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CORRIGENDUM

NOTE ON THE DIVISIBILITY OF THE CLASS NUMBER OF CERTAIN IMAGINARY QUADRATIC FIELDS – CORRIGENDUM

By YASUHIRO KISHI

Department of Mathematics, Fukuoka University of Education, Munakata, Fukuoka 811-4192, Japan
e-mail: ykishi@fukuoka-edu.ac.jp

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In the proof of [1, Lemma 2.3], the lines 12–14 on page 190 – 'Then p must be equal to 3, and hence we have $2^{k+3} = 1 + 3b^2D$ by (2.2)' – is incorrect. The author would like to thank Akiko Ito for pointing out this error to him. As a consequence, the statement of [1, Lemma 2.3] is lacking in the condition $(k, n) \neq (2, 3)$. The following revised version of [1, Lemma 2.3] is correct.

LEMMA. *Let k and n be positive integers with $2^{2k} < 3^n$, $n \geq 3$ and $(k, n) \neq (2, 3)$, and put $\alpha := 2^k + \sqrt{2^{2k} - 3^n} \in \mathbb{Q}(\sqrt{2^{2k} - 3^n})$. Then $\pm\alpha$ is not a pth power in $\mathbb{Q}(\sqrt{2^{2k} - 3^n})$ for any prime p.*

Proof. Let p be a prime number. In the same way of the proof of [1, Lemma 2.3], let us lead a contradiction by assuming that α is pth power in $\mathbb{Q}(\sqrt{2^{2k} - 3^n})$. Let α denote

$$\alpha = \left(\frac{a + b\sqrt{D}}{2}\right)^p \quad (a, b \in \mathbb{Z},\ a \equiv b \pmod 2),$$

where D is the square-free part of $2^{2k} - 3^n$. The proofs in [1] are not wrong for the cases where '$p = 2$', '$p \geq 3$ and a even', and '$p \geq 3$, a odd and $k = 1$'. Now we consider the case where $p \geq 3$, a odd and $k \geq 2$. In this case, it must hold that $p = 3$ as we have seen in the proof of [1, Lemma 2.3]. Then we have

$$2^k + \sqrt{2^{2k} - 3^n} = \left(\frac{a + b\sqrt{D}}{2}\right)^3. \tag{1}$$

Noting that $a = \pm 1$, we have

$$2^{k+3} = a(1 + 3b^2D).$$

Since D is negative, a must be equal to -1. Then we have

$$3b^2D = -2^{k+3} - 1. \tag{2}$$

Taking the norm of both sides of equation (1), on the other hand, we have

$$3b^2D = 3 - 4 \cdot 3^{(n+3)/3}. \tag{3}$$

By equation (2) and equation (3), we get the equation

$$2^{k+1} - 3^{(n+3)/3} = -1.$$

We note here that the equation $2^x - 3^y = \pm 1$ has only three positive integer solutions $(x, y) = (1, 1), (2, 1), (3, 2)$ (see [1, Lemma 2.1]). Hence it must hold that $(k, n) = (2, 3)$. This implies that we get a contradiction if $(k, n) \neq (2, 3)$. Thus the lemma is now proved. □

Therefore the statement of [1, Theorem 1.2] must be changed as follows:

THEOREM. *For any positive integers k and n with* $2^{2k} < 3^n$ *and* $(k, n) \neq (2, 3)$*, the class number of the imaginary quadratic field* $\mathbb{Q}(\sqrt{2^{2k} - 3^n})$ *is divisible by n.*

REMARK. In case of $(k, n) = (2, 3)$, the class number of $\mathbb{Q}(\sqrt{2^{2k} - 3^n}) = \mathbb{Q}(\sqrt{-11})$ is equal to 1.

REFERENCE

1. Y. Kishi, Note on the divisibility of the class number of certain imaginary quadratic fields, *Glasgow Math. J.* **51** (2009), 187–191.

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