Bull. Aust. Math. Soc. **106** (2022), 163–164
doi:10.1017/S0004972722000557

SOLUTIONS TO DIFFERENTIAL EQUATIONS VIA FIXED POINT APPROACHES: NEW MATHEMATICAL FOUNDATIONS AND APPLICATIONS

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(Received 20 April 2022; first published online 9 June 2022)

2020 *Mathematics subject classification*: primary 34A08; secondary 34B15, 34B60, 47H10.

Keywords and phrases: boundary value problem, Thomas–Fermi equation, Emden equation, Rus's contraction mapping theorem, fluid dynamics, fixed point technique, fractional differential equation.

The central aim of this thesis is to construct a fuller and firmer mathematical foundation for the solutions to various classes of nonlinear differential equations than that currently available in the literature. This includes boundary value problems (BVPs) that involve ordinary differential equations, and initial value problems (IVPs) for fractional differential equations. In particular, we establish new conditions that guarantee the existence, uniqueness and approximation of solutions to second-order BVPs, third-order BVPs and fourth-order BVPs for ordinary differential equations. The results enable us, in turn, to shed new light on problems from applied mathematics, engineering and physics, such as: the Emden and Thomas–Fermi equations; the bending of elastic beams through an application of our general theories; and laminar flow in channels with porous walls. We also ensure the existence, uniqueness and approximation of solutions to some IVPs for fractional differential equations. An understanding of the existence, uniqueness and approximation of solutions to these problems is fundamental from both pure and applied points of view. Our methods involve an analysis of nonlinear operators through fixed-point theory in new and interesting ways. Part of the novelty involves generating new conditions under which these operators are contractive, invariant and/or establishing new *a priori* bounds on potential solutions. As such, we draw on Banach fixed-point theorem, Schauder fixed-point theorem, Rus's contraction mapping theorem, and a continuation theorem due to A. Granas and its constructive version known as the continuation method for contractive maps. The ideas in this thesis break new ground at the intersection of pure and applied mathematics. Thus, this work will be of interest to those who

Thesis submitted to the University of New South Wales in June 2021; degree approved on 2 November 2021; supervisors Christopher Tisdell and Chris Goodrich.

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https://doi.org/10.1017/S0004972722000557 Published online by Cambridge University Press

are researching the theoretical aspects of differential equations, and those who are interested in better understanding their applications.

Some of the research from the thesis has been published in [1–6].

References

[1] S. S. Almuthaybiri and C. C. Tisdell, 'Global existence theory for fractional differential equations: new advances via continuation methods for contractive maps', *Analysis (Berlin)* **39**(4) (2019), 117–128.

[2] S. S. Almuthaybiri and C. C. Tisdell, 'Establishing existence and uniqueness of solutions to the boundary value problem involving a generalized Emden equation, embracing Thomas–Fermi-like theories', *J. Engrg. Math.* **124** (2020), 1–10.

[3] S. S. Almuthaybiri and C. C. Tisdell, 'Existence and uniqueness of solutions to third-order boundary value problems: analysis in closed and bounded sets', *Differ. Equ. Appl.* **12**(12) (2020), 291–312.

[4] S. S. Almuthaybiri and C. C. Tisdell, 'Sharper existence and uniqueness results for solutions to third-order boundary value problems', *Math. Model. Anal.* **25**(3) (2020), 409–420.

[5] S. S. Almuthaybiri and C. C. Tisdell, 'Sharper existence and uniqueness results for solutions to fourth-order boundary value problems and elastic beam analysis', *Open Math.* **18**(1) (2020), 1006–1024.

[6] S. S. Almuthaybiri and C. C. Tisdell, 'Uniqueness of solutions for a coupled system of nonlinear fractional differential equations via weighted norms', *Commun. Appl. Nonlinear Anal.* **28**(1) (2021), 65–76.

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https://doi.org/10.1017/S0004972722000557 Published online by Cambridge University Press