TRAPPING TIME OF RESONANT ORBITS IN PRESENCE OF POYNTING-ROBERTSON DRAG.

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ABSTRACT

We study numerically the competition between the Poynting-Robertson drag and the gravitational interaction of grains with Jupiter near orbital resonances. The computations are based on the plane elliptic restricted three body problem. Numerical investigations show that the grains always cross the resonance region without any oscillation, except in the special case where the grains were initially inside the resonance. Such grains are temporarily trapped, then due to the drag they are ejected out of the resonance. The trapping time of a particle turns out to be much more important in the 3/2 and 2/1 commensurabilities than in the others.
A numerical exploration of numerous orbits for different initial conditions and different sizes of grains has been performed. The trapping time appears to be closely connected to the size of the librator-type orbits regions; it increases with the initial eccentricity of the orbit, and is also proportional to the radius and the density of the particle.

1. INTRODUCTION

In a previous paper (R. Gonczi, Ch. Froeschlé and C. Froeschlé 1982) henceforward referred to as Paper I, we studied the effect of the Poynting-Robertson drag on grain orbits near the resonance 2/1 with Jupiter. Our calculations were based on the plane elliptic restricted three body problem. We found two kinds of orbital behaviour. If the grains were initially inside the resonance they were trapped temporarily, otherwise they crossed the resonance without any oscillation. We also explained the variation of the osculating elements of the orbits by Greenberg's and Schubart's theories. These results are summed up in Section 2 of the present paper. In Section 3, we calculate the size of the librator orbit region around the principal resonances, which we call the resonance width. We determine in Section 4, the trapping time of grains for the commensurabilities 3/2, 2/1, 3/1 and 5/2. We study

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the variation of this time both according to the initial values of the eccentricity e_o and semimajor axis a_o, and according to the radius and density of the grains. It is shown that the trapping time is closely connected to the width of the resonance.

2. REVIEW OF THE PREVIOUS RESULTS .

Let us briefly recall the equations of the planar three body elliptic restricted problem in the presence of a non gravitational force $\vec{f}$. We consider a particle of negligible mass moving in the gravitational field of the sun (mass m_1) and of Jupiter (mass m_2), which are both assumed to be point masses orbiting around their common centre of mass G. Denoting by r_1, r_2 and r the distances of the particle respectively to the sun, Jupiter and G, the equation of motion for the particle reads as :

$$\ddot{\vec{r}} = - Km_1 \frac{\vec{r}_1}{r_1^3} - Km_2 \frac{\vec{r}_2}{r_2^3} + \vec{f} \qquad (1)$$

where K is the gravitational constant.

As we consider the effect of the Poynting-Robertson drag (1903,1937), $\vec{f}$ reads as :

$$\vec{f} = - \frac{\alpha \vec{V}}{r^2}$$

where $\vec{V}$ is the particle velocity, and α a parameter depending on the radius s and the density ρ of the particle :

$$\alpha = \frac{2.5 \times 10^{11}}{s\rho} \ cm^2 \ s^{-1}$$

The numerical integration of Eq (1) is performed by a Burlich-Stoer (1966) method. After each step the elements of the osculating orbits are computed with respect to the sun. In Paper I, numerical explorations were done for the resonance 2/1, considering a particle of radius $s = 10^{-3}$ cm and density $\rho = 2g/cm3$ (i.e. $\alpha = 2 \times 10^{-5}$ (a.u.)^2/year). The initial osculating elements of the orbit were : $e_o = 0.14$ and $a_o = 3.36$ a.u. and we took as variable the critical argument σ defined by :

$$\sigma = \ell_J \ (p + q)/q - \ell - \tilde{\omega}$$

where ℓ and ℓ_J are respectively the particle and the Jovian mean longitude, p and q are integers which determine the commensurability $\frac{p+q}{p}$ and ω the pericenter longitude of the particle.

It was found that the orbits always cross the resonance region without any oscillation (Fig. 1a) except in the special case where the grain

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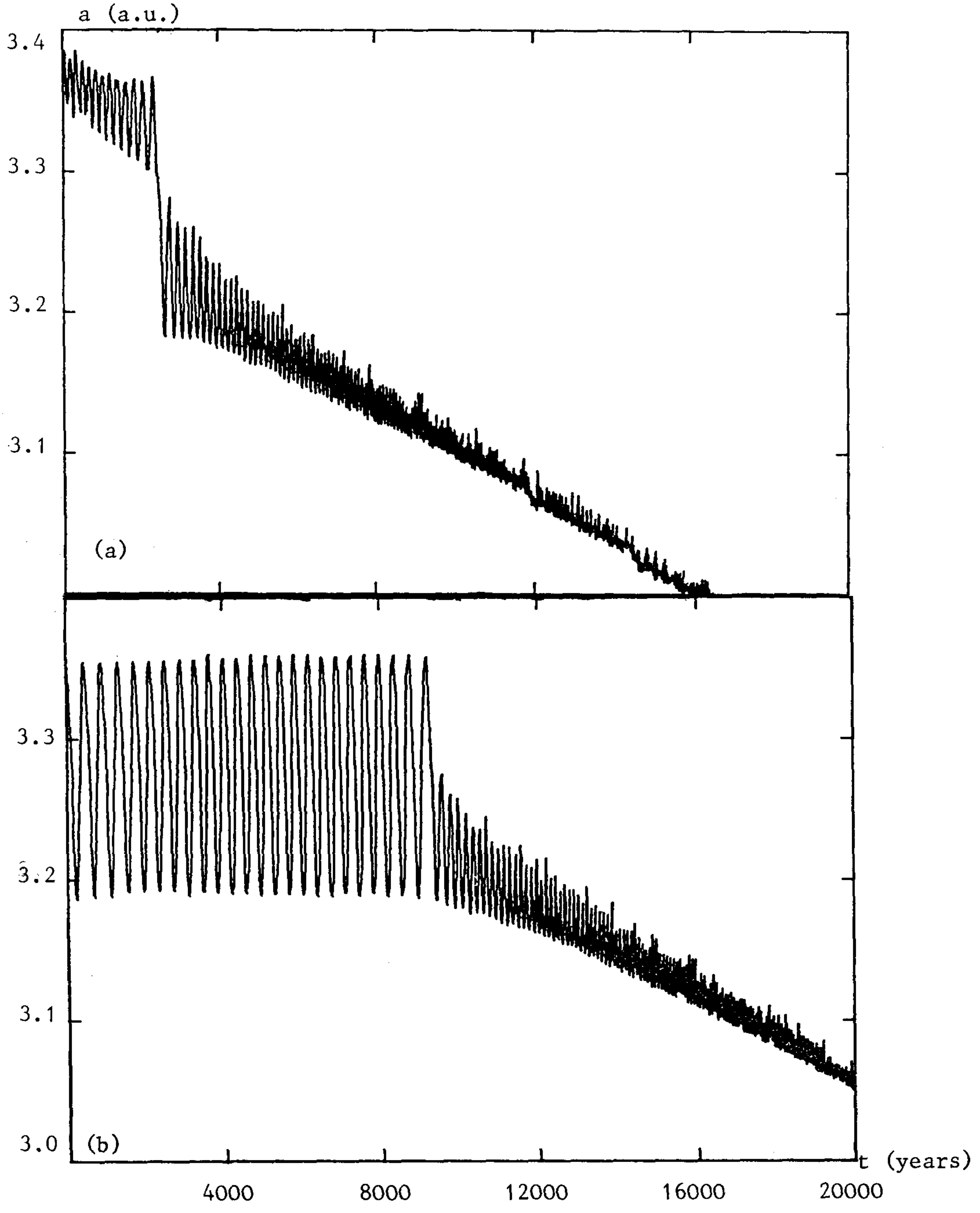

Fig. 1 : variation of the semimajor axis for an orbit near the 2/1 resonance with Jupiter; the initial conditions are :

a) a_o = 3.36 a.u.; e_o = 0.14; $\sigma_o = -2\pi/3$; $\alpha = 2\mathrm{x}10^{-5}$ $(a.u)^2$/yr

b) a_o = 3.36 a.u.; e_o = 0.14; $\sigma_o = -2\pi/10$; $\alpha = 2\mathrm{x}10^{-5}$ $(a.u.)^2$/yr

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was initially inside the resonance. In that case (Fig. 1b) the particle remains inside the resonance for a long time, until the drag ejects it toward the sun. These two cases have been explained using Schubart's theory. The orbits of the second type (Fig. 1b) are librators, which become circulators after ejection of the particle from the resonance.

3. DETERMINATION OF THE WIDTH OF THE RESONANCES.

As we have seen before that only librators can be trapped into a resonance, in order to calculate the trapping time of such a particle, we have first to estimate the size of the libration region around the commensurabilities.

First a systematic exploration of the resonance region is performed : on the (a,e) diagram we construct a lattice defined by 8 values of "e" regularly spaced between 0.0 and 0.4 and 50 values of "a" regularly spaced between two values a_{min} and a_{max} surrounding the commensurability. Each point of this lattice represents the initial conditions a_o and e_o of one orbit; moreover, the initial critical argument σ_o takes the value σ^* corresponding to the most favourable geometrical configuration for getting a librator : $q\sigma^* = 0°$ in the 2/1, 3/2, 5/2 commensurabilities, and $q\sigma^* = 180°$ in the 3/1 case (Schubart, 1964). Each orbit is numerically integrated until $q\sigma$ crosses 6 times either 0° or 180°.

It is then considered as :

- a circulator if $q\sigma$ passes alternatively through 0° and 180°;
- a librator if $q\sigma$ only crosses the value $q\sigma^*$ and never its opposite $q\sigma^* + \Pi$;
- an alternator (C. Froeschlé and H. Scholl, 1977) in all the other cases.

The results for the resonances 3/2, 2/1, 3/1 and 5/2 are given on Figs.2. We see that the resonances 3/1 and 5/2 have very few librators, which is not surprising since they are known as gaps in the distribution of asteroids. On the other hand, the resonance 3/2 and 2/1 show a large librator region. This is again in agreement with the observations for the 3/2 resonance, but not for the 2/1 one.

The boundaries of the libration region turn out to be well defined, except in the 3/2 case where many alternators and hyperbolae appear, probably due to the influence of the closeness of Jupiter.

Figs. 2a, 2b, 2c also clearly show that for all the commensurabilities studied here, the resonance width increases with the orbit eccentricity; in particular, there are very few librators with $e_o < 0.1$ in the 3/2 resonance, which is confirmed by the observed asteroids in the Hilda group.

To complete this study and eventually eliminate the incidence of the

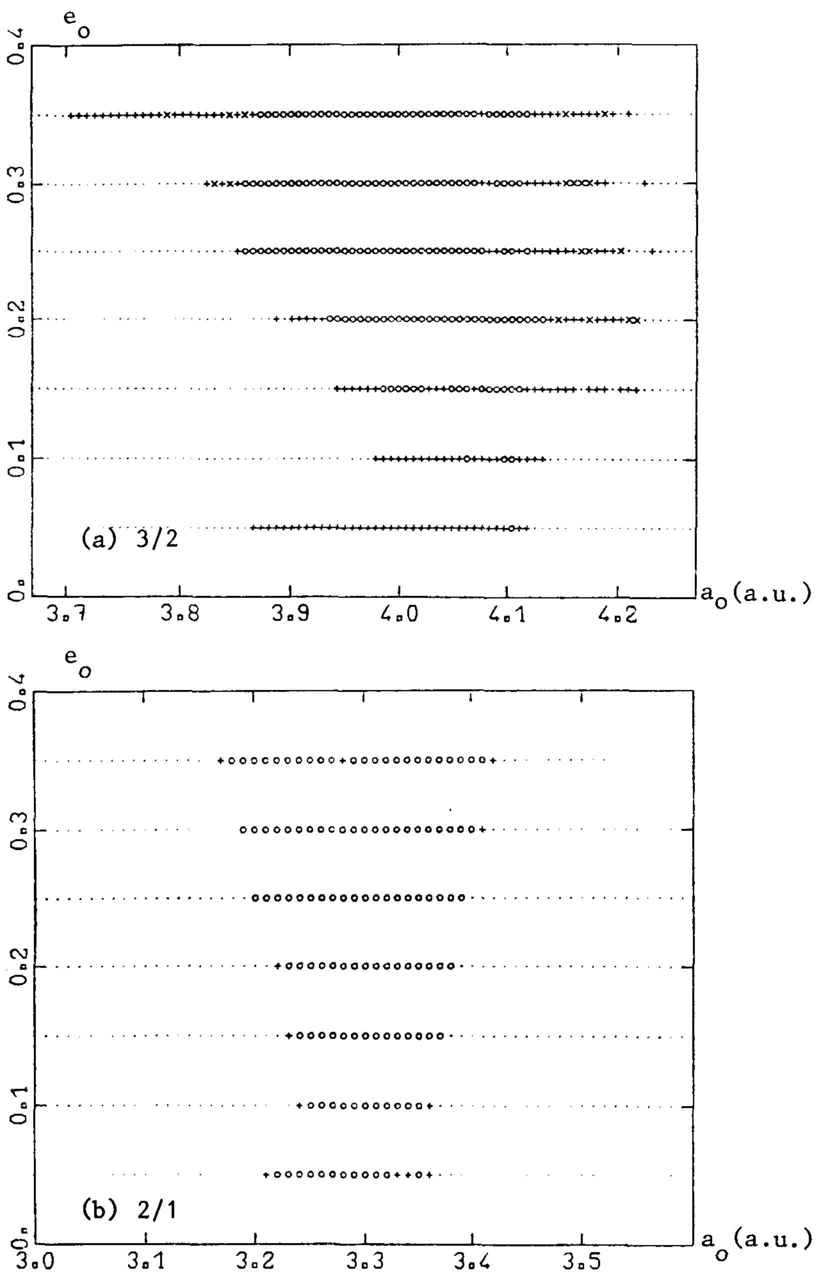

Fig. 2 : repartition of librators and circulators around resonance regions.
Initial conditions of each orbit : a_o, e_o and $\sigma_o = \sigma^*$ is the most favourable value for getting a librator.
∘ librator; . circulator; + alternator; x orbit suffering a close approach.
a) orbits near the 3/2 commensurability; b) orbits near the 2/1 commensurability; c) orbits near the 3/1 and 5/2 commensurabilities

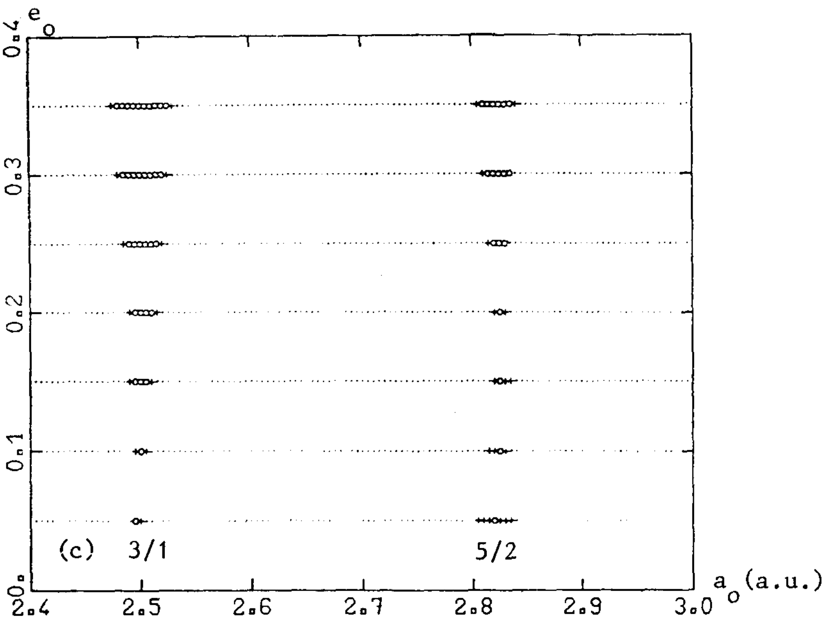

Fig. 2 - continued.

particular choice of initial condition $\sigma_o = \sigma^*$, we now investigate by a Monte-Carlo method, orbits whose initial conditions a_o, e_o and σ_o are randomly chosen within the extremes :

$$a_{min} < a_o < a_{max} \; ; \quad 0. < e_o < 1. \; ; \quad -\Pi < \sigma_o < \Pi$$

The integration and classification of each orbit into circulator, librator and alternator are performed as before and plotted on Figs. 3a, 3b, 3c.

For each resonance we have also calculated (Table 1) the percentage of each type of orbits obtained with both estimations. The qualitative comparison between Figs. 2 and 3, as well as the quantitative results of Table 1 do not show significant differences between the two numerical experiments. Indeed the peculiar case $\sigma_o = \sigma^*$ is a good representative of the global picture of the phase space.

4. DETERMINATION AND VARIATION OF THE TRAPPING TIME.

For a given resonance we consider orbits chosen among the librators determined in Section 3. We introduce now the Poynting-Robertson drag $\vec{f}$: we know that the particle will not librate indefinitely but

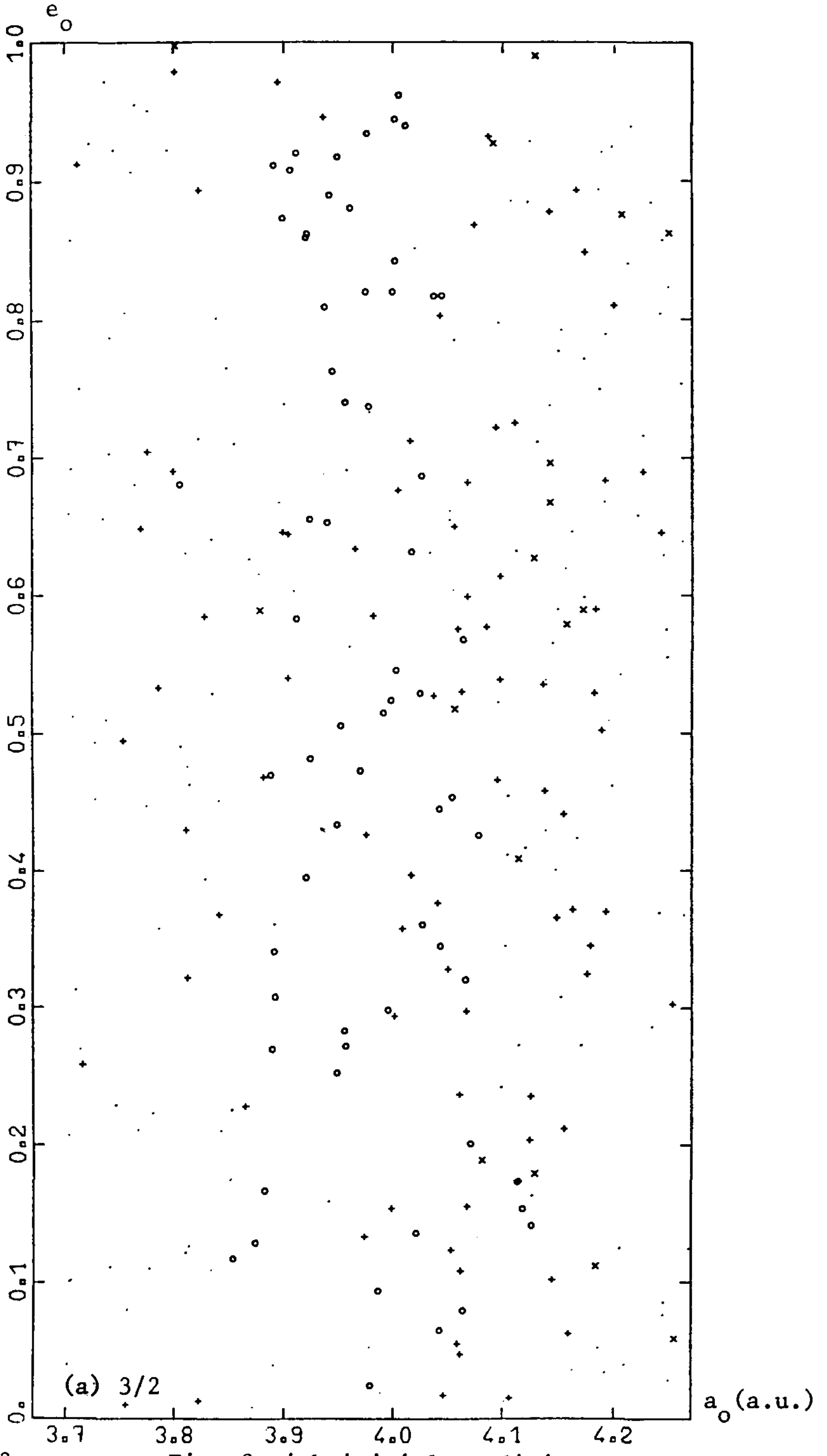

Fig. 3 : same as Fig. 2 with initial conditions a_o, e_o, σ_o chosen at random.

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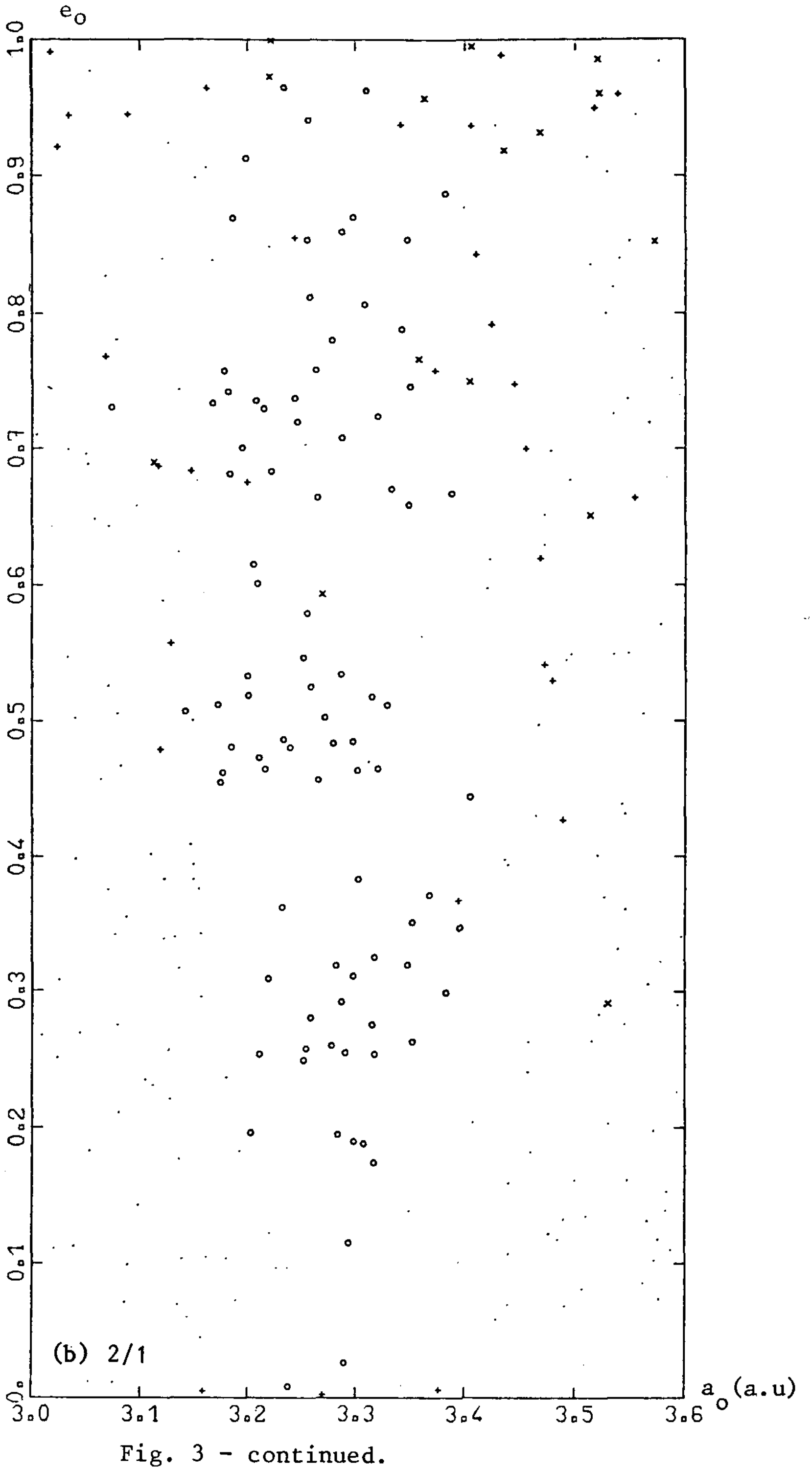

Fig. 3 - continued.

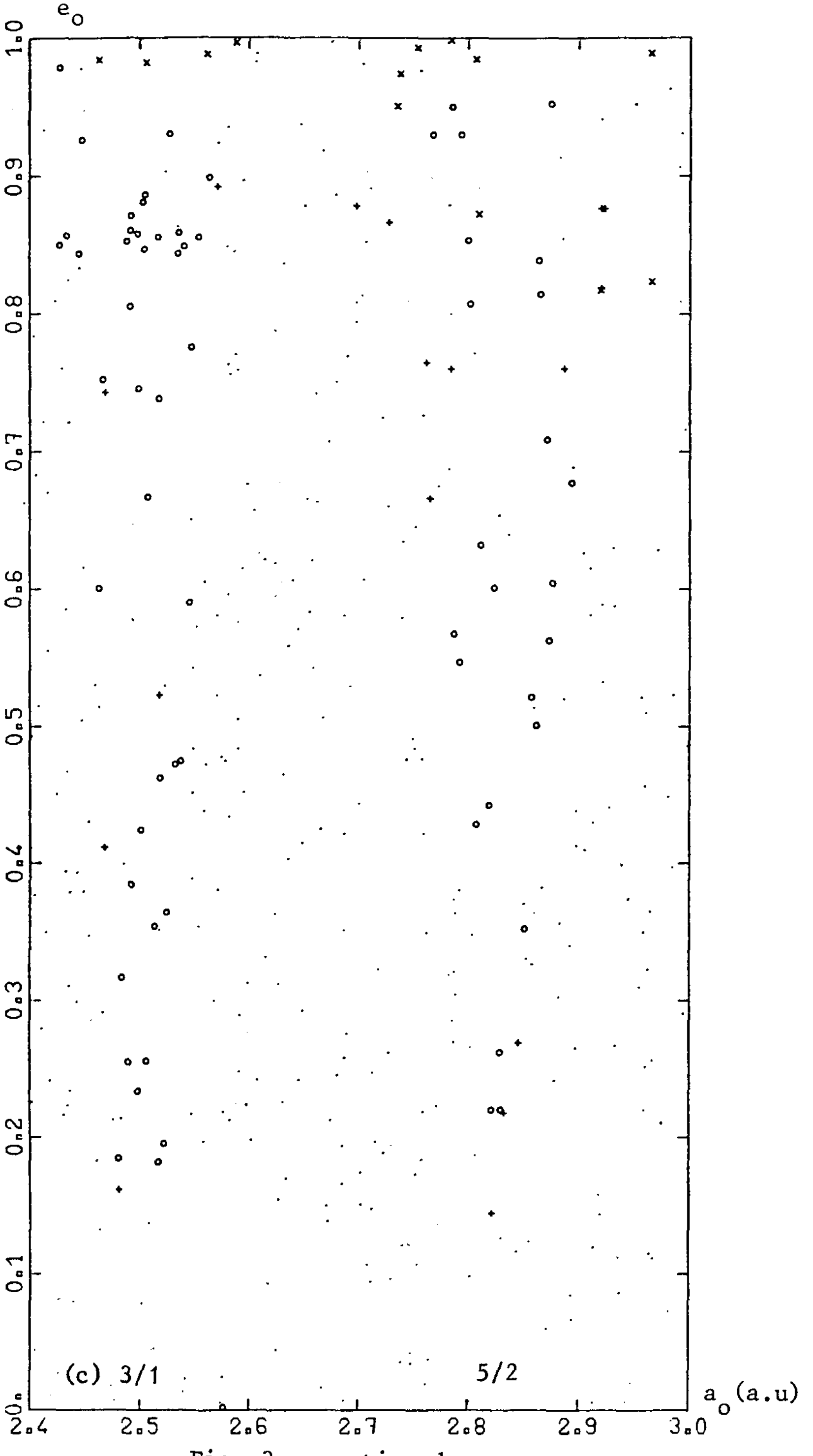

Fig. 3 - continued.

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Com.	Initial conditions	Librators	Circulators	Alternators	Close approach	Total
3/2	case 1	25.8	45.0	26.0	3.2	100.
	case 2	21.3	44.4	30.6	3.7	100.
2/1	case 1	28.5	68.5	2.9	0.0	100.
	case 2	23.4	72.6	3.2	0.8	100.
3/1	case 1	8.2	88.9	2.9	0.	100.
	case 2	10.6	89.4	0.0	0.	100.
5/2	case 1	5.5	89.0	5.5	0.	100.
	case 2	5.1	91.0	3.8	0.	100.

Table 1 : Percentages of each type of orbit obtained around the commensurabilities 3/2, 2/1, 3/1 and 5/2.

case 1 : systematic exploration for the initial conditions e_o and a_o, σ_o is equal to σ^* (see Figs. 2)

case 2 : e_o, a_o and σ_o are chosen at random (see Figs. 3)

will be ejected out of the resonance and become a circulator. The time an orbit is trapped inside the resonance (trapping time) depends on several parameters. In this section we investigate the variation of the trapping time with p, q, a_o, e_o and α.

For each orbit, Eq. (1) is numerically integrated and we note the first time $t = t_1$ for which $q\sigma$ takes the value $q\sigma^*$. In order to be sure, that the orbit is a circulator we note also the time $t = t_n$ of the n^{th} crossing of $q\sigma$ through $q\sigma^*$. We choose n = 6, and assume, that the trapping time t_T lies between t_1 and t_6, which are respectively the lower and the upper bounds of t_T.

4.1. Variation of t_T with a_o and e_o.

Let α and e_o be fixed, we calculate the trapping time for orbits close to the commensurabilities : 3/2, 2/1, 3/1 and 5/2, and for different values of the initial semimajor axis a_o.

On Fig. 4, the time t_T is plotted as a function of $d = a_o - a_{res}$ (where a_{res} is the value corresponding to the exact commensurability), for the fixed values $e_o = 0.14$ and $\alpha = 2 \times 10^{-5}$ (a.u.)^2/year. We notice a great difference in magnitude of the time t_6 between the 3/2 and 2/1 resonance and the others, as already observed in the previous

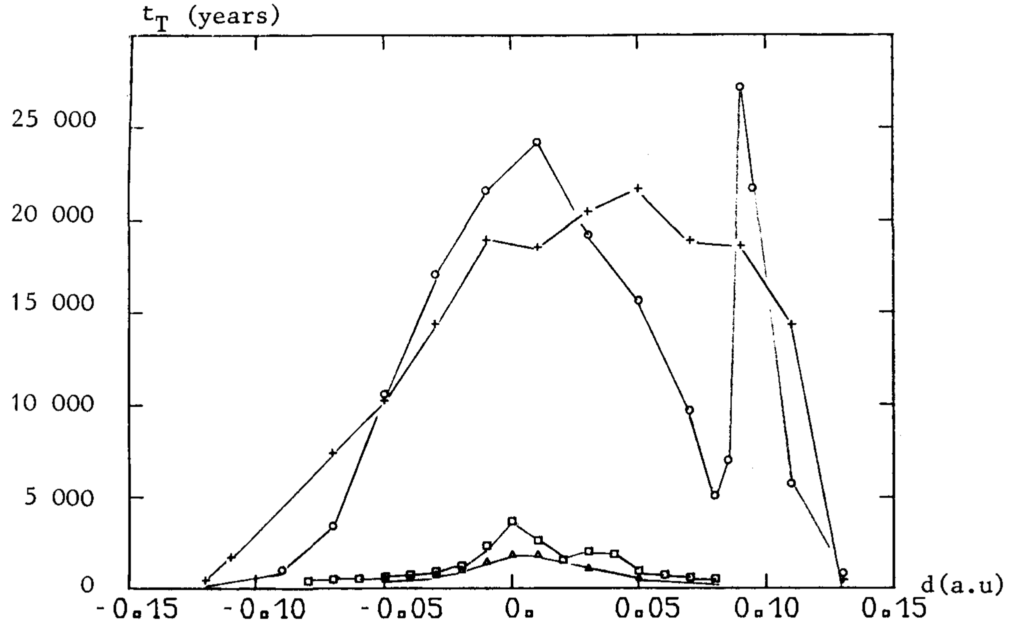

Fig. 4 : trapping time as a function of $d = a_o - a_{res}$ for the commensurabilities :
3/2 (+); 2/1 (o); 3/1 (□); 5/2 (△)
α, e_o and σ_o are fixed: $\alpha = 2\times10^{-5}$ (a.u.)^2/yr; $e_o = 0.25$; $\sigma_o = \sigma^*$

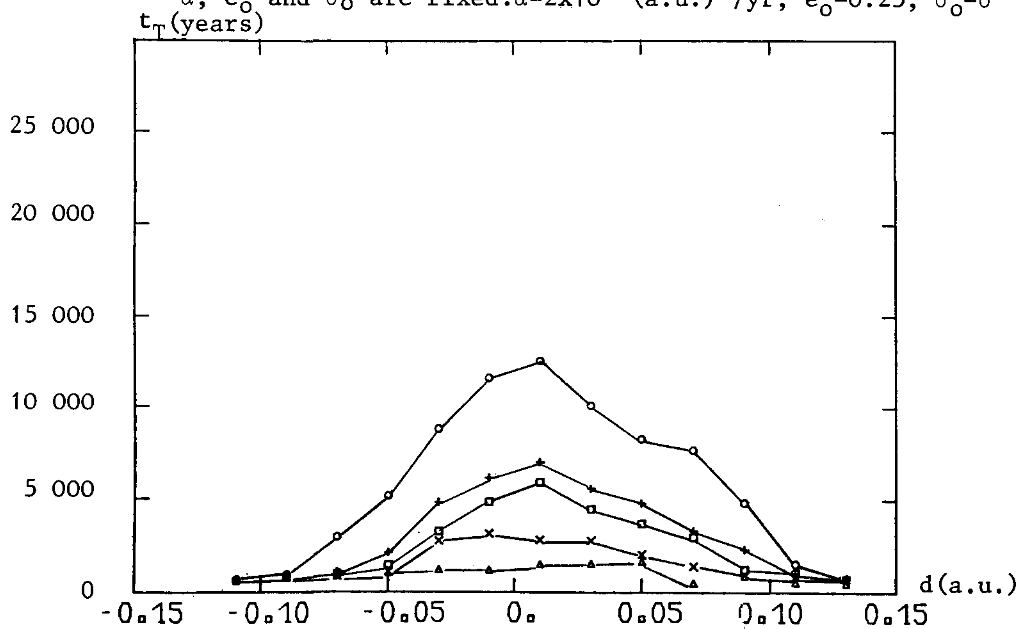

Fig. 5 : trapping time as a function of $d = a_o - a_{res}$ for the initial eccentricities :
e_o = 0.25 (o); e_o = 0.17 (+); e_o = 0.14(□); e_o = 0.07 (×); e_o = 0.0 (△)
α, p and q are fixed : $\alpha = 4\times10^{-5}$ (a.u.)^2/yr, p = 1; q = 1.

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section for the width of these resonances. Indeed in the 3/1 and 5/2 resonance which have a very small width, the trapping time is very short, while for the other two which have a comparably large width, the trapping time is longer and of the same order of magnitude.

The quasi symmetry of the curves is a consequence of the well known fact that the semimajor axis of librators oscillates from one edge of the resonance to the other. The centre of symmetry is slightly different from the exact resonance d = 0. It would rather correspond to the centre of the librator region in the Schubart's plot.

As $|d|$ increases, i.e. the initial value a_o moves away from the exact resonance, the time t_6 obviously decreases, to become almost zero near the outer edges of the resonances. We have no explanation for the secondary peak which appears in the resonance 2/1, we can only say that this peak disappears for other values of α (see Fig. 5).

We have done several similar numerical explorations for different values of e_o, and different resonances. On Fig. 5, we have plotted for the 2/1 commensurability the trapping time t_6 versus $d = a_o - a_{res}$ for five initial values of e_o. Again this is closely related to the width of the librator region (Figs. 2) which is larger for higher eccentricities. While for both the two body problem (Wyatt and Whipple (1950)) and the restricted three body problem (out of the resonance), the falling time of a particle into the sun decreases when e_o increases, we have here an opposite behaviour.

4.2. Variation of the trapping time with α.

It is known that for the two body problem, the Poynting-Robertson drag is more efficient for small particlesthan for larger ones, and that the time of fall into the sun is a linearly increasing function of the product $s\rho$. Here we study the trapping time t_T as a function of α between 10^{-6} and 10^{-4} $(a.u)^2/yr$ (i.e. between 10^{13} and 10^{15} cm^2/s).

As $\alpha = \frac{2.5\times10^{11}}{s\rho}$ cm^2/s, these values correspond, for a particle of density $\rho \cong 2 g/cm^3$, to a radius from 10^{-4} cm to 10^{-2} cm.

Several orbits have been computed. The results obtained for one orbit near the 3/2 resonance and two orbits near the 2/1 one are shown on Figs. 6a, 6b, 6c. The trapping time t_T (or more precisely the lower and upper bound t_1 and t_6) plotted versus α in a log-log scale shows a good linear behaviour, implying that the trapping time is proportional to the product $s\rho$.

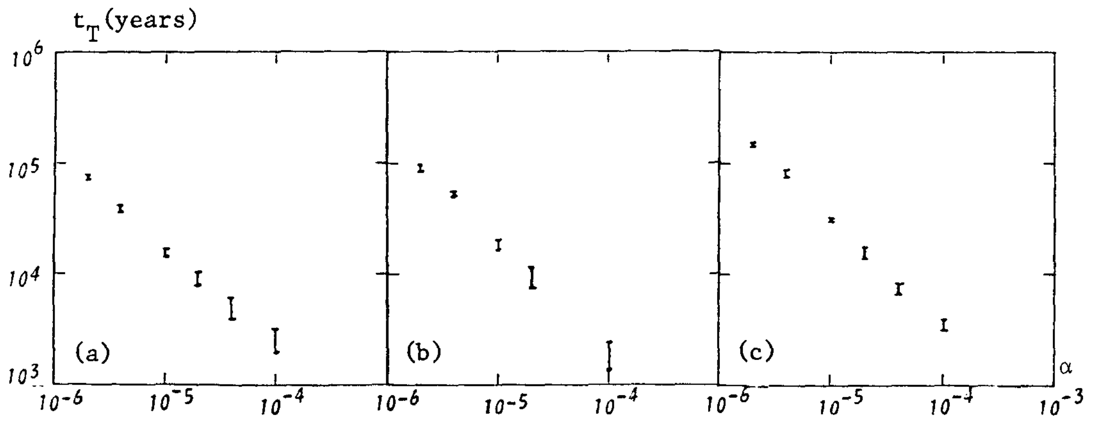

Fig. 6 : trapping time as a function of α for initial conditions :

a) a_o = 3.98 a.u. ; e_o = 0.20 ; σ_o = 0.; p = 2 ; q = 1.

b) a_o = 3.29 a.u. ; e_o = 0.14 ; σ_o = 0.; p = 1 ; q = 1.

c) a_o = 3.29 a.u. ; e_o = 0.20 ; σ_o = 0.; p = 1 ; q = 1.

5. CONCLUSION

We have performed a systematic exploration of the trapping time for grains librating initially inside resonances. This time appears to be closely connected to the size of the librator type orbit region. It has been found to increase with the initial eccentricity of the orbit and is proportional to the radius and to the density of the particle.

Except for the 2/1 commensurability, the results are in agreement with either the gaps or the concentrations of observed asteroids, not only qualitatively but even quantitatively : for the 3/2 (Hilda group) resonance, the agreement with the observed eccentricities is quite good.

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