

**SESSION 9**

**CONVECTIONS AND OSCILLATIONS  
IN ACTIVE REGIONS**

## SUNSPOT SEISMOLOGY: THE INTERACTION OF SOLAR $p$ -MODES WITH A SUNSPOT

JOHN H. THOMAS

Department of Mechanical Engineering, Department of Physics and Astronomy, and C. E. K. Mees Observatory, University of Rochester, Rochester, NY 14627

**ABSTRACT** Progress in sunspot seismology is briefly reviewed. New observations of the absorption of  $p$ -modes by a sunspot (Bogdan, Brown, Lites, and Thomas 1993) show that the absorption coefficient (averaged over frequency and azimuthal order) increases with horizontal wavenumber to a maximum near  $k = 0.9 \text{ Mm}^{-1}$  and then decreases monotonically to zero at about  $k = 1.5 \text{ Mm}^{-1}$ . The absorption along individual  $p$ -mode ridges shows a sinusoidal-like modulation, which may be an important feature for comparison with theory. Results for the quiet Sun show no net absorption, but do show apparent emission along  $p_1$  over the same range of  $k$  where the sunspot shows its greatest absorption. This behavior, and the overall decrease in absorption at higher wavenumbers, can be explained as an effect of the decreasing lifetime of  $p$ -modes with increasing wavenumber  $k$  (increasing mode degree  $l$ ).

### INTRODUCTION

Sunspot seismology is the branch of helioseismology in which we attempt to deduce the structure of a sunspot below the solar surface by comparing observations and theoretical models of the interaction between a sunspot and the resonant acoustic oscillations ( $p$ -modes) of the Sun. A recent, comprehensive review of sunspot seismology has been given by Bogdan (1992).

Although sunspots have been known to be a magnetically dominated phenomenon since 1908, when Hale discovered the intense magnetic field in a sunspot, we still do not have a clear picture of the configuration of this magnetic field below the solar surface. Sunspot seismology offers the hope of settling the current controversy as to whether a sunspot's magnetic flux is fairly uniformly distributed over the cross-section of a large, "monolithic" flux tube, or instead is clumped into a "cluster" of individual flux tubes separated by field-free gas. (For a discussion of this and other issues in the

physics of sunspots see Thomas and Weiss 1992.) Achieving a satisfactory understanding of sunspot structure has broad implications in astrophysics because a sunspot is perhaps the optimal proving ground for the theory of ideal magnetohydrodynamics, upon which rests much of our understanding of other magnetic fields in the universe.

The concept of sunspot seismology was introduced a decade ago by Thomas, Cram, and Nye (1992). We compared spatially unresolved observations of oscillations in a sunspot umbra with a simple theoretical model of the interaction of the sunspot with the  $p$ -modes in the surrounding atmosphere. The temporal power spectrum of the umbral oscillations had a number of reproducible peaks in the five-minute band which were interpreted as the response of the sunspot to specific  $p$ -modes with energy concentrated at different depths. The theoretical model then gave a measure of the radius of the sunspot flux tube (assuming a monolithic tube) at each of these depths. The results showed the flux tube radius decreasing slowly with depth in close agreement with the magnetostatic model of Deinzer (1965). However, because of the lack of spatial resolution, the identification of specific  $p$ -modes was uncertain and these results should be viewed only as suggestive and illustrative of the method.

The first spatially resolved measurements in sunspot seismology were carried out by Abdelatif, Lites, and Thomas (1986). They compared the power spectrum of oscillations in a sunspot umbra with the power spectrum of oscillations in an equivalent area in the quiet Sun outside the sunspot and found the following: (i) the dominant 5-minute  $p$ -mode oscillations have reduced amplitude inside the umbra, with an rms velocity about half that in the surrounding quiet photosphere; (ii) there is a general shift of oscillatory power to longer horizontal wavelengths in the umbra; and (iii) the umbra acts as a selective filter by transmitting certain frequencies of the incident  $p$ -modes in preference to other frequencies. These results were given a simple theoretical interpretation by Abdelatif and Thomas (1987) on the basis of a model consisting of a vertical cylindrical magnetic flux tube embedded in a field-free, unstratified gas, with resonant  $p$ -modes produced by upper and lower reflecting horizontal boundaries. Horizontally propagating  $p$ -modes incident upon the flux tube are, in general, partially transmitted into the flux tube and partially reflected. The transmitted wave is a magnetoacoustic wave having greater horizontal phase speed, and hence longer horizontal wavelength, than the incident wave, which explains the shift of power to longer wavelengths in the umbra. The transmission coefficient as a function of horizontal wavelength exhibits peaks and valleys, because of the preferred transmission of waves for which an integral number of wavelengths fit across the diameter of the flux tube; this explains the selective filtering.

A different approach to sunspot seismology, first proposed by Bogdan and Zweibel (1987) on theoretical grounds, is to treat the interaction

of the sunspot with the  $p$ -modes as a classical scattering problem, using measurements of oscillations only in an annular region of quiet Sun surrounding the sunspot. This approach was first employed in observational studies by Braun, Duvall, and LaBonte (1987, 1988, hereafter BDL; see also Braun *et al.* 1992), who made the surprising and important discovery that sunspots absorb an appreciable fraction (up to 50 percent at some wavelengths) of the energy of the incident  $p$ -modes. Their discovery has stimulated a flurry of theoretical effort to identify the physical mechanism responsible for this acoustic absorption.

In the remainder of this paper I shall present some very recent observational results using the scattering method, which I obtained in collaboration with Thomas Bogdan, Timothy Brown, and Bruce Lites of the High Altitude Observatory (Bogdan, Brown, Lites, and Thomas 1993). Our results generally confirm the absorption of  $p$ -modes by a sunspot discovered by BDL, but with some significant differences. I will also discuss a simple but elegant interpretation of these results, due to Bogdan, in terms of the finite lifetimes of the  $p$ -modes, and the implications that this interpretation has for future measurements in sunspot seismology.

## NEW OBSERVATIONS

Our observations (Bogdan *et al.* 1993) were made with the vacuum tower telescope at the National Solar Observatory (Sacramento Peak) using the universal birefringent filter and multi-diode array. We used a technique developed by November (1984, 1991) in which a polarizing beam splitter is introduced, producing beams with two bandpasses, each  $1/4 \text{ \AA}$  in width and separated by  $1/4 \text{ \AA}$ , allowing us to record simultaneous images in the red and blue wings of a spectral line. The difference between these two images gives us a sensitive measure of the line-of-sight Doppler velocity. A short exposure time (20 ms) freezes much of the seeing motions, and a rapid cadence (one image every 6 s) builds up a high signal to noise ratio. We produce a time series of Doppler velocity maps of a  $425$  by  $425$  arcsecond area of the solar surface that are as free of instrumental and seeing effects as we could achieve.

We use a spherical harmonic decomposition of the  $p$ -modes instead of the Fourier-Bessel (cylindrical) decomposition used by BDL. Consider a spherical coordinate system  $(r, \theta, \phi)$  with its origin at the center of the Sun and with its north pole placed at the center of a sunspot. We interpolate on the doppler velocity maps to obtain a time sequence of velocity  $v(\theta, \phi, t)$  as a function of position on a discrete grid in  $\theta$  and  $\phi$  at discrete times  $t$ . Now consider this velocity over an annular region  $\theta_1 < \theta < \theta_2$  centered on the sunspot with  $\theta_1$  chosen just large enough to exclude the sunspot from the annulus. The velocity can be represented as a superposition of inward and

outward propagating waves on this annulus in terms of solutions of the associated Legendre equation, in the form

$$v(\theta, \phi, t) = \sum_l \sum_m \sum_n V(l, m, n) [P_l^m(\cos\theta) - \frac{2i}{\pi} Q_l^m(\cos\theta)] \exp[i(m\phi + 2\pi\nu_n t)] + \text{c.c.}$$

where c.c. denotes the complex conjugate. Values of the separation constant  $l$  are chosen so that the wavefunctions are mutually orthogonal on the annulus. The azimuthal index  $m$  takes on only integer values (positive and negative), and the index  $n$  denotes the discrete frequencies  $\nu_n$  measured by the discrete time series. By considering the asymptotic form of the associated Legendre functions for large  $l$ , it is found that positive values of  $n$  correspond to waves propagating radially inward across the annulus and negative values of  $n$  correspond to outward propagating waves. Hence, we can define an absorption coefficient

$$\alpha(l, m, n) = \frac{|V(l, m, |n|)|^2 - |V(l, m, -|n|)|^2}{|V(l, m, |n|)|^2},$$

which gives the fractional absorption of wave energy as a function of degree  $l$ , azimuthal order  $m$ , and frequency  $\nu_n$ . One can then look at the absorption in less detail and, for example, sum the absorption coefficient  $\alpha(l, m, n)$  over several azimuthal orders and look at absorption as a function of degree  $l$  and frequency  $\nu_n$ .

In order to compare our observations with those of BDL, we compute an overall absorption coefficient, averaged over azimuthal orders  $m$  from  $-5$  to  $+5$  and frequencies from  $1.74$  to  $5.04$  mHz, in the same way as BDL. Figure 1 shows a plot of this average absorption coefficient for our best data set, for a sunspot observed very near disk center on 19 March 1989. The absorption increases monotonically with increasing horizontal wavenumber up to a peak value of about  $\alpha = 0.35$  at  $k = 0.9 \text{ Mm}^{-1}$  and then decreases monotonically to zero at about  $k = 1.5 \text{ Mm}^{-1}$ . Also shown in Figure 1 are the results of BDL for their 18 January 1983 sunspot. The agreement between the two sets of results up to  $k = 0.9 \text{ Mm}^{-1}$  is remarkably good, particularly considering that they represent different sunspots and different techniques of measurement and analysis. However, for wavenumbers in the range  $1.0 < k < 1.5 \text{ Mm}^{-1}$  the results disagree; we find the absorption decreasing monotonically to zero over this range, whereas BDL find it to remain roughly constant. This difference could possibly be due to different behavior of the two sunspots, but we feel that it is more likely due to our better spatial resolution and signal-to-noise ratio. Besides, in the next section

I will present a simple physical argument for why one should expect the measured absorption to drop monotonically to zero over this range of wavenumbers, irrespective of the details of the absorption mechanism.

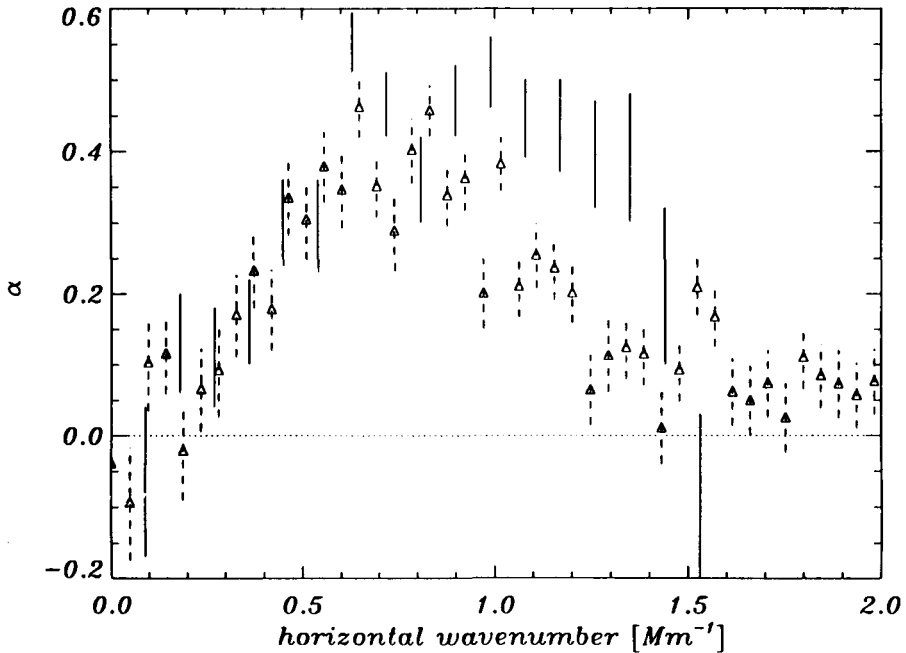


Fig. 1. The absorption coefficient  $\alpha$  (averaged over frequency and azimuthal wavenumber) for the absorption of solar  $p$ -modes by a sunspot, as a function of mode degree  $l$ . Triangles: results of Bogdan *et al.* 1992. Vertical lines: results of Braun *et al.* 1988.

The high spatial resolution and high signal-to-noise ratio of our data allow us to measure the variation of the absorption along individual  $p$ -mode ridges for radial orders up to  $n = 5$ . We find that the overall absorption is greatest for  $p_1$  and falls off with increasing  $n$ . For an individual radial order, the absorption shows a sinusoidal-like variation with increasing degree  $l$ . For example, the upper panel in Figure 2 shows the variation of the sunspot's absorption coefficient with degree  $l$  for the  $p_1$  and  $p_2$  modes. (The four diamonds at each  $l$  correspond to four different ways of defining the limits of the ridge of power associated with the  $p_1$  mode; they correspond to two different choices for the width of the ridge, with or without subtracting a background, "continuum" level of power, in computing the absorption coefficient.) Note that the absorption coefficient follows the general trend of the overall absorption coefficient shown in Figure 1.

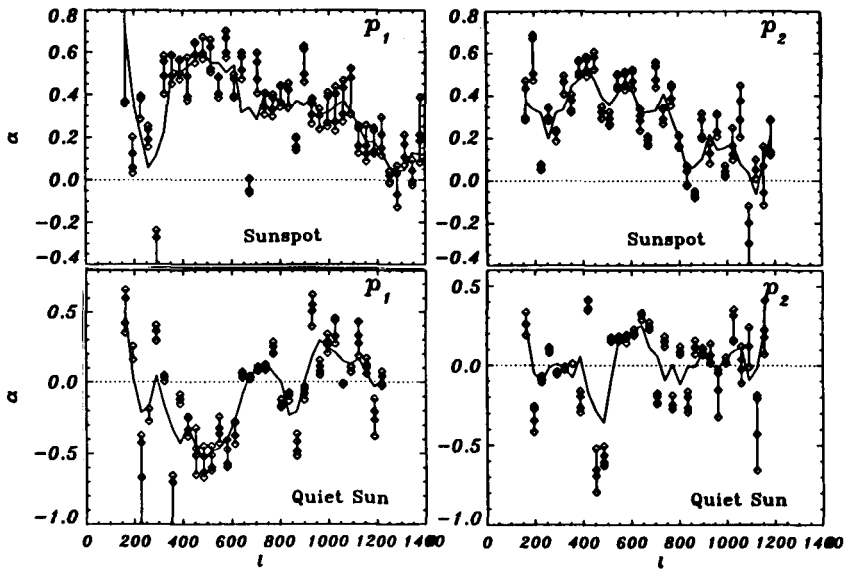


Fig. 2. Absorption coefficient  $\alpha$  as a function of mode degree  $l$  along the  $p_1$  and  $p_2$  ridges, for a sunspot (upper panels) and for the quiet Sun (lower panels). (From Bogdan *et al.* 1993).

The details of the modulation of the absorption along individual  $p$ -mode ridges may well turn out to be a very important key to sunspot seismology, to help identify the physical mechanism for the absorption and to determine the configuration of a sunspot's subsurface magnetic field. I will not attempt to review the many different physical mechanisms that have been proposed for the absorption of  $p$ -modes by a sunspot; this has been done recently by Bogdan (1992). I would like to point out, however, that the mechanism proposed by Spruit (1991) and Spruit and Bogdan (1992) does indeed show a sinusoidal-like variation of absorption with horizontal wavenumber along individual  $p$ -mode ridges. Their mechanism is based on the coupling of  $p$ -modes to slow magnetoacoustic modes ( $s$ -modes) that are guided along a sunspot's magnetic flux tube (or its separate tubes in a cluster model) and propagate downward out of the  $p$ -mode cavity, thus providing a "leak" of wave energy and hence an apparent absorption. Their theoretical model is too idealized to justify a detailed comparison with our observational results, but the qualitative agreement is suggestive and it would certainly be of interest to develop more realistic models for the coupling of  $p$ -modes to the leaky  $s$ -modes in a sunspot.

## LOCAL VERSUS GLOBAL MODES

The phenomenon of quiet-Sun emission at values of  $l$  where the sunspot shows its maximum absorption, as well as the overall dependence of the sunspot's acoustic absorption on degree  $l$ , can be broadly understood as a simple consequence of the finite lifetime (or, equivalently, the finite range) of individual  $p$ -modes (Bogdan *et al.* 1993, based on an idea of Bogdan). The lifetime of a solar  $p$ -mode may be estimated from the measured full width at half maximum (FWHM) of its power ridge in an  $l$ - $\nu$  diagram; the lifetime decreases with increasing degree  $l$ . We can estimate the *range*  $r_l(\nu)$  of a particular  $p$ -mode, that is, the distance across the solar surface that the mode propagates during its lifetime, as

$$r_l(\nu) = \frac{2\pi R_o}{\sqrt{l(l+1)}} \frac{\nu}{\text{FWHM}} = \lambda Q ,$$

where  $\lambda$  is the horizontal wavelength and  $Q = \nu/\text{FWHM}$  is the quality of the mode. Then we can distinguish between *global* modes which have range  $r_l(\nu)$  much greater than the outer radius of the annulus over which we measure the scattering properties (425 arcseconds in our case), and *local* modes with range  $r_l(\nu)$  much less than the outer radius of this annulus. Global modes will show absorption by the sunspot, but local modes will not because they do not cross the annulus to be absorbed by the sunspot. In effect, the local modes in the annulus do not "feel" the presence of the sunspot and in a statistically steady state there will be a balance of ingoing and outgoing power for these modes. Extrapolating from the measurements of the FWHM of Korzennik (1990), we find that the mode  $p_1$  is essentially global for  $l < 300$  and local for  $l > 900$ . The decrease in absorption that we measure over this range of  $l$  (Fig. 2) is simply due to the fact that the mode changes its character from global to local. Similar results hold for modes of other radial orders  $n$ , and the decrease in the overall absorption over the range  $1.0 < k < 1.5 \text{ Mm}^{-1}$  (Fig. 1) is simply a consequence of the transition from global to local character of the various  $p$ -modes over this range. This decrease in measured absorption is *independent of the physical mechanism for the absorption*; it is just a consequence of the method of measurement. Sunspots may well absorb acoustic energy at higher wavenumbers ( $k \geq 1.5 \text{ Mm}^{-1}$ ) but this absorption is undetected by the scattering method.

The distinction between global and local modes also helps us to understand the apparent emission in the quiet-Sun data for values of  $l$  where the sunspot shows its greatest absorption (e.g., for  $p_1$  in the range  $200 < l < 600$ , Fig. 2). In a statistically steady state the emission and absorption of  $p$ -modes will be in detailed balance, and there will be net emission in quiet-Sun regions to compensate for the absorption by sunspots outside the annulus but within mode range of the annulus. For our quiet-Sun data, then, we would



expect to measure net emission of global modes, which sense the sunspot outside the annulus, at wavelengths corresponding to strong absorption by the sunspot. However, we would expect to measure no emission for local modes in the annulus, which do not interact with sunspots outside the annulus.

## CONCLUSIONS

The variation of the absorption coefficient along individual  $p$ -mode ridges shown in our observational results (Bogdan *et al.* 1993) is a promising diagnostic for sunspot seismology. More precise measurements of this variation, along with more realistic theoretical models of the absorption for comparison, are now called for.

On the other hand, we have seen that the scattering approach to sunspot seismology has rather severe limitations for modes of high wavenumber  $k$ , which are essentially "local" in character. The "absorption" measured by the scattering method will drop to zero with increasing wavenumber regardless of the physical mechanism responsible for the true absorption, simply as a consequence of the increasingly local character of the  $p$ -modes. In view of this limitation of the scattering approach, we should also direct efforts toward observing and analyzing the oscillations near a sunspot and within the sunspot itself.

## ACKNOWLEDGEMENTS

I am grateful to Tom Bogdan, Doug Braun, Tim Brown, and Bruce Lites for many helpful discussions of the results and ideas expressed here. I thank the AAS and the IAU for travel grants that allowed me to attend this colloquium. My work on sunspot seismology has been supported by NASA grants NAGW-2123 and NAGW-2444.

## REFERENCES

- Abdelatif, T. E., Lites, B. W., and Thomas, J. H. 1986, *Ap. J.*, **311**, 1015.
- Abdelatif, T. E., and Thomas, J. H. 1987, *Ap. J.*, **320**, 884.
- Bogdan, T. J. 1992, in *Sunspots: Theory and Observations*, ed. J. H. Thomas and N. O. Weiss (Dordrecht: Kluwer), p. 345.
- Bogdan, T. J., Brown, T. M., Lites, B. W., and Thomas, J. H. 1993, *Ap. J.*, in press
- Bogdan, T. J., and Zweibel, E. G. 1987, *Ap. J.*, **312**, 444.
- Braun, D. C., Duvall, T. L., Jr., and LaBonte, B. J. 1987, *Ap. J.*, **319**, L27.

- Braun, D. C., Duvall, T. L., Jr., and LaBonte, B. J. 1988, *Ap. J.*, **335**, 1015.
- Braun, D. C., Duvall, T. L., Jr., LaBonte, B. J., Jefferies, S. M., Harvey, J. W., and Pomerantz, M. A. 1992, *Ap. J.*, **391**, L113.
- Deinzer, W. 1965, *Ap. J.*, **141**, 548.
- Korzennik, S. G. 1990, Ph.D. dissertation, University of California, Los Angeles.
- November, L. J. 1984, in *Small-Scale Dynamical Processes in Quiet Stellar Atmospheres*, ed. S. L. Keil (Sunspot, NM: National Solar Observatory), p. 74.
- November, L. J. 1991, in *Solar Polarimetry*, ed. L. J. November (Sunspot, NM: National Solar Observatory), p. 149.
- Spruit, H. C. 1991, in *Challenges to Theories of the Structure of Moderate Mass Stars*, Lecture Notes in Physics No. 388, ed. J. Toomre and D. O. Gough (Berlin: Springer-Verlag), p. 121.
- Spruit, H. C., and Bogdan, T. J. 1992, *Ap. J.*, **391**, L109..
- Thomas, J. H., Cram, L. E., and Nye, A. H. 1982, *Nature*, **297**, 485.
- Thomas, J. H., and Weiss, N. O. (eds.) 1992, *Sunspots: Theory and Observations* (Dordrecht: Kluwer).