EVOLUTIONARY TRACKS FOR CENTRAL STARS OF PLANETARY NEBULAE

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1. INTRODUCTION

Our understanding of the evolution of Central Stars of Planetary Nebulae (CPN) has made considerable progress during the last years. This was possible since consistent computations through the asymptotic giant branch (AGB), with thermal pulses and (in some cases) mass loss taken into account, became available (Schönberner, 1979, 1983; Kovetz and Harpaz, 1981; Harpaz and Kovetz, 1981; Iben, 1982, 1984; Wood and Faulkner, 1986). It turned out that the evolution depends very sensitively on the inital conditions on the AGB. More precisely, the evolution of an AGB remnant is a function of the phase of the thermal-pulse cycle during which this remnant was created on the tip of the AGB by the planetary-nebula (PN) formation process (Iben, 1984, 1987). This was first shown by Schönberner (1979), and then fully explored by Iben (1984). In short, two major modes of PAGB evolution to the white dwarf stage are possible, according to the two main phases of a thermally pulsing AGB star: the hydrogen-burning or helium-burning mode. If, for instance, the PN formation, i.e. the removal of the stellar envelope by mass loss, happens during a luminosity peak that follows a thermal pulse of the helium-burning shell, the remnant leaves the AGB while still burning helium as the main energy supplier (Härm and Schwarzschild, 1975). On the other hand, PN formation may also occur during the quiescent hydrogen-burning phase on the AGB, and the remnant continues then to burn mainly hydrogen on its way to becoming a white dwarf.

In order to classify the different internal structures of an AGB star over a thermal-pulse cycle, we define that phase zero be at the surface-luminosity peak occurring shortly after the helium shell flash. The following classification is then possible:

i) Phase 0.....0.15, the star is burning helium, hydrogen is shut off ("helium burner");
ii) Phase 0.3 .... 1.0, the star is burning hydrogen, heliums burns only on a low level, $L_{He}/L_H \cong 0.01$ ("hydrogen burner");
iii) Phase 0.15... 0.3, helium and hydrogen are burning at comparative levels.

The timing of the PN formation, which is of crucial importance for

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our understanding of the late phases of stellar evolution, is a priori not known owing to our poor knowledge of the mass-loss processes on the AGB. The high sensitivity of PAGB evolutionary tracks to the initial phase ϕ_i at the tip of the AGB allows, however, a distinction to be made between the helium-burning (ϕ_i = 0) and hydrogen-burning ($\phi_i \gtrsim$ 0.3) mode of evolution by observations. Using hydrogen-burning models (with $\phi_i >$ 0.5), Schönberner (1981), Schönberner and Weidemann (1981) and Schönberner (1984) demonstrated that the temporal evolution of central stars can be very well explained by models with masses between 0.55 and 0.64 $M_\odot$. The conclusion then follows that obviously the PN ejection is generally <u>not</u> initiated by a thermal pulse, but occurs during the quiescent hydrogen-burning phase on the AGB. Also, the observed shape of the luminosity function of CPN could only be explained by hydrogen-burning PAGB models (e.g. Fig. 9 of Schönberner, 1981, and discussion in Schönberner and Weidemann, 1983). One special feature of this luminosity function is a deficit ("gap") of CPN with $M_V \approx 5$. This "gap" can be explained by hydrogen-burning PAGB models of ≈ 0.6 $M_\odot$ because they experience a rapid luminosity drop of $\approx$ 1 dex within only about 10^3 years when hydrogen burning starts to cease. Conversion into a luminosity function leads to a pronounced dip between $M_V \approx 4.5$ and 6.0, the exact position depending somewhat on the mass of the models. Such a luminosity drop is not found in models that leave the AGB while burning helium (cf. Fig. 1 in Iben, 1984), and this fact clearly indicates that at least the majority of CPN must be hydrogen burners. Additional observational support for a fast luminosity drop during the CPN evolution comes from the variation of the nebular ionization during the later phases of evolution. Schönberner (1986) showed that a correlation exists between the luminosities of the CPN and the degree of nebular ionization, in that PN with a lower ionization also belong to intrinsically faint CPN, whereas highly ionized PN also have luminous central objects (see also Schmidt-Voigt and Köppen, 1987).

In this review, I will concentrate only on models that are evolving off the AGB in thermal equilibrium under the influence of hydrogen shell burning and mass loss. The possibility of a final helium shell flash, and its consequences, is extensively discussed in Iben (1984, 1987).

2. POST-AGB EVOLUTION

The structure of an AGB star is rather complicated. It has a hydrogen-exhausted core, M_H, which contains two burning shells, namely the hydrogen-burning shell at the core's surface and the helium-burning shell further inwards. The helium-exhausted inner part of the core consists of carbon and oxygen and is electron degenerated. The core is actually nothing else than a very hot white dwarf which is surrounded by a huge, nearly fully convective envelope, M_e, containing the unprocessed stellar matter. The stellar radius exceeds the core radius by factors up to about 10^4! In the course of evolution along the AGB, the hydrogen-exhausted core is growing in mass at the expense of the envelope due to nuclear burning in the hydrogen-burning shell, while its radius is shrinking. The core of an AGB star may contain up to more than 99% of the stellar mass! The evolutionary track of an AGB star in the H-R diagram is entirely due to the response of the envelope to the masswise growing core: expansion of the envelope along the AGB, and finally contraction to white-dwarf dimensions if the envelope mass becomes too small (PAGB evolution).

The evolution along the AGB is terminated if either M_e becomes very small by the combined effect of nuclear burning in the hydrogen-burning shell and mass loss from the surface, or M_H approaches 1.4 $M_\odot$. The second possibility leads to an SN explosion and will not be discussed here. The transition from an AGB star to a white dwarf can be split into two steps:

i) If M_e is only of the order of several percent of the stellar mass, the envelope starts to shrink, but is still able to release enough gravitational energy to maintain the burning temperatures at its base. Consequently, the luminosity stays about constant ("plateau" luminosity), and the star evolves horizontally across the HR diagram. The core evolution is still independent from that of the envelope.

ii) If $M_e/M_\odot$ becomes about 10^{-4}, the hydrogen-burning shell starts to cool, the effective temperature reaches its maximum value (turn-around point), the luminosity drops rapidly and the star enters the white-dwarf regime, living mainly from its gravitational energy (Iben and Tutukov, 1984; Koester and Schönberner, 1986).

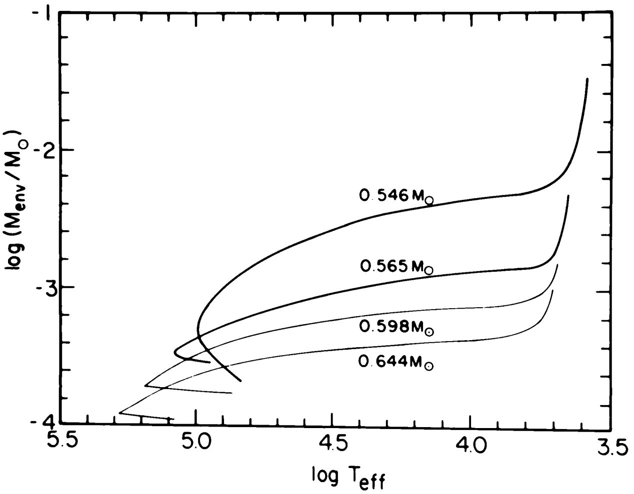

Fig. 1: Envelope mass M_e vs. T_{eff} for PAGB models of different core masses M_H according to Schönberner (1983).

Fig. 1 shows the variation of the envelope mass M_e with effective temperature for different PAGB models as given by Schönberner (1983). Note that in these evolutionary phases, M_H practically equals the total stellar mass M because of the smallness of M_e ($M = M_e + M_H$). For a given M_H, a unique relation $T_{eff}(M_e)$ exists for the horizontal evolution from the AGB till the turn-around point at $T_{eff} > 10^5$ K. The shapes of the relations $T_{eff}(M_e)$ are similar, but M_e increases with decreasing M_H. Similar relations for a larger range of M_H are given in Paczynski (1971).

The timescale for the crossing of the HR diagram with the "plateau" luminosity L is determined by the total amount of the available fuel ΔM_e and the fuel consumption $\dot{M}_e$:

$$\Delta t = \Delta M_e / \dot{M}_e .$$

Following Schönberner (1987), we define a horizontal "speed" as follows:

$$\dot{T}_{eff} = \dot{M}_e \ (dT_{eff}/dM_e),$$

where $\dot{M}_e$ consists of two terms, one of which being due to nuclear burning, $\dot{M}_H$, at the bottom of the envelope, the other describing mass loss from the surface by a stellar wind, $\dot{M}_W$:

$$\dot{M}_e = -(\dot{M}_H + \dot{M}_W).$$

With $\dot{M}_H = L/E_H X_e$, where E_H (= 6 10^{18} erg g^{-1}) is the energy release per gram of hydrogen, and X_e the hydrogen abundance (by mass) in the envelope, we have for a typical PAGB star of 0.6 $M_\odot$ with L = 6000 $L_\odot$: $\dot{M}_H \approx 10^{-7}$ $M_\odot yr^{-1}$. This value may be compared with typical mass-loss rates as they are found in the CPN regime which are, in most cases, well below 10^{-7} $M_\odot yr^{-1}$ (Cerruti-Sola and Perinotto, 1985). Thus, it appears that only the nuclear term controls the horizontal speed of hydrogen-burning PAGB stars throught the CPN region.

The situation is different at the cool side of the H-R diagram. Without mass loss, all models evolve extremely slowly in the vicinity of the AGB, as can be understood from the shape of the $T_{eff}(M_e)$ relation. Observed mass-loss rates at the tip of the AGB seem to reach values of $\dot{M}_W \approx 10^{-4}$ $M_\odot yr$ (e.g. Knapp, 1987), about 3 orders of magnitude larger than the nuclear term $\dot{M}_H$. Even a Reimers-like wind (Reimers, 1975) with its $\dot{M}_W \approx 10^{-6}$ $M_\odot yr^{-1}$ exceeds $\dot{M}_H$ by a large amount. Thus, it is the mass loss which terminates the AGB evolution and also controls the evolutionary speed in the vicinity of the AGB. The different transition times from the AGB to the CPN region for a hydrogen-burning remnant of 0.6 $M_\odot$ are collected in Table 1 for 3 different cases. Case 1 means no mass loss at all, $\dot{M}_W = 0$. Case 2 means that mass loss is included according to the Reimers formula ($\eta = 1$), which is assumed to hold, for convenience, also for hotter stars (Schönberner, 1979, 1983). Finally, Case 3 is the model adopted by Schönberner (1983): $\dot{M}_W$, as in Case 2, with the exception that $\dot{M}_W = 10^{-4}$ $M_\odot yr^{-1}$ for $T_{eff} \leq 10^{3.7}$ K ("superwind", Renzini, 1981).

Table 1 demonstrates clearly the sensitivity of the transition time from the tip of the AGB to 30000 K to the assumed mass-loss model. Especially the details of the "superwind" are important for this transition time, since if the "superwind" stops too early (i.e. at a lower

Table 1: Transition times $\Delta t = \Delta M_e/(\dot{M}_H + \dot{M}_W)$ in different parts of the H-R diagram for a PAGB model with $M_H = 0.6\ M_\odot$ and $\dot{M}_H = 9\ 10^{-8}\ M_\odot yr^{-1}$ from Schönberner (1979).

$\Delta\log T_{eff}$	$M_e/M_\odot$	t/yr		
		Case 1	Case 2	Case 3
3.55 .. 3.7	$5\ 10^{-2}$	$5.5\ 10^5$	$6\ 10^4$	$5\ 10^2$
3.7 ... 4.5	$6\ 10^{-4}$	$7\ 10^3$	$3\ 10^3$	$3\ 10^3$
4.5 ... 5.0	$3\ 10^{-4}$	$3.5\ 10^3$	$3.5\ 10^3$	$3.5\ 10^3$

T_{eff} than assumed in Case 3), the remnant will spend too much time in the vicinity of the AGB ("lazy" CPN, cf. Renzini, 1981). With reasonable assumption about $\dot{M}_W$ (cf. Case 3), it is possible to get short

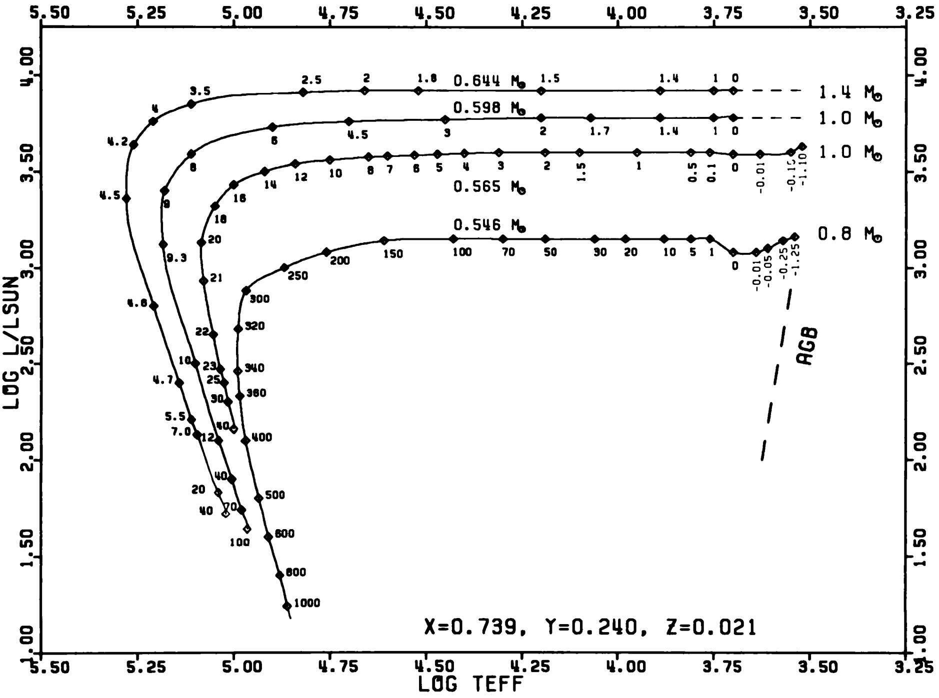

Fig. 2: Evolutionary tracks of four hydrogen-burning post-AGB models (Schönberner, 1981, 1983). The numbers give the ages in 1000 yr; age zero is assumed at $T_{eff} = 10^{3.7}$ K.

transition times which are consistent with the observations. Fortunately, the mass-loss term appears to be unimportant at hotter temperatures (see above), and this fact facilitates the modelling of CPN evolution by hydrogen-burning PAGB remnants without the details of previous mass-loss phases being known. A fuller discussion of mass loss and PAGB evolution can be found in Schönberner (1987).

Fig. 2 shows the evolutionary tracks of four hydrogen-burning post-AGB models according to the computations of Schönberner (1979, 1983), generated from Pop I stars with inital masses varying from 0.8 to 1.4 $M_\odot$. The two lower-mass remnants were generated according to Case 3 of Table 1, the remaining two according to Case 2. Except for the lowest remnant mass (0.546 $M_\odot$), the transition times are rather short and not in contradiction with the observations. This is due to the inclusion of mass loss which considerably accelerates the evolution below 10000 K. The horizontal evolution through the CPN region is highly mass-sensitive: $\Delta t \sim M_H^{-10}$. The reason is the larger luminosity (core-mass luminosity relation) and the smaller available amount of fuel (see Fig. 1) if M_H is increased. A similar mass dependence holds for the luminosity drop when hydrogen burning extinguishes.

This rapid drop of the stellar luminosity is possible because the hydrogen-burning shell is so thin (in mass, $\approx 10^{-4}$ $M_\odot$) and the helium-burning shell so weak ($L_{He} \approx 0.01$ L_H). This luminosity drop is expected to be larger and faster the smaller the hydrogen-shell mass and the lower the helium-shell luminosity is. The former decreases with increasing core mass M_H (i.e. with increasing luminosity), and the minimum of L_{He} during a thermal-pulse cycle decreases with increasing pulse number (Gingold, 1974, Fig. 2). Thus, we expect that only post-AGB stars which went on the AGB through full-amplitude helium shell flashes experience fast luminosity drops when hydrogen burning ceases.

Indeed, the 0.546 $M_\odot$ model is still below the threshold for the occurrence of thermal pulses, and helium burning still contributes about 30% to the stellar luminosity. The 0.565 $M_\odot$ model experienced 4, the 0.598 $M_\odot$ model 10 and the 0.644 $M_\odot$ model 24 thermal pulses. The initial thermal-pulse cycle phase for the post-AGB evolution of the latter three models is about 0.7. In passing, we note that helium-burning models do not show rapid luminosity drops (Iben, 1984), obviously because the mass contained in the helium-burning shell is about 100 times larger than that of the hydrogen-burning shell.

3. A "STANDARD" 0.6 $M_\odot$ PAGB MODEL

In this section I will try to extract the properties of a typical 0.6 $M_\odot$ hydrogen-burning PAGB model, as they follow from computations of different authors. The models are the following:

1: 0.598 $M_\odot$, Z = 0.02, Schönberner (1979);
2: 0.593 $M_\odot$, Z = 0.02, Kovetz and Harpaz (1981);
3: 0.599 $M_\odot$, Z = 0.001, Iben (1984);
4: 0.6 $M_\odot$, Z = 0.02, Iben and MacDonald (1986);
5: 0.6 $M_\odot$, Z = 0.02, Wood and Faulkner (1986).

Important features of these evolutionary models are compiled in

Table 2, as there is the "plateau" luminosity L, the envelope mass ΔM_e burnt between 30000 K and 100000 K, the envelope mass at the turn-around point, M_e(TA), the luminosity drop within 10^3 yr, $\Delta \log L/L_\odot$, starting at the turn-around point, the absolute magnitude, M_V, after that drop, the absolute magnitude, M'_V, after 50000 yr, and the number of thermal pulses, N, on the AGB.

Table 2 shows that all the pop I models (Nos. 1,2,4,5) have practically the same "plateau" luminosity (the table entries are not corrected for the slightly different model masses). Despite its larger envelope mass, the evolution of the pop II model of Iben (No. 3), beyond the turn-around point is essentially identical with that of model No. 1 and 2. Of course, the horizontal evolution of model 3 is about 2 times slower because it burns more matter. The larger envelope mass for a given effective temperature, and the slightly lower luminosity, are obviously due to the lower metallicity, as the pop I model (No. 4) of Iben and McDonald demonstrates (computed with the same evolutionary code). Only model No. 5 of Wood and Faulkner disagrees in all its properties (except for the luminosity) from the other (pop I) models. A possible explanation will be given at the end of this section.

Table 2: Important properties of 0.6 $M_\odot$ hydrogen-burning PAGB models

Mod.	$\log L/L_\odot$	$M_e/M_\odot$	$M_e(TA)/M_\odot$	$\log L/L_\odot$	M_V	M'_V	N
1	3.78	$3\ 10^{-4}$	$1.9\ 10^{-4}$	0.8	6.0	6.8	10
2	3.81	$3\ 10^{-4}$	$1.0\ 10^{-4}$	1.2	6.2	7.0	5
3	3.73	$7\ 10^{-4}$	$2.7\ 10^{-4}$	0.9	6.2	7.2	10
4	3.80	$2.5\ 10^{-4}$	$1.2\ 10^{-4}$	-*	-*	-*	-*
5	3.79	$6\ 10^{-4}$	$3\ 10^{-4}$	0.5	4.9	6.5	12

* No information given.

Neglecting for the moment the model of Wood and Faulkner (1986), the following properties emerge for a typical hydrogen-burning PAGB model of 0.6 $M_\odot$:

i) the "plateau" luminosity is 6200 $L_\odot$;
ii) the transition time from the AGB to 30000 K depends on the assumed mass-loss rates but may be as small as 3000 yr;
iii) the evolution from 30000 K till the turn-around point occurs in 6000 yr ($\dot{M}_W = 0$);
iv) the luminosity drops by $\approx$1 dex within 1000 yr when hydrogen burning stops;
v) the limiting CPN magnitude is predicted to be $M_V \approx 7$ (or $L \approx 80\ L_\odot$);

vi) it follows from the evolutionary rates that at least 75% of a complete sample of CPN should be fainter than $M_V \approx 6$ (or $L \approx 300\ L_\odot$).

I will close this review with a discussion on the discrepant behaviour of Wood and Faulkner's (1986) 0.6 $M_\odot$ hydrogen-burning PAGB model. It has already been shown above (cf. Table 2) that the computations of Schönberner (1979), Kovetz and Harpaz (1981) and also Iben (1984) - if opacity differences are taken into account - give practically the same results. The model of Wood and Faulkner (1986) differs considerably in that it burns more hydrogen, resulting in a reduced horizontal speed of evolution. Furthermore, the final luminosity drop is only one third as large (0.5 dex) and the limiting magnitude brighter by 0.5. Overall, the temporal evolution of Wood and Faulkner's (1986) hydrogen-burning 0.6 $M_\odot$ PAGB model mimics the corresponding 0.565 $M_\odot$ model of Schönberner (1983). Before going further into detail, it should be noted that theory predicts in fact a variation of the evolutionary speed with Φ_i in the sense that the speed increases slightly with Φ_i (Wood and Faulkner, 1986). This effect, however, cannot explain the discrepancies discussed here.

One might speculate that a possible explanation for these differences comes from the model histories on the AGB. In the Schönberner, Kovetz and Harpaz, and Iben calculations, mass loss was either included according to Reimers' formula (1975) or simply neglected. Wood and Faulkner, however, applied a rate as high as $\approx 1\ M_\odot \mathrm{yr}^{-1}$ till the star was stripped down to $M_e = 0.015\ M_\odot$. Then a much lower rate was used ($3.10^{-5}\ M_\odot \mathrm{yr}^{-1}$). A rate of $\approx 1\ M_\odot \mathrm{yr}^{-1}$ certainly destroys the thermal equilibrium in the deeper layers. For instance, Schönberner (1983) found that already rates of $\dot{M}_W \approx 10^{-4}\ M_\odot \mathrm{yr}^{-1}$ lead to small de-adjustments of the nuclear-burning regions (cf. also Fig. 2 above). Much larger effects are expected for even higher mass-loss rates. A not thermally adjusted PAGB model has a larger hydrogen-burning shell mass and, as a consequence, also a larger envelope mass for a given effective temperature. Also such a model should have a larger gravitational energy release. Both effects result in reduced evolutionary speed and luminosity drop. It would be desirable to make a direct comparison between the internal structures of the Wood and Faulkner models and those of the other authors.

For the time being, the following conclusions can be drawn: since the properties of PAGB models are extremely sensitive to the previous treatment on the AGB, realistic models for central stars are only expected if

i) the applied mass-loss rate does not largely exceed $\approx 10^{-4}\ M_\odot \mathrm{yr}^{-1}$,
ii) all thermal pulses are taken properly into account,
iii) the initial masses are roughly consistent with an empirical initial-final mass relation.

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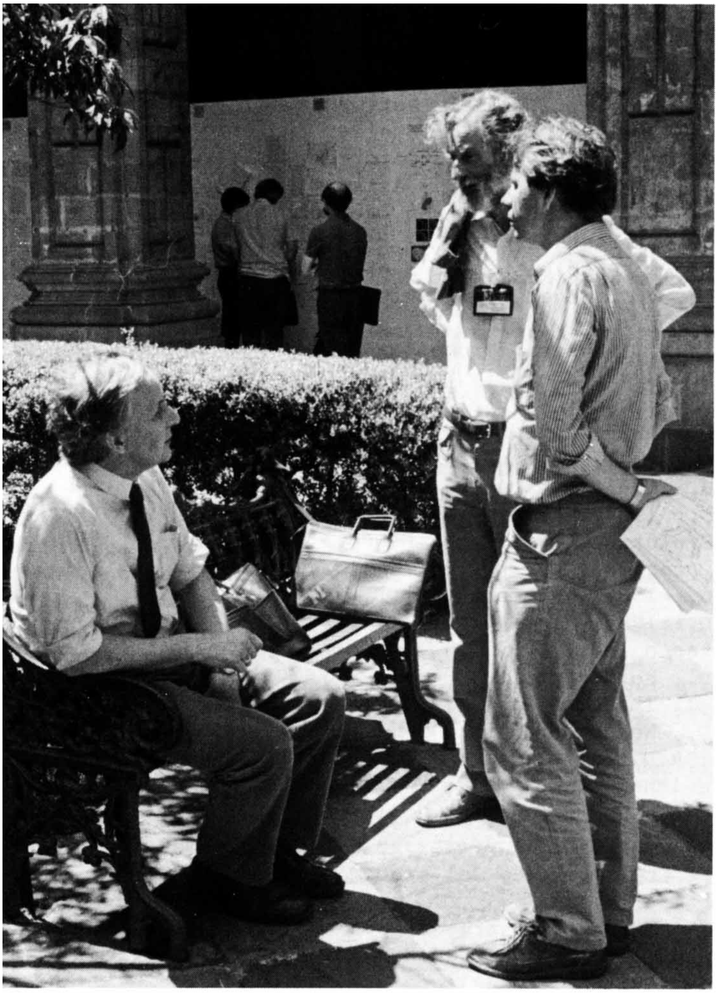

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