tools designers cannot usually estimate the costs of tolerance definition accurately in the early phase of a product's development process. The economic influence of tolerance should not be underestimated, especially in view of the fact that the costs of correcting design errors increase tenfold from phase to phase during product development (Klein and Mannewitz, 1993).

The aim of this paper is to provide a wide-ranging overview of the current state of scientific research on tolerance-induced costs. The literature survey focuses on the approaches to the description and reduction of costs that can be found in the literature, and assesses their applicability today. The questions that arise here are: (1) What standards are used as the basis for the various investigations undertaken in the literature? (2) In particular what reference is made to the standards system of Geometrical Product Specifications (GPS), which can be regarded as state of the art today?

2 STATE OF THE ART

This section describes the development of tolerancing and provides a brief introduction to Geometrical Product Specifications. For this purpose, the standards system and symbols are briefly described. Also existing application problems are pointed out.

2.1 Evolution of tolerancing

Human productivity dates back approximately 1 million years. Since then, various technical and social developments have influenced the evolution of our current tolerancing system, as shown in Figure 1.

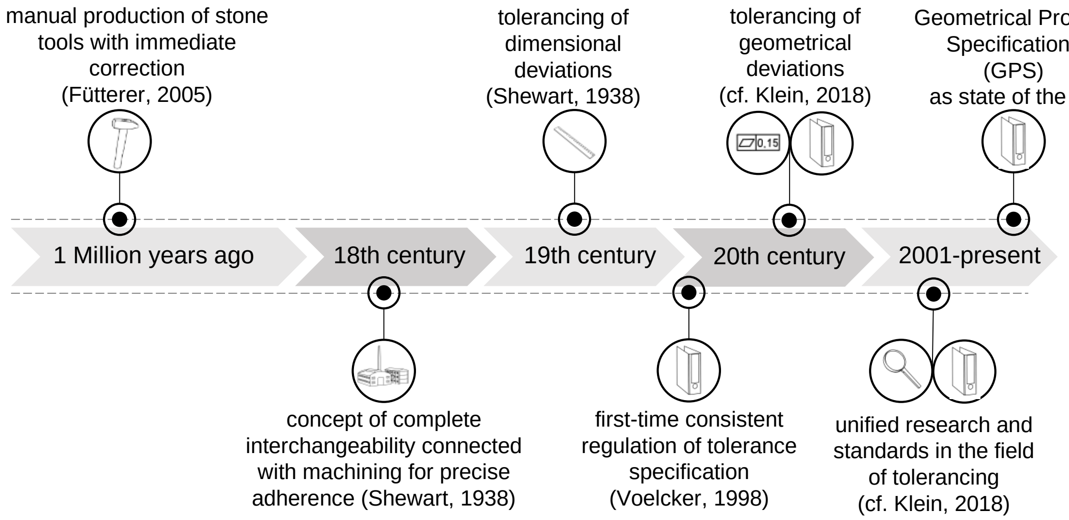

Figure 1: Evolution of tolerancing

From the beginning of human productive power until the middle of the 18th century, products were manufactured by hand and individual components were matched to each other by immediate post-processing (Fütterer, 2005). In the middle of the 18th century, in the French weapons industry, efforts were made to design interchangeable components (Alder, 1997). During the industrial revolution in the mid-19th century, dimensional deviations were tolerated for the first time (Shewart, 1938). At the beginning of industrial mass production in the early 20th century, consistent regulations for the specification of tolerances were introduced (Voelcker, 1998). In the 1960s, the focus of consideration in the area of tolerancing was expanded to geometrical deviations as a supplement to the already known dimensional deviations (Klein, 2018). Over the last 20 years, research has become more unified and previous national standards have been harmonized (Klein, 2018). This has resulted in the development of the GPS standards system, which can be regarded as state of the art today.

2.2 Geometrical product specifications

The GPS system is a set of common tolerance standards. It was developed by Technical Committee 213 "Dimensional and geometrical product specifications and verification" of the International Organization for Standardization (ISO), founded in 1996 (ISO/TC 213, 2018). According to the fundamental standard ISO 14638:2015-01, GPS serves to describe "the geometrical requirements of workpieces [...] and the requirements for their verification" in a technical drawing. Geometric properties are descriptive features, as are physical and chemical properties. The geometrical specifications of a workpiece include its length and angle dimensions, its shape and position descriptions, as well as a definition of the surface texture (Weidemann, 2018a).

2.2.1 Symbolism of geometrical product specifications

The allocation of GPS to a drawing can be regarded as a language to describe and control the function of a part (Nielsen, 2012). This language consists of symbols, numbers and letters that can be understood by employees in design, production and metrology all over the world.

Figure 2 shows an example of a tolerated component and illustrates how the symbolism of GPS is based on various standards of the GPS system. The specifications represent a fragment of a completely dimensioned and tolerated technical drawing. Dimensional, shape and position tolerances are used in the system of Geometrical Product Specifications to limit deviations of components from the ideal concept. In the example, the position of the median line of the hole and the flatness of the bottom surface are limited (ISO 1101:2017-02, ISO 12781:2011-04). For the application of position tolerance, the allocation of datums is necessary. In this context, reference is made to ISO standard 5459:2011-08, which defines datums and datum systems. In order to determine the theoretically exact location of the median line of the hole, theoretically exact dimensions are specified (TED) (ISO 1101:2017-02). Dimensional tolerances for the width and length of the component are entered in accordance with ISO 14405-1:2016-08 for dimensional tolerancing. The indication of surface texture in drawings, and the application of the ISO code system for tolerances on linear sizes, are specified in the standards ISO 1302:2002-02, ISO 286-1:2010-04 and ISO 286-2:2010-06.

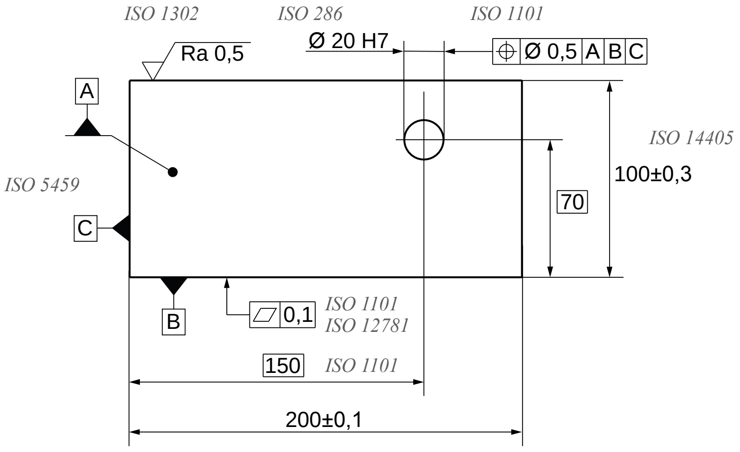

Figure 2: Interaction between GPS standards in the specification of a component, based on Weidemann (2018b)

2.2.2 Application of geometrical product specifications

The GPS standards system is a complex construct of interacting standards that are subject to high dynamics. Its complexity is reflected in the number of standards published by ISO/TC213. Since its foundation in 1996, a total of 146 ISO standards have been published. 24 ISO standards are under development (International Organization for Standardization (ISO), 2019). The two factors "complexity" and "dynamics" make it difficult to use the tools provided in the latest GPS standards for specification in technical drawings.

A survey conducted by Sersch and Gust (2018) of German mechanical engineering companies confirms that there are application problems with the system of Geometrical Product Specifications. According to the participants in the study who use GPS, the main problem is a lack of know-how and skills. Nevertheless, more than half of the participants who know about the GPS system see the economic potential of its application.

3 ECONOMIC EFFICIENCY OF TOLERANCES

As described in the Introduction, tolerances have a decisive influence on the choice of manufacturing process and resulting production costs. A consideration of the costs caused by tolerances can be an incentive for companies to look more closely at the relevant standards. With regard to the topic of responsible design, it also makes sense to consider the cost-effectiveness of tolerances. In addition to the above-mentioned reduction of reject rates and the associated saving of material resources, the sustainable use of financial resources must also be considered important.

For this reason, the literature survey was conducted to determine the extent to which the costs attributable to tolerances have already been examined from a theoretical point of view. Approaches to the description of tolerance-induced costs were identified and existing methods for their reduction described.

3.1 Methodology of the literature survey

The literature survey was carried out with the help of search engines for scientific publications such as Google Scholar, and via systematic keyword searches in library catalogues. After a brief review of the text and analysis of the abstract, the text sources were selected. Following the inherent characteristics of a backward search, sources of older origin were identified based on current textbooks. From a thematic point of view, the research was directed away from writings with more general content and toward texts with a specialized thematic focus. There were no restrictions on the type of literature or the time horizon of the search. In addition to published sources such as textbooks and articles from specialist journals and conference proceedings, grey literature such as dissertations and university reports were also included. The reference to the ISO or DIN standards system and the language of the source were limiting criteria. Only texts written in either German or English were considered. Even within the limits of these boundary conditions, the literature survey does not claim to be exhaustive.

3.2 Results of the literature survey

This section first identifies the approaches to the description of tolerance-based costs found during the literature survey, as well as various options for cost optimization. Then the fundamental influence of tolerances on production costs in the form of mathematical tolerance-cost functions is discussed. On this basis, the statistical aspect of tolerancing is presented. The final part of the section gives an introduction to various computer-aided methods of tolerance optimization that take economic considerations into account.

3.2.1 Tolerance-cost functions

In general, the relationship between tolerance and the costs associated with it can be described in the form of a mathematical equation, the so-called *tolerance-cost function*. According to Wittmann (2001), equation 1 is a simple example of such a mathematical term.

$$\mathrm{FK} \approx \mathrm{A} + \mathrm{B} \cdot \mathrm{T}^{-\mathrm{e}} \tag{1}$$

This equation is only one of many possible approaches to the description of production costs (*FK*). According to Chase *et al.* (1990), what the approaches all have in common is that the costs, in most cases including fixed costs (*A*), can always be described based on the production costs of the considered workpiece dimensions (*B*) as a function of the tolerance (*T*) corresponding to them. In this concrete example, the exponent *e* represents the tolerance exponent according to Spotts, which depends on the type of surface treatment (cf. Wittmann, 2001). The reciprocal exponential influence of the tolerance on the production costs leads to a hyperbolic function graph. Due to the characteristics of this function graph, the term *cost hyperbola* is also used in this context. However, as investigations by ABB and General Electric have shown, this can only be observed for chip-removing manufacturing processes, since a more linear cost behavior has emerged for forming and primary shaping processes (cf. Klein, 2018). Figure 3a, based on Jorden and Schütte (2017), illustrates the characteristics of machining processes and also establishes a connection to the value of a product. As can be seen from the figure, with wide tolerances, production costs drop, but on the other hand the value of the product is limited. Due to the narrowing of tolerances, a degressive increase in value can be observed, with simultaneous progressive cost development. This results in tolerance values ranging between an unfavorable price and tolerances at which acceptable quality can be achieved at reasonable costs.

The continuous hyperbolic curve of the cost graph in Figure 3a is simplified for reasons of clarity. In fact, as Figure 3b illustrates, the cost curve is composed of separate sections that form a discrete graph. This is because the selection of a suitable manufacturing process depends on the tolerance selected. There are price discrepancies between the different processes, among other things due to increasing complexity and decreasing processing speed, factors which lead to these cost jumps. According to Jorden and Schütte (2017), economically optimized tolerances, as shown by tolerance T_1 in Figure 3b, are located just above a point of discontinuity. While the value of the product in this example is almost identical for tolerances T_1 and T_2, the production costs for tolerance T_1 are significantly below those for tolerance T_2 due to the different machining processes.

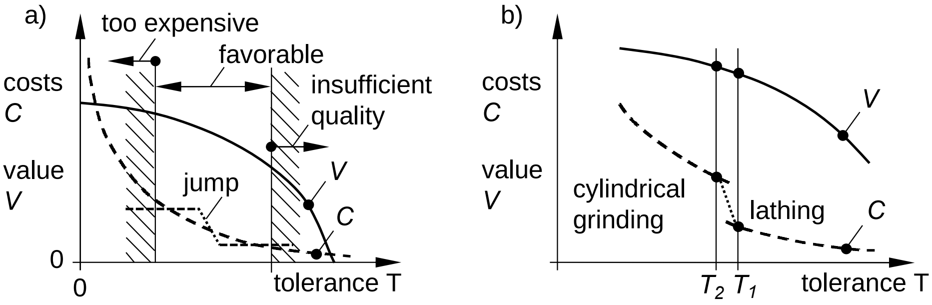

Figure 3: Connection between tolerance and value, based on Jorden and Schütte (2017)

This kind of graphic representation of production costs can, according to Klein (2018), help the engineer design with direct consideration of economic aspects, using a relative cost worksheet as part of a cost information system in accordance with DIN 32991-1 [costs informations, costs information-documents, principles of presentation (withdrawn)]. This enables comparison of relative production costs not only at different levels of tolerance but also with different materials and machines.

In addition to graphic visualization of cost information, tolerance-cost functions also make it possible to identify an economically favorable tolerance value using mathematical optimization methods. Chase *et al.* (1990) provide a good overview of various approaches to tolerance-cost functions and the optimization methods associated with them. However, optimization by means of Lagrange multipliers remains the most commonly used method.

And although tolerance-cost functions substantially simplify real cost behaviors, production costs are often estimated using the so-called *Michael-Sidall-function* (cf. Klein, 2018). Finally, an approach proposed by Diplaris and Sfantsikopoulos (2000) should also be mentioned, in which the dimensions of the workpiece are taken into account when determining production costs. Especially in machining production, the volume relevant for machining has a decisive influence on the machining effort and thus a direct impact on costs (Diplaris and Sfantsikopoulos, 2000).

3.2.2 Statistical tolerancing

According to Klein and Mannewitz (1993), one way to achieve economic and resource-saving tolerancing is to abandon the principle of *absolute interchangeability*. By exploiting stochastic relationships, larger tolerances can be allocated without limiting the quality or functionality of a product.

The reasoning behind this is that all industrial production is affected by fluctuations caused by external influences, which are subject to stochastic laws. Hence manufactured dimensions vary and have a frequency distribution characteristic of the individual process (Klein and Mannewitz, 1993).

These distributions, following Klein (2011), can range from rectangular distributions for continuous tool wear to triangular distributions for small series production, and Gaussian normal distributions for large series production. They are derived for assemblies by folding the dimensional distributions of the individual components. If possible, the tolerance should be selected in such a way that the entire scatter range of dimensional distribution is covered.

Statistical tolerance is based on the idea that a larger dimensional deviation does not necessarily mean that the finished part is to be regarded as a reject. Instead, it is assumed that, due to the range of manufactured dimensions, there is usually a combination that satisfies all functional requirements and can therefore be taken as fulfilling the quality objective (Klein and Mannewitz, 1993).

In the context of Six-Sigma, statistical tolerancing in accordance with Klein (2011) is an essential quality tool. If the tolerance limits can be set at a distance of six times the standard deviation around the nominal dimensions, this quality objective is achieved and the product meets the requirements of Robust Design. From a traditional point of view, a component is equally good within its tolerance range and equally bad outside its tolerance limits, no matter how large the actual deviation from the nominal dimension is. If, however, the quality loss of the feature is considered in a more differentiated way within the tolerance range, the Taguchi quality loss function can be used to record the monetary loss and determine the tolerance limits under economic criteria (Klein, 2011).

3.2.3 Computer-aided tolerance optimization

The approach, shown in Figure 4 and described by Chase *et al.* (1990), is to determine a cost-optimized production sequence for an assembly of three components via data processing. Taking into account overall functionality, and aiming for minimal manufacturing costs of the assembly as a whole, the tolerances and manufacturing process should be determined individually for each component. It is assumed that components 1 and 2 each require two different manufacturing processes, while three different processes are available for component 3. A cost-tolerance relationship exists for each process (cf. Section 3.2.1). Hence there are twelve possible combinations for manufacturing the assembly. In order to determine the most favorable manufacturing combination, Chase *et al.* (1990) apply the Lagrange optimization method to each of the possible combinations. The optimization approach can, however, be varied in accordance with the existing tolerance-cost relationship. A final comparison of all twelve possibilities will yield the most cost-effective tolerance values and manufacturing process for the assembly of the respective components. To save computing time, the optimization can be simplified: Instead of calculating all combinations (*Exhaustive Search*), the most favorable process for each component (*Univariate Search*) can be determined sequentially (Chase *et al.*, 1990).

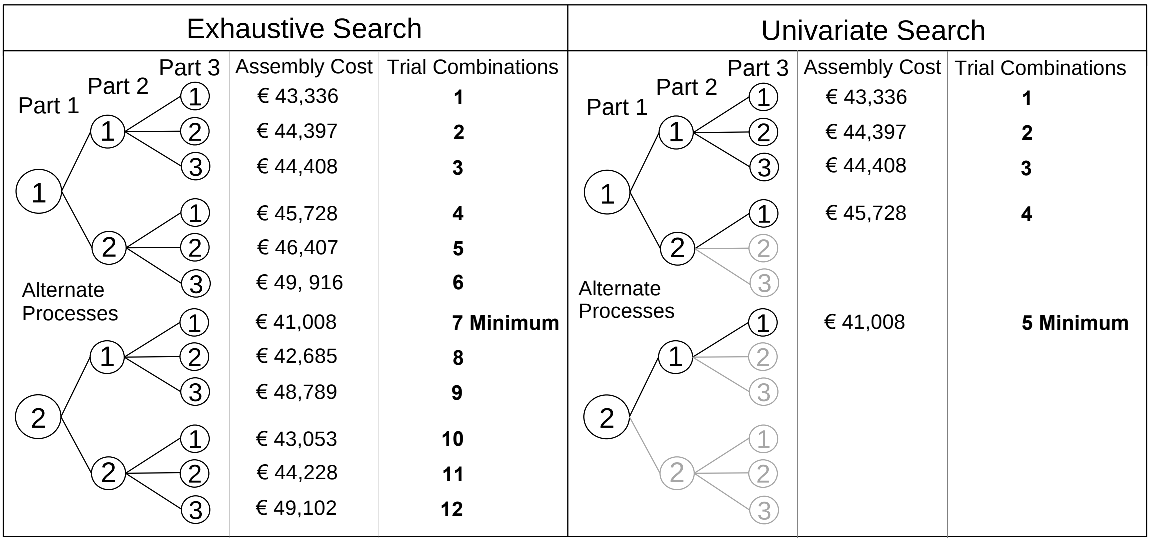

Figure 4: Exhaustive and Univariate Search for process selection of tolerance-cost optimized assemblies, based on Chase et al. (1990)

Jourdan (2001) describes another possibility of computer-aided tolerance allocation and tolerance cost calculation. The aim of his approach is to calculate the dimensional tolerances of the individual components from the closed component of an assembly by solving a mathematical equation system defined in advance. The required manufacturing sequence of the component is then defined for the predetermined tolerance values. In order to do this, the computer compares the required accuracies with the accuracies to be achieved on the existing machines in a company-specific database. Based on the machine parameters stored in this database, the processing time for each component is calculated and the manufacturing costs are determined via the cost rates of the machines. A cost-optimized tolerance allocation can be realized if the necessary dimensional accuracies are allocated while taking into account the expected manufacturing costs (Jourdan, 2001).

An alternative method for economic tolerance allocation in early design phases is the *feature-based integrated tolerance information system (FIT)* described by Wittmann and Scheer (2000). In their approach, the designer is provided with relevant additional information on the individual geometric elements (features) in the usual working environment. This information allows the designer to check the specification from an economic point of view and, if necessary, to modify the design in order to reduce costs. The aim of this information system is to gather together information from different departments, such as design, manufacturing, metrology and accounting, and make it available to everyone involved in the product planning and manufacturing process (Wittmann and Scheer, 2000).

Wu *et al.* (2009) describe a procedure for tolerance specification in which a combination of Monte Carlo simulation and genetic algorithms can be used to determine a cost-optimized component tolerance. By specifying initial dimensional deviations (population) and simulating a limited number of deviating components (individuals), a targeted variation of the observed deviations of each individual can be applied, using genetic algorithms to create a distribution for the functional fulfillment of the observed population; costs can then be calculated using cost functions. The objective is to find a population, i.e. a range of dimensional deviations, with low costs and high functional performance (Wu *et al.*, 2009). In this simulative experimental method, economically optimized tolerance values can be identified even if there is no analytical approach for optimization.

A different approach to the determination of component tolerances was introduced by Kopardekar and Anand (1995). Here, artificial neural networks serve to identify favorable component tolerances for individual components of an assembly based on different input values, such as machine capabilities, (i.e. the maximum accuracy that can be achieved on a machine), and the total tolerance of the assembly.

3.3 Discussion

One fundamental finding of the literature survey is that tolerance-induced costs have generally been considered from a theoretical point of view. Figure 5 illustrates and distinguishes the cost determination and optimization approaches identified in the context of this literature research.

Approaches to the determination of tolerance-related costs

Tolerance-cost functions depend heavily on the quality of the data on which they are based and represent a simplified description of the real relationship between the tolerance specification and manufacturing costs. Although this simplification may result in deviations from the actual costs incurred, it is still a good way to estimate the manufacturing costs caused by tolerances. In addition, these functions can be used to determine cost-optimized tolerances using familiar mathematical optimization methods.

As an alternative to cost-tolerance functions, the manufacturing costs, as described by Jourdan (2001), can also be determined in the traditional way by means of business cost accounting, taking into account the expected time effort. In addition, such an approach makes possible the reduction of tolerance costs by means of computer-based determination of the most favorable manufacturing process. The focus of this approach is not primarily on the optimization of tolerance values, but rather on the optimization of the manufacturing process. Because it integrates parameters deriving from the available machines, this approach is strongly company-related and can therefore only be used in a limited way to find a generally valid cost optimum. However, according to Jourdan (2001), this limitation does not represent a disadvantage, since he believes that tolerance optimization should always be adapted to the conditions operating in the particular company.

Klein (2018) describes a method for capturing quality-related tolerance costs via the quality-loss function introduced by Taguchi. By taking into account different dimensional deviations within the tolerance range, the corresponding financial impacts can be analyzed. This approach can mainly be adopted in connection with statistical tolerancing, since variance in manufactured dimensions represents a calculation factor especially for the assessment of quality loss in series production. As well as having this limitation, this approach makes it possible to define the tolerance limits for statistical tolerancing via a previously set maximum economic quality loss.

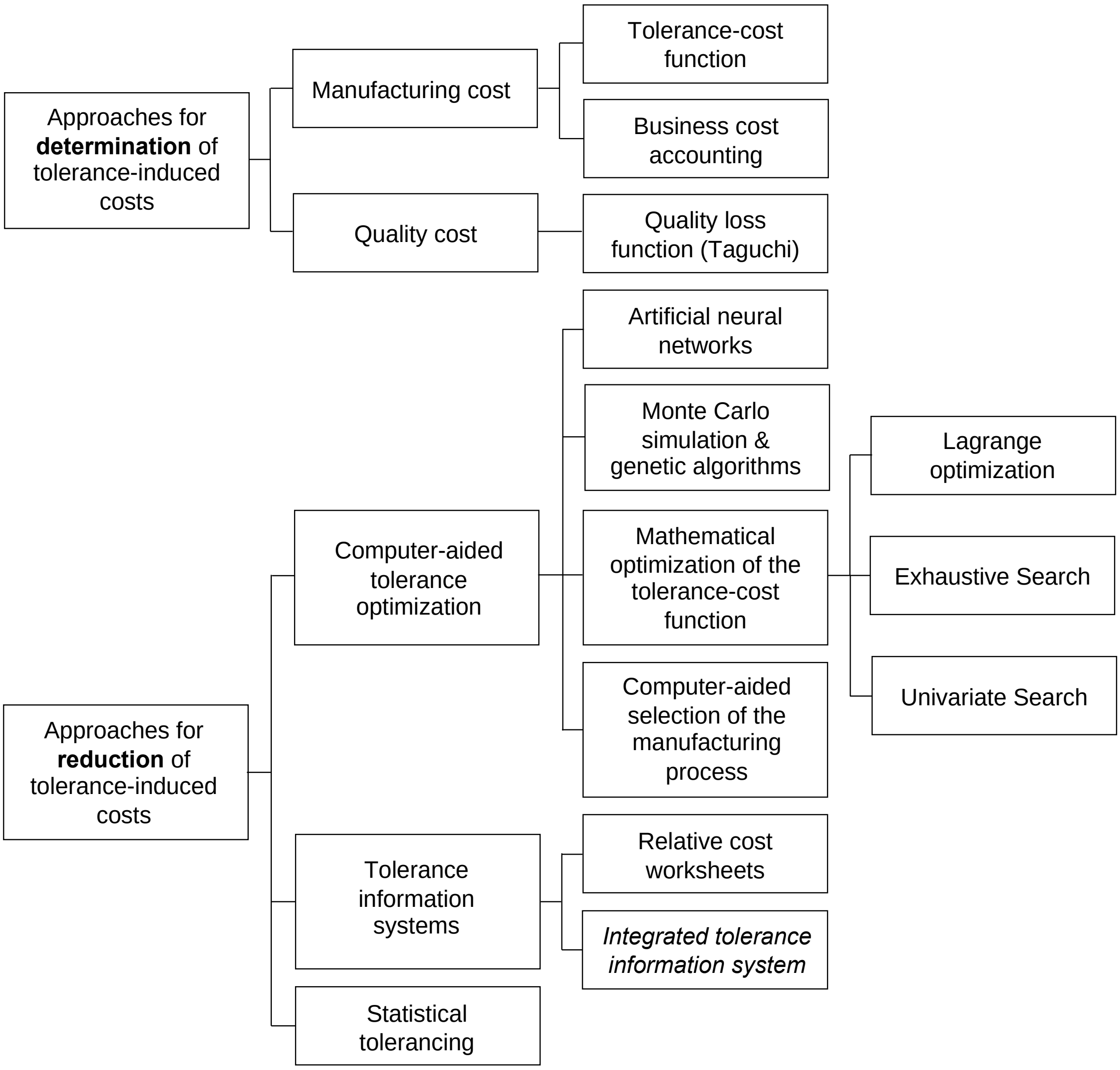

Figure 5: Overview of approaches to tolerance-induced costs

Approaches to the reduction of tolerance-related costs

In order to reduce tolerance-induced costs, component specification must be optimized under economic aspects. Here mathematical optimization methods can be implemented for tolerance-cost functions. The Lagrange multiplier method should be mentioned first, as – due to its uniform methodology – it has a particularly advantageous effect on computer-aided optimization (Chase *et al.*, 1990). Previously defined boundary conditions can also be taken into account. The disadvantage of this method is that it can only be applied to continuous tolerance-cost correlations.

Computer-aided procedures in particular are of great importance for the economic allocation of tolerances. As described by Chase *et al.* (1990), the application of Lagrange optimization can reduce the manufacturing costs of an assembly. While an Exhaustive Search considers every possible process combination and accordingly requires a lot of computing time, the sequential process selection for individual components in the Univariate Search can significantly reduce the number of combinations considered, and thus save time (cf. Figure 4). Depending on the order in which the components are viewed, certain combinations can be excluded in advance, so that the absolute cost minimum cannot always be identified with certainty (cf. Chase *et al.*, 1990).

Tolerance optimization by means of genetic algorithms and Monte Carlo simulation, based on Wu *et al.* (2009), has the advantage that, due to the experimental character of this method, optimization is possible even if there is only a discrete tolerance-cost dependency or no description of this dependency at all. However, the quality of the calculation results is highly dependent on the test quantity, i.e. the size of the genetic population. More precise results can therefore only be achieved with increasing calculation effort. A further consideration is that the Monte Carlo simulation method is no longer up to date; a current approach is Latin Hypercube Sampling (LHS).

The use of artificial neural networks avoids long calculation times, due to the possibility of parallel calculations. A further advantage of this method is that, by simple extension of the model, influences not investigated in the past can be considered for the first time (Kopardekar and Anand, 1995). Nevertheless, a neural network in its mode of operation is a black box. It is not clear exactly how the output variables are calculated from the input variables. In addition, the neural network must be trained, using known test data. The quality of the calculated results depends significantly on the amount and rigor of this training.

As well as by targeted calculation of an economically optimized tolerance, costs can also be reduced by providing cost information to the designer. With this method, the designer still makes the tolerance specifications according to own criteria, but can make decisions on the basis of cost information. While relative cost worksheets, as described by Klein (2018), offer a good and rapid overview, the integrated tolerance information system of Wittmann and Scheer (2000) also has the advantage that additional data is displayed to the designer in his/her familiar working environment. This data is managed electronically, so several people can at the same time access the most current data as adapted to the relevant company structure. However, the inclusion of incorrect data in the databases could negatively affect the quality of the calculations.

According to Klein and Mannewitz (1993), using stochastic relations enables an improvement in economic efficiency by widening the tolerance limits in statistical tolerancing. The Six Sigma quality objective can be achieved by the targeted definition of tolerance limits with appropriate conditions. However, one possible result of statistical tolerancing is that certain dimensional combinations of manufactured components cannot be assembled into functional units. This can lead to an increased reject rate, as well as to considerable loss of time during assembly (Klein and Mannewitz, 1993).

4 CONCLUSION

With reference to the research questions outlined at the beginning of this article, it can be said that this topic has already seen some consideration from a theoretical perspective. Most, but not all, of the approaches are limited to the description and optimization of tolerance-induced manufacturing costs. Only one approach is suitable for determining tolerance-related quality costs. Concerning the type of tolerance considered, all the approaches consider only dimensional tolerances and not form or position tolerances. It was not possible to assign an underlying standard to any of them, but it can be assumed that the approaches identified can be applied to dimensional tolerance in accordance with the currently valid Geometrical Product Specifications standards. A conclusive assessment of potential applicability in the limitation of geometrical features is not possible in the current state of the investigation.

Tolerances clearly have economic impact, and this paper has presented a number of approaches to the quantification and reduction of costs resulting from tolerance specification based on literature research. Further studies on the economic effects of dimensional and geometrical specifications are needed in order to quantitatively evaluate the economic influence of tolerance allocation. In the case of design-to-cost, this topic also has research potential in relation to Design to X.

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