Magnetic Fields and Filling Factors in Late-Type Stars: Predictions from Dynamo Theory

B. Montesinos, J.L. Fernández-Villacañas, C. Jordan

Department of Theoretical Physics, University of Oxford, 1 Keble Road,
Oxford OX1 3NP, England, UK

Abstract: In this paper we examine the link between observed magnetic fields and filling factors in main-sequence stars of types G and K, and the results obtained for these parameters from a simple dynamo model. We explore how the predicted magnetic fluxes, fB, for a selected sample of stars, vary with rotation, adopting two theoretical approaches to estimate the filling factor, and considering different expressions for the variation of the stellar angular velocity, ω, with depth.

1. Introduction

In the last decade, the detection of magnetic fields in late-type stars has opened a new window for a deeper understanding of all the phenomena broadly known under the name of *stellar activity*. The techniques for estimating the magnetic field strengths and the fraction of stellar surface covered by strong fields – the so-called *filling factor* – have been greatly improved since the first positive detections by Robinson (1980) up to the more sophisticated methods applied by Saar (1986, 1990) and Marcy and Basri (1989). A state-of-the-art review of the topic can be found in the work by Saar (1990, see also this volume).

On the other hand, there exists a wide range of theoretical approaches attempting to explain how the magnetic fields are generated in main-sequence stars, how they rise from the deep layers of the star to the surface and how they interact with the gas in the stellar atmosphere. The interaction between differential rotation and convection in the sub-photospheric convection zone *–dynamo action–* is so far the most plausible mechanism to explain the origin of the observed magnetic fields in the Sun and in late-type stars (see the review by Moss, 1986). The fields are supposed to rise by mechanisms combining buoyancy, Coriolis and Lorentz forces, reaching the photosphere and depositing their energy in the corona through MHD waves or by reconnection processes.

We find that there is a large gap between the observational work, which is providing a great deal of material to be analysed, and the *purely* theoretical work which creates complex models from the mathematical point of view, but which are often unable to predict what should be observed if the models were applied to a real star.

This contribution aims to help to fill that gap. Since there are many parameters of the stellar interior that are unknown we have carried out exploratory work, by taking a simple kinematic dynamo model and applying it to a sample of main-sequence late-type G and K stars which have the most reliable detections of magnetic fields and filling factors. Some extreme approaches have been chosen both for the dependence of rotation with depth, $\omega(r)$, and for the estimate of the filling factor. The model applied is summarised in Section 2 and the results are given in Section 3.

2. The model

Although a detailed description of all the characteristics of the model applied will be published elsewhere (Montesinos *et al.*, 1991) we summarize here its main points.

The available information consists of magnetic field strengths B, and filling factors, f. Our goal is to find the theoretical approach which gives an accurate prediction of both parameters, or some other related quantity, and their dependence on stellar rotation. We have chosen to study the magnetic flux fB, and we have decided to apply our formalism only to G and K stars, since F and M stars have very shallow and very deep convective zones, respectively, for which our formulation may not be appropriate.

2.1 The convection zone magnetic field

The field is assumed to be generated at the bottom of the convection zone by dynamo action. The generation region has been located one pressure scale height above the bottom of the convection zone and we assume that the depth of that region is also one scale height. The estimate of the convection zone magnetic field is described in the papers by Durney and Robinson (1982), Montesinos *et al.* (1987), and Montesinos *et al.* (1991), where further details can be found.

Two approaches have been taken to describe the variation of the stellar angular velocity with depth. These are based on helioseismological data, but some of the constants, such as the equatorial rotation velocity, are scaled to apply to each particular star. The first law, hereafter denoted as [1] is based on the approximation found by Durney (1985) and has the property that the shear in the angular velocity $d\omega/dr$ *is proportional* to the surface stellar angular velocity; the second one, [2], has been taken following a simplification of some results by Brown *et al.* (1989) and, in contrast to [1], the shear in ω *does not* depend on the angular velocity itself.

2.2 The filling factor

Starting from a toroidal configuration of tubes at the bottom of the convective zone and assuming that they rise up to the photosphere by buoyancy, it is possible to estimate the fraction of the stellar surface covered by magnetic tubes. Two approximations have been considered for the estimate of f. The first one, outlined by Durney and Robinson (1982), takes into account the diffusion time for the magnetic fields as they reach the photosphere. The expression for f is in this case

$$f \simeq \frac{3\pi^2}{16} \frac{\varphi}{l} \left(\frac{R_g}{R_*}\right)^2 \frac{B_{cz}}{B} \frac{u}{v_{ph}} L_{ph} \tag{1}$$

The second approach, which has been developed by us, only takes into account the number of flux tubes that at a given moment are present on the stellar photosphere. Whereas the first approximation can be considered as a 'time-average' estimate, our approximation can be called the 'snapshot' or 'static' approach. The expression for f is

$$f \simeq \frac{\pi^2}{16} \frac{\varphi}{l} \left(\frac{R_g}{R_*}\right)^2 \frac{B_{cz}}{B} L \tag{2}$$

In both expressions R_g is the radius of the generation zone, R_* is the stellar radius, B_{cz} and B are the convection zone and the photospheric magnetic fields, u is the rate of rise of a tube in the generation zone, L and L_{ph} are the pressure scale heights at the bottom of the convection zone and at the photosphere, respectively, and v_{ph} is the convective velocity near the photosphere. There are some unknowns in equations (1) and (2) which have to be fixed by analogy with the Sun, namely φ, the angle around the stellar equator where the bunch of magnetic tubes is supposed to be concentrated, as in the solar case, and l, the length of an eruptive section of magnetic tube when it starts rising from the bottom of the convection zone. Thus φ is taken as $\sim 2\pi/5$ and l is normalised to reproduce the solar filling factor or that of other chosen star. The photospheric field B is computed through the equipartition theorem which seems to be a good approximation to the observed values (Saar, 1990).

3. Results

Figure 1 gives the results of the calculations. The two upper graphs, (a) and (b), show the values for fB versus rotation period computed using the 'time-average' estimate for f and the two lower graphs, (c) and (d), give those computed using the 'snapshot' approach. The results in (a) and (c) have been computed using law [1] for ω, whereas those in (b) and (d) were estimated using law [2]. Open circles are observational results and solid circles are our theoretical predictions. The two open circles near the bottom of each graph and joined by a vertical bar represent the Sun. Theoretical predictions and observational results for each star lie in the same vertical line.

Although limits on space do not allow us to discuss in detail the results of the different approaches, we draw attention to several interesting results. In graph (a)

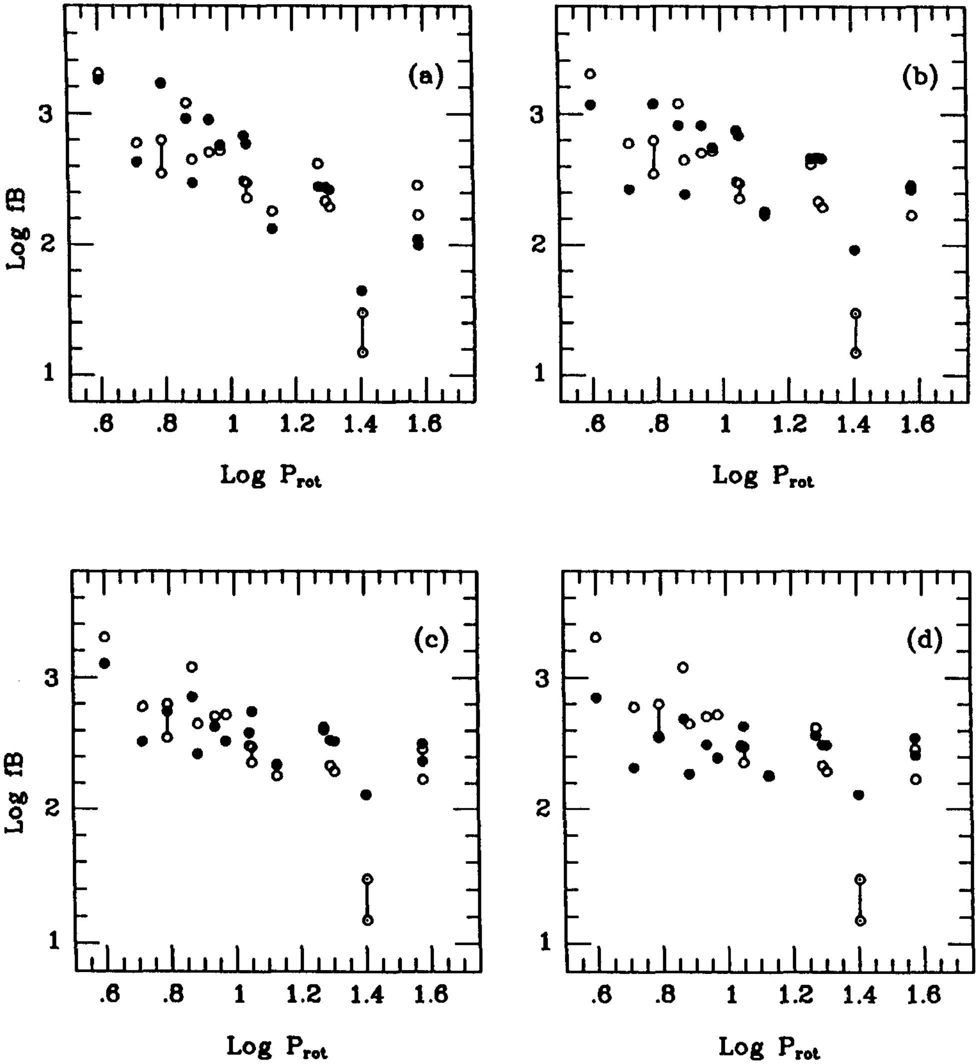

Fig. 1. Predicted values of fB (solid circles) and observational results (Saar, 1990) (open circles) versus rotation period for a sample of G and K main-sequence stars. See text for details.

both the solar and stellar values of fB are reproduced with reasonable accuracy. The agreement between the stellar values of fB observed and those computed is also good in (b) but the estimate for the Sun is a factor $\sim$ 4 higher than the observed value. In graphs (c) and (d) the spread of the results is similar to the spread in the observational data but again the results for the solar fB-value are higher than the observed one. The different spread in the results between the top graphs and the bottom graphs is due to the dependence of v_{ph} and L_{ph} on spectral type. As far as the behaviour of the relationship $fB \propto P_{rot}^{-\alpha}$ is concerned, the slope of the observed values of fB with rotation depends on whether or not the Sun and the two slowest rotators are included in the correlation. The 'time-average' approximation for the filling factor – graphs (a) and (b) – leads to larger values of α than the 'snapshot' approach, and for a given method of computing f, larger slopes are found from adopting an explicit dependence of $d\omega/dr$ on ω, *i.e.* using approximation [1]. These points will be discussed in detail in Montesinos *et al.* (1991).

As we can see in Figure 1, the extreme approximations taken both for the estimate of the internal angular velocity and the filling factor open a range of possibilities that must be explored further. More determinations of f and B for other active stars, including both rapid and slow rotators, are necessary to enlarge the sample of data available. This will allow the behaviour of these parameters with rotation to be found more accurately. With improved data constraints on the possible dependence of the internal angular velocity and on the mechanism by which the fields originate and rise up to the photosphere could be found.

References

Brown, T.M., Christensen-Dalsgaard, J., Dziembowski, W.A., Goode, P., Gough, D.O., Morrow, C.A.: 1989, *Astrophys. J.* **343**, 526

Durney, B.R.: 1985, *Astrophys. J.* **297**, 787

Durney, B.R., Robinson, R.D.: 1982, *Astrophys. J.* **253**, 290

Marcy, G.W., Basri, G.S.: 1989, *Astrophys. J.* **345**, 480

Montesinos, B., Fernández-Figueroa, M.J., Castro, E.: 1987, *Mon. Not. R. astr. Soc.* **229**, 627

Montesinos, B., Fernández-Villacañas, J.L., Jordan, C.: 1991, in preparation

Moss, D.: 1986, *Physics Reports*, Vol. **140**, No. 1

Robinson, R.D.: 1980, *Astrophys. J.* **239**, 961

Saar, S.H.: 1986, *Astrophys. J.* **324**, 441

Saar, S.H.: 1990, In *Solar Photosphere: Structure, Convection and Magnetic Fields, IAU Symp. No. 138*, ed. J.O. Stenflo, Kluwer, Dordretch, p. 427